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WASHINGTON-PARIS LONGITUDE BY RADIO SIGNALS,

By F. B. LITTELL AND G. A. HILL.

[Communicated by Captain J. A. HOOGEWERFF, U. S. Navy, Superintendent U. S. Naval Observatory.]

A proposal from Captain J. L. JAYNE, U.S.N., Superintendent of the Naval Observatory, to determine the difference of longitude between the Naval Observatory at Washington and the Observatoire de Paris at Paris, France, by the use of radio signals from the naval station at Radio, Virginia, and from the Eiffel Tower station in Paris, having been favorably received by the French authorities, preparations were begun by the direction of the Navy Department early in the spring of 1913. The methods adopted were those developed by the French and described in *Détermination par la Télégraphique sans Fil de la Différence de Longitude entre Paris et Bizerte* and other publications.

The longest distance over which the method had previously been applied was in the above mentioned determination between Paris and Bizerte, which distance is about 960 miles, and in order to make sure that the method could be extended to meet the requirements of the present case, the French government sent over a preliminary expedition with four observers in March, 1913. This party was equipped with the astrolabe as modified and improved by Messrs. CLAUDE and DRIENCOURT for the determination of time, and although it was late in the season for successful radio transmission over so great a distance, the party succeeded in securing a sufficient number of radio clock comparisons on two nights to convince them that the radio method would be entirely practicable at a more favorable season of the year, and to give a good approximate longitude between the stations occupied. For a report of their work which should be regarded as a reconnaissance preliminary to the more extensive work of the following season, see *Comptes Rendus*, 21 Juillet 1913.

Warm weather being unfavorable for transmitting radio signals to great distances, it was planned to begin operations as early as possible in the Fall. For the American observers, two new transits were ordered from

M. PRIN, of Paris. They were to be practically duplicates of those in use by the French observers, of three inches aperture, thirty-three inches focal length, with self-registering right ascension micrometers driven by electric motor and controlled by hand, reversible on each star, with hardened steel pivots, and electric lighting. A meridian mark and lens of approximately one hundred and fifty feet focal length was provided with each instrument. One instrument was delivered in Washington and one in Paris in time for some preliminary practice before work was begun near the end of October. The director of the Observatoire de Paris, M. BAILLAUD, also very kindly sent over one of his instruments early in the summer for the American observers to use for practice. As it was planned that the work should be done in duplicate by French and American observers working simultaneously, it was necessary to erect two small buildings in the Observatory grounds for the shelter of the instruments. These were built of sheet iron louver work, and were provided with removable roofs. The piers were made of concrete.

Captain H. G. BULLARD, U.S.N., Superintendent Naval Radio Service, and his staff, coöperated heartily in making the necessary arrangements for the emission and reception of signals at Radio, and in securing the minimum of interference from other radio stations.

At Paris the American longitude parties were under the direction of Commander H. H. HOUGH, U.S.N., to whose energy and address the success of the preliminary arrangements is largely due. The director and staff of the Observatoire de Paris and the commandant and staff of the Tour Eiffel radio station made every effort to perfect the arrangements and their coöperation is acknowledged with profound gratitude. The American observers connected with the work were, for the first period at Washington, Prof. F. B. LITTELL, and Lieut. W. T. MALLISON, U.S.N., astronomical observers, and Ensigns H. E.

SAUNDERS, and R. A. LAVENDER, U.S.N., radio observers; and at Paris, Assistant Astronomer G. A. HILL and Lieut. C. W. MAGRUDER, U.S.N., astronomical observers, and Lieut. R. B. COFFMAN, and Ensign G. S. GILLESPIE, U.S.N., radio observers. For the second period, all the observers and astronomical instruments were interchanged.

The astronomical program was to observe from 7.30 p.m. to 1.30 a.m. local mean time at both Paris and Washington. As each star was observed with instrument direct and reversed, the collimation error was eliminated except for a small correction depending on the length of the signals which was applied, the level was determined by the striding level between every two star observations, and the azimuth by means of readings on the meridian marks which were also made between every two star observations. With this program about five clock stars per hour could be observed. When possible, three or four azimuth stars were observed each night. The observing list for each station was made up so that in any period of three hours the stars were fairly well balanced as to the zenith. Nearly all the stars north of the Washington zenith and south of the Paris zenith were common to both observing lists. The two astronomical observers of each party alternated in observing the first and last half of the evening.

At Washington the RIEFLER standard sidereal clock, which is situated in a constant temperature vault, and is in an airtight case under constant pressure, sends its signals on the even seconds (omitting for identification the zero second of each minute) to a small break circuit relay which operates a large 7-point break circuit relay which distributes the signals to the various instruments of the Observatory. Only one of the seven points being available, it was used to operate a 4-point relay, one of whose points was used to send the clock signals to Radio, one to the French, and one to the American chronograph. As the French chronograph could operate only on a make circuit, and as the American could operate on either, the circuits were arranged as make circuits. By the above arrangement the 4-point relay became practically the observing clock, and it was only necessary to determine the lag of the point used for the chronograph relative to that used to send the signals to Radio. This was done by recording on the chronograph sidereal time signals alternately through the different points, while at the same time mean time signals were being recorded through an independent circuit. When the French observers arrived, Prof. H. ABRAHAM, of the Université de Paris, brought with him two galvanometer pen chronographs designed by himself and especially adapted for the recording of such lags, and he very kindly loaned one to the Naval Observatory. It was more convenient in use than the

other arrangement and gave practically the same results. There was a small systematic difference of 0.004 between the results obtained by the two methods which was attributed by Prof. ABRAHAM to the superior sensitivity of his chronograph. For a short period at the beginning the relative lag between the Radio and American chronograph points was 0.010. The point then in use having been shown by records on the much more sensitive photographic galvanometer chronograph of Prof. ABRAHAM to be defective, the chronograph circuit was changed to the other available point of the relay and after adjustment the lag was 0.000 and remained so throughout the work. In general two records of the lag were made each night. The time from the closing of the contact at the Observatory to the reception of the signal in the telephone receiver at Radio was measured by the American radio observers and by Prof. ABRAHAM and was found to be negligible.

At Paris the RIEFLER clock which is situated in a chamber in the catacombs at a depth of about twenty-eight meters below the surface of the ground where the temperature change is practically zero, sends its signals on the even seconds (omitting for identification the 58th second of each minute) and synchronizes a "clock relay" which distributes the signals to the chronographs. The synchronizing circuit apparently operates when the pendulum of the clock relay is at its lowest point and the signal circuit is closed near the end of the swing of the pendulum so that the lag is either approximately +0.5 or +1.5 according to the direction in which the pendulum is swinging when it is synchronized, and as the clock relay omits no signals for identification and the signal nearest the zero of the RIEFLER is usually called zero the result is that the lag is either approximately +0.5 or -0.5. If the control is out of action for a time or if the relay is stopped and started again, it is readily seen that the lag may change from one value to the other. From October to December it had the former value and from January to March the latter. It was measured several times each night by an ABRAHAM galvanometer pen chronograph and also usually by the observers at the Eiffel Tower by coincidence and suitable electric connections having been provided for this purpose. The results of the two methods are in substantial agreement.

During the first period of the work, the weather conditions at Paris were very unfavorable for astronomical work and observations of stars were secured on comparatively few nights. The clock corrections are based upon the positions of Boss' Catalogue of 6188 stars. Corrections to these places have been deduced from the observations themselves but as their effect on the longitude is very small, they have not been applied.

Each star has been reduced with the mean of the level and azimuth constants resulting from the measures made before and after the observation. The mean lag correction for the night has been applied. The work of each night has been reduced to the mean observer by the application of relative personal equations determined from the observations, though this correction affects the results but little except in the infrequent case where one observer secured all the observations. The following are the personal equation corrections resulting from a preliminary investigation and as used, the signs being applicable to clock corrections:

	I	II
LITTELL	0 ^s .000	+0 ^s .012
MALLISON	0.000	-0.012
HILL	+0.003	-0.005
MAGRUDER	-0.003	+0.005

The probable error of a clock correction from a single star, including the errors of star places were as follows:

	Washington	Paris	All	No. Obs.
LITTELL	±0 ^s .032	±0 ^s .040	±0 ^s .034	812
MALLISON	.040	.041	.041	618
HILL	.041	.046	.043	336
MAGRUDER	.032	.044	.037	384

The above probable errors would indicate a probable error of ±0^s.008 at Washington and ±0^s.010 at Paris for a clock correction from an average night's work of about twenty stars. The intercomparison of results by the different observers on the same instruments and of the results of the French and American observers on different instruments indicates a probable error of clock correction of ±0^s.013 for Washington, and ±0^s.015 for Paris, which is considerably larger. The difference is due perhaps to variable personal equations or imperfect instrumental action.

As a large part of the discordances of the clock corrections seems to be due to the errors in the determination rather than to irregular running of the clocks, an attempt was made to get a better determination of the clock corrections on the dates on which observations were made, as well as for the dates on which there were radio observations but no star observations, by making a least square solution using equations of the form

$$C_x = C_0 + N \cdot R + N^2 \Delta R$$

where C_x is the clock correction on any date x , C_0 is the clock correction on the epoch date, N is the number of days between these dates, R is the daily rate, and ΔR is an assumed constant acceleration of the daily rate. However, the running of the clocks is not apparently in con-

formity with so simple a law, and the results were not satisfactory as the observations were only satisfied with a probable error of ±0^s.021 for a clock correction.

The clock corrections were then plotted and it was apparent that with the aid of group means, it would be possible to draw a rather smooth curve which would represent the observations of a series much better than the least square solution. The curves are too irregular to admit of general analytical treatment but they are fairly well defined and it is believed that they represent the clock corrections, on the average, better for a given series of nights, than do the results of the individual night's observations. This is indicated by a considerable diminution of the probable error for the longitude from the seventeen nights on which time observations were secured at both stations when the clock corrections are taken from the curve, as compared with that obtained by using the observed value for each night. The curves satisfy the observations for Washington with a probable error of ±0^s.012 and for Paris with a probable error of ±0^s.015. The use of these curves also furnishes a means of utilizing all the radio observations on whatever nights they were obtained. It seems probable that most of the causes which operate, to produce the small deviations from regularity noted in the running of good clocks, installed so as to be effectively protected from changes of temperature and atmospheric pressure, act gradually and spread their effects over considerable periods of time. Among such causes are doubtless the progressive changes taking place in the composition of the lubricating oil and the variable friction due to the disintegration of material in the bearings, the latter especially being quite irregular in its action yet probably gradual in its results as affecting the clock corrections.

In all the preliminary reductions daily clock rates derived from the least square solution have been used and it was assumed that there was no diurnal variation in the rate. However, in the second Paris series it is found that almost invariably the second observer of a night obtained a clock correction smaller than the first observer while the general clock rate would require the reverse to be true. The difference which averages about 0^s.04 for a three-hour interval, is too large to be accounted for by periodic errors in the star places, and the lag was measured four or five times each night and does not account for it. Nor does it seem plausible to suppose that there is a diurnal variation in the clock rate. However, in order to test what effect such a variation would have upon the longitude each night's observations were divided into two groups averaging about three hours apart and the mean of the rates thus determined was compared with the mean derived from the daily rates, the results being given below. Apparently there is little indication

of variation at Washington, while it is strongly indicated for Paris II. For all the series except Paris I, it was possible to eliminate the effect of personal equation, but for that series it was necessary to apply as a correction the personal equation determined from Washington II.

	Washington		Paris	
	I	II	I	II
Mean of evening rates (per hour)	$-\overset{s}{0}.0212$	$-\overset{s}{0}.0226$	$+\overset{s}{0}.0066$	$-\overset{s}{0}.0134$
Mean of daily rates (per hour)	$-\overset{s}{0}.0222$	$-\overset{s}{0}.0184$	$+\overset{s}{0}.0010$	$+\overset{s}{0}.0020$
Corrections to daily rates (per hour)	$+\overset{s}{0}.0010$	$-\overset{s}{0}.0042$	$+\overset{s}{0}.0056$	$-\overset{s}{0}.0154$

As the epoch of the Washington time observations was about $2\frac{1}{2}$ hours after that of the radio observations and that of the Paris time observations was about $2\frac{1}{2}$ hours before that of the radio observations, the effect of the above differences on the resulting longitudes may be approximated as follows:

I	II	Mean
$+\overset{s}{0}.017$	$-\overset{s}{0}.049$	$-\overset{s}{0}.016$

At each station the program for the radio work was essentially the same. At Radio, for example, a LEROY clock controlled the emission apparatus so as to send out three series of 420 signals omitting for identification the multiples of sixty, the intervals between the successive signals being approximately $0^s.99$, and each signal being $0^s.5$ long. The radio observer observed by ear by means of telephone receivers the coincidences of the outgoing signals with the beats of a mean time chronometer, and as the chronometer beat half seconds, he observed a coincidence about once in fifty seconds, or usually seven or eight in a series. He also noted the omitted signals in such a way that the serial numbers of the signals at the coincidences could be determined. With this data it is possible to determine the chronometer time of any desired beat in the series of 420 emitted by the LEROY clock. Of the three series emitted each night, one was intended for practice work at the other station, one for the French observer, and one for the American observer. However, it soon became apparent that the practice series was not needed for that purpose, and that two or even three observers could listen at the same time, so that when conditions were favorable all three series were observed by both the French and American observers.

The incoming signals from the over-sea station were observed with considerable difficulty, as a rule, especially in the first period, both on account of their feebleness and on account of the interference of static or magnetic storms and of messages from near-by commercial stations. The latter were especially troublesome at Paris.

Occasionally the over-sea signals came in with great strength.

The radio signals having been compared with the comparing mean time chronometer, this was in turn compared directly with the standard RIEFLER sidereal clock of the Observatory by the same coincidence methods, securing usually four coincidences for each comparison, and two comparisons each night, one just before and one just after the radio work. The intervals between these comparisons were usually from $1^h.0$ to $1^h.5$. During that interval, of course, reliance is based entirely on the rate of the chronometer.

From the comparisons of the signals from the emitting clock with those from the comparing chronometer, it is possible to obtain a chronometer time from each coincidence for any signal in the series selected as a reference signal, and by means of the comparisons of the comparing chronometer with the RIEFLER clock and the determined corrections to this clock, it is possible to obtain the local sidereal time of this reference signal. In order to eliminate the errors in the assumed rate of gain of the emitting clock on the comparing chronometer due to errors of observation, the signal whose number in the series corresponds to the mean of the numbers of the signals at which coincidences were observed should be taken as the reference signal. As this number is different for the two stations, the signal corresponding to the mean of the two numbers was used as the reference signal, and in this way these errors were rendered negligible though not completely eliminated. Having determined the local sidereal time of the reference signal at Washington and at Paris, the difference of these times is the difference of longitude.

The following are the probable errors of the deduced time of the reference signal depending on a single coincidence for the different radio observers, as derived from the results of the separate coincidences:

	RIEFLER Comparison	Emission Signals	Reception Signals
COFFMAN	$\pm 0^s.0032$	$\pm 0^s.0057$	$\pm 0^s.0111$
GILLESPIE	.0031	.0070	.0150
SAUNDERS	.0026	.0050	.0093
LAVENDER	.0038	.0061	.0109

It will be noticed that the principal source of error lies in the observation of the reception signals, as was to be expected on account of their extreme feebleness. In combining the work of the different observers, that of GILLESPIE was given one half weight.

The above probable errors give for the probable error due to the radio work of a longitude from one series of signals observed under average conditions with four RIEFLER coincidences, seven emission coincidences, and

four reception coincidences, $\pm 0^s.006$, but by intercomparing the results from different series on the same night the following much larger probable errors for a series were obtained, from which it is inferred that there is considerable systematic error in each series, perhaps due to the fact that the observers after having secured the first coincidences knew just when to expect the next ones and were unconsciously somewhat influenced by that knowledge. It would perhaps have been a better plan to have recorded the coincidences and emissions by the use of an auxiliary chronograph and to have avoided counting entirely.

COFFMAN	$\pm 0^s.023$
GILLESPIE	$\pm .029$
SAUNDERS	$\pm .018$
LAVENDER	$\pm .020$

From the nature of the comparisons, a long signal with a short chronometer tick, there is evidently the possibility of considerable personal equation. In fact, there seems to have been a range at each station of several seconds between the observers in the average times of getting coincidences. Assuming that these personal equations remained constant their effect is eliminated by the interchange of observers. Even if they did not remain constant, it is probable that the changes have been relatively small so that their effect has been very largely eliminated.

As MESSRS. COFFMAN and GILLESPIE frequently observed the same series it is possible to determine their relative personal equations directly. It is also possible to deduce the relative equations of MESSRS. SAUNDERS and LAVENDER by indirect methods. The following are the results, in general, COFFMAN and LAVENDER getting coincidences later than GILLESPIE and SAUNDERS. These relative personal equations have not been used in the reductions.

	C.—G.	L.—S.
I Emission	$+0^s.014$	$+0^s.017$
I Reception	$+ .021$	$+ .003$
II Emission	$+ .007$	$- .001$
II Reception	$+ .007$	$.000$

The nights on which radio observations were made at both stations are especially adapted for the determination of the transmission time, as the results from such nights are free from the errors in the adopted clock corrections, though not from those of the adopted clock rates, as may be seen from the following:

Let λ_0	= the true longitude.
λ_P	= the longitude from signals received at Paris.

λ_W	= the longitude from signals received at Washington.
D	= the distance from Paris to Washington.
V	= the velocity of transmission of the signals.
T_P and T_W	= the clock times of a signal from Washington as observed at Paris and Washington.
T_P' and T_W'	= the clock times of a signal from Paris as observed at Paris and Washington.
ΔT_{0P} and ΔT_{0W}	= the adopted clock corrections at Paris and Washington for any time T_0 .
δT_P and δT_W	= the adopted clock rates at Paris and Washington.
X_P and X_W	= the corrections to reduce clock corrections to true values.
Y_P and Y_W	= the corrections to reduce clock rates to true values.
CG_{EP} and CG_{RP}	= the average personal equations of COFFMAN and GILLESPIE for emission and reception signals at Paris.
SL_{EW} and SL_{RW}	= the average personal equations of SAUNDERS and LAVENDER for emission and reception signals at Washington.

$$\text{Then (1) } \lambda_P = T_P + \Delta T_{0P} + \delta T_P (T_P - T_0) - T_W - \Delta T_{0W} - \delta T_W (T_W - T_0)$$

$$(2) \quad \lambda_0 = \lambda_P - \frac{D}{V} + CG_{RP} + X_P + Y_P (T_P - T_0) - SL_{EW} - X_W - Y_W (T_W - T_0)$$

$$(3) \quad \lambda_W = T_P' + \Delta T_{0P} + \delta T_P (T_P' - T_0) - T_W' - \Delta T_{0W} - \delta T_W (T_W' - T_0)$$

$$(4) \quad \lambda_0 = \lambda_W + \frac{D}{V} + CG_{EP} + X_P + Y_P (T_P' - T_0) - SL_{RW} - X_W - Y_W (T_W' - T_0)$$

And from (2) and (4)

$$\lambda_P - \lambda_W = \frac{2D}{V} + CG_{EP} - CG_{RP} + Y_P (T_P' - T_P) + SL_{EW} - SL_{RW} - Y_W (T_W' - T_W)$$

and after the interchange of observers

$$\lambda_P - \lambda_W = \frac{2D}{V} + SL_{EP} - SL_{RP} + Y_P' (T_P''' - T_P'') + CG_{EW} - CG_{RW} - Y_W' (T_W''' - T_W'')$$

The effect of errors in the adopted clock rates is small because of the short interval between the mean times of

the observations of signals in opposite directions. These errors are also accidental in character. The effect of personal equation is not eliminated but it is probably true

that the personal equation of each observer was nearly the same for emission and reception signals, in which case the effect of these errors is small.

OBSERVED VALUES OF THE DOUBLE TRANSMISSION TIME.

1913	Double Transmission Time	Interval	Wt.	1914	Double Transmission Time	Interval	Wt.
Nov. 19	+0 ^s .200	0 ^{hr} .2	0.5	Jan. 22	-0 ^s .001	0 ^{hr} .2	0.8
20	+ 40	0 .5	1.3	23	+ 39	0 .4	1.5
21	+ 50	0 .6	0.8	24	+ 134	0 .2	0.3
24	+ 123	0 .4	0.5	26	+ 45	0 .2	2.2
25	+ 22	0 .2	1.3	28	+ 52	0 .0	1.7
26	+ 88	0 .2	1.1	29	+ 32	0 .2	1.5
27	- 18	0 .1	0.8	30	+ 66	0 .0	2.7
Dec. 3	- 4	0 .3	1.0	31	+ 69	0 .2	2.5
8	+ 36	0 .4	1.3	Feb. 3	+ 19	0 .2	1.2
10	- 8	0 .2	0.9	7	+ 52	0 .0	1.5
11	+ 36	0 .2	1.9	9	+ 52	0 .0	1.7
12	- 17	0 .2	1.5	10	+ 18	0 .2	2.2
15	+ 84	0 .2	1.9	16	+ 23	0 .0	1.1
16	+ 46	0 .0	3.2	17	+ 37	0 .2	1.2
18	+ 69	0 .1	2.0	18	+ 20	0 .3	1.2
19	+ 41	0 .1	1.7	20	+ 31	0 .1	0.4
20	+ 111	0 .3	2.0	25	+ 40	0 .3	1.0
23	+ 75	0 .2	1.8	26	+ 47	0 .4	1.7
29	- 82	0 .0	0.4				
30	- 44	0 .2	0.9				
31	+ 44	0 .0	2.7				

Double transmission time from I	0 ^s .0461	±0 ^s .0068	21 nights.
Double transmission time from II	.0431	±0 .0033	18 nights.
Double transmission time from All	.0437	±0 .0039	39 nights.

The distance from Paris to Washington being 3830 miles, the above transmission time corresponds to a velocity of 175,000 ±16,000 miles per second.

The transmission time from Paris to Bizerte (see *Différence de Longitude entre Paris et Bizerte*, p. 107), was determined to be 0^s.0071, and as the distance is 963 miles, this corresponds to a velocity of 136,000 miles per second.

The correction for time of transmission as determined above has been applied to all the longitude results. All of the available radio observations for each night, omitting series in which but a single coincidence was observed, have been combined to form a single longitude determination. There were nine nights in the first period and eight nights in the second period when astronomical observations were secured at both stations and when radio observations were made at one or both stations. There were also five additional combinations of nights in the first period, and five in the second period when independent longitude determinations can be obtained by carrying the clock correction at one or both stations for from one to three days by means of the clock rate. In view of the excellent installation and good performance of

the clocks at both observatories, this is considered a safe proceeding in the present case. The weight assigned for the longitude of a night is based on the number of series of radio observed, the number of coincidences observed in the series, the number of stars observed, and the number of days the clock corrections have been carried by the rates. The following table gives the data available and the results obtained on these nights.

From the data of the following table values of the observed longitudes have been deduced for the first and second periods of the work:

I	5 ^h 17 ^m	36 ^s .616	±0 ^s .0115
II	5 17	36 .811	±0. 0054

By applying the correction -0^s.056 to each in order to reduce to the adopted meridians of Washington and Paris respectively, and taking the mean the following longitude, Washington-Paris, is obtained.

$$5^h 17^m 36^s.658 \pm 0^s.0059 \quad (1)$$

The probable error assigned is based on the assumption that the difference between I and II is due chiefly to

LONGITUDES BASED ON OBSERVED CLOCK CORRECTIONS.

I						II									
1913		No. Stars		No. Sets Radio		Longitude	Wt.	1911		No. Stars		No. Sets Radio		Longitude	Wt.
		W	P	W	P					W	P	W	P		
Oct.	31	18		3	} 5 ^h 17 ^m 36 ^s .679	1.1	Jan.	21	9	16	3	} 5 ^h 17 ^m 36 ^s .858	1.1		
Nov.	1		27					24		30	1			1	.818
	4	32		2	} .619	0.9		26	12		3	2	} .856	1.7	
	5		25					31	24	21	5	2			.808
	7	11	29	1	} .602	0.6	Feb.	2	29	25	5		} .842	1.4	
	11		23					3	21	25	1	2			.781
	13	8		3	} .697	1.0		4	10	29	1		} .828	1.5	
	16		9					5		28		3			
	17	32		2	} .678	0.9		7	24		2	2	} .771	0.7	
	19	19	26	1			1	.606	1.0	9	23	4			3
	25	12			} .706	1.1		10		22	4	2	} .790	1.6	
	27			2			1	.706	1.1	11	27				1
	29		11		} .535	1.5		17	25	20	2	3	} .828	1.0	
Dec.	8	21	17	1			3	.535	1.5	21	27				2
	13	24	30	1	} .560	1.0		24		22			} .798	1.0	
	19	36	18	2			4	.574	1.6	24	16				
	20	34	35	2	5	.543	1.6	25		7	2	1	.767	1.6	
	23	20	9	2	4	.645	1.3	26	21	20	2	2			
	31	7	8	3	3	.662	0.9	27	25	26		1			

differences of personal equation between the astronomical and radio observers of the two parties and other similar errors and that the effects of those errors are eliminated in the mean. The extent to which this is the case will be shown further on.

If the longitude is based on the seventeen nights on which astronomical observations were made at both stations, the result is

$$5^h 17^m 36^s.642 \pm 0^s.0065 \quad (B).$$

If the longitude is based on clock corrections derived from the common stars observed the same nights at both stations, thus eliminating the errors of the star places, the number of nights is reduced to fifteen, and the result is

$$5^h 17^m 36^s.628 \pm 0^s.0061 \quad (C).$$

If the longitude is based on clock corrections derived from the three hour group nearest the time of the radio comparisons at each station thus reducing the effect of errors in the adopted clock rates, the number of nights is reduced to fourteen and the result is

$$5^h 17^m 36^s.629 \pm 0^s.0073 \quad (D).$$

If the longitude is based on the seventeen nights as in (B) but using clock corrections from the curve instead of the observed clock corrections for each night, the result is

$$5^h 17^m 36^s.652 \pm 0^s.0042 \quad (E).$$

By the use of the clock corrections derived from the curve, all the radio observations, made on 67 nights, can be utilized. The following table gives the data available and the results obtained by this method.

LONGITUDES BASED ON PLOTTED CLOCK CORRECTIONS.

I					II				
1913	No. Sets Radio		Longitude	Wt.	1911	No. Sets Radio		Longitude	Wt.
	W	P				W	P		
Oct. 31			$5^h17^m36^s.657$	1.2	Jan. 21		3	$5^h17^m36^s.831$	1.3
Nov. 4			.659	1.0	22	1	2	.782	1.4
7			.683	0.7	23	1	3	.804	1.6
13			.703	1.2	24	1	1	.880	0.8
15	2		.725	1.2	26	3	2	.842	1.6
17		2	.685	1.0	27		2	.852	1.2
19	1	1	.619	1.0	28	2	3	.820	1.6

LONGITUDES BASED ON PLOTTED CLOCK CORRECTIONS. (Continued.)

I					II				
1912	No. Sets Radio		Longitude	Wt.	1914	No. Sets Radio		Longitude	Wt.
	W	P				W	P		
Nov. 20	2	1	5 ^h 17 ^m 36 ^s .665	1.5	Jan. 29	2	2	5 ^h 17 ^m 36 ^s .809	1.5
21	1	2	.687	1.3	30	4	3	.816	1.7
22	1		[.823]*	[0.7]	31	5	2	.812	1.7
24	1	1	.639	1.0	Feb. 2	5		.839	1.6
25	2	1	.590	1.5	3	1	2	.817	1.4
26	2	2	.611	1.5	4		1	.815	1.0
27	2	1	.655	1.4	5		3	.807	1.4
28		3	.576	1.3	7	2	2	.831	1.5
29		2	.631	0.9	9	3	2	.829	1.6
Dec. 3	1	1	.617	1.3	10	4	2	.813	1.6
4		1	.609	0.3	11	1		.833	0.7
5	2		.647	1.3	14	4		.815	1.4
6		2	.572	0.7	16	2	2	.805	1.5
8	1	2	.561	1.5	17	2	3	.820	1.6
9	1		.562	1.0	18	2	2	.805	1.4
10	1	2	.589	1.3	19		3	.818	1.3
11	3	2	.578	1.6	20	1	2	.784	1.4
12	2	2	.626	1.5	21		2	.777	1.3
13	1		.574	1.0	25	2	1	.765	1.3
15	3	2	.549	1.6	26	2	2	.795	1.6
16	3	6	.615	1.7	27		1	.791	1.0
17		1	.610	1.0	Mar. 5		2	.800	1.3
18	3	2	.557	1.6					
19	2	4	.604	1.6					
20	2	5	.565	1.6					
22	2		.576	1.2					
23	2	4	.601	1.6					
26	2		.600	1.0					
27	1		.544	1.0					
29	3	1	.619	1.4					
30	3	2	.609	1.6					
31	3	3	.633	1.7					

* Rejected: radio observer's note, "Very poor."

From the above data the following values of the observed longitude for the first and second periods of the work have been deduced:

$$I \quad 5^h 17^m 36^s.614 \pm 0.0049.$$

$$II \quad 5 \quad 17 \quad 36.813 \pm 0.0027.$$

By applying the correction $-0^s.056$ to each to reduce to the adopted meridians of Washington and Paris

respectively, and taking the mean, the following longitude, Washington-Paris, is obtained:

$$5^h 17^m 36^s.658 \pm 0.0029 \quad (F)$$

This value is considered the best of the six given above, which, though preliminary, will not differ materially from the definitive values to be published in an appendix to Volume IX. *Publications of the U. S. Naval*

	I	II
Correction due to diurnal variation in clock rates	+0.017	-0.049
Correction due to lag determination for Paris I	+ .010	
Correction due to variation of longitude	+ .001	- .009
Correction due to systematic differences in levels	+ .026	- .030
Correction due to difference of radio observers personal equations	+ .020	- .020
Correction due to difference of astronomical observers personal equations	- .028	+ .008
Total	+0.046	-0.100

Observatory, Second Series. With respect to the probable error assigned, it is to be repeated that it is based on the assumption that the difference between I and II is principally due to errors whose effect is eliminated in the mean. There seem to have been a number of causes which have contributed to produce this difference. The corrections due to these causes are in some cases somewhat uncertain. As far as they have been investigated they are tabulated in order to show that there has been a fairly complete elimination of such errors in the mean value of the longitude which has been adopted above.

The first correction has already been explained. If the results of the French observations should confirm the existence of the diurnal variations in the clock rates, the longitude results should be corrected accordingly.

The second correction is due to a slight change made in the installation for determining the lag at Paris at the beginning of period II, in order to make the installation there uniform with that at Washington. The amount of this correction is based on observations but is subject to considerable uncertainty.

The third correction has been estimated for period II, as the results of the observations for this correction have not yet been published for the year 1914.

The fourth correction is one indicated by the results of comparisons made after the instruments had been returned to Washington. Such systematic differences between levels have been found in the past at this observatory.

The fifth correction is based on statements made by the radio observers as to their relative times of obtaining coincidences during the work and during preliminary practice when all of them worked together.

The sixth correction is based on a determination of the relative personal equation between LITTELL and HILL made after the return to Washington. As a result of three nights' work, it was found that at that time LITTELL observed later than HILL by $0^s.025 \pm 0^s.007$. The corrections given are the differences between those resulting from this determination and those already applied to reduce to the mean of the two observers at each station.

The following values of the observed longitude are obtained by applying the above corrections:

$$\text{I } 5^h 17^m 36^s.660 \pm 0^s.0056.$$

$$\text{II } 5 \ 17 \ 36.713 \pm 0^s.0043.$$

By applying the correction $-0^s.056$ to each to reduce to the adopted meridians of Washington and Paris respectively, and taking the mean the following longitude, Washington-Paris, is obtained:

$$5^h 17^m 36^s.630 \pm 0^s.0037.$$

These probable errors are based on the residuals formed by comparing the corrected results of the individual nights with the general mean.

Previously determined values of the transatlantic longitudes are as follows:

From Eclipses and Occultations		Washington, Old Obs'y — Greenwich	Washington, New Obs'y — Paris
1. WALKER	—1843	5 ^h 8 ^m 11 ^s .14	5 ^h 17 ^m 35 ^s .70
2. PEIRCE	1851	11 .57	36 .13
3. PEIRCE	1839	11 .45 $\pm 0^s.3$	36 .01
4. PEIRCE	1856-1861	13 .13	37 .69
From Moon Culminations			
5. WALKER	1843-1845	10 ^s .01	34 ^s .57
6. LOOMIS	1838-1844	9 .3	33 .86
7. GILLISS	1838-1842	10 .04	34 .60
8. WALKER	1845	9 .60	34 .16
9. NEWCOMB	1846-1860	11 .6 $\pm 0^s.4$	36 .16
10. NEWCOMB	1862-1863	9 .8	34 .36
From Chronometers			
11.	—1849	12 ^s .46	37 ^s .02
12. BOND	1849	11 .14	35 .70
13. WALKER	1849	12 .00	36 .56
14. BOND	1849	12 .20 $\pm 0^s.20$	36 .76
15. BOND	1855	13 .43 $\pm 0^s.19$	37 .99

By Cable		Cambridge — Greenwich	Washington, New Obs'y — Paris
16. GOULD	1866	4 ^h 44 ^m 30 ^s .891 ±0 ^s .090	5 ^h 17 ^m 36 ^s .56
17. DEAN	1870	31.065 ±0.056	36.73
18. HILGARD	1872	31.016 ±0.041	
Cambridge — Paris			
18. HILGARD	1872	4 ^h 53 ^m 51 ^s .951 ±0 ^s .051	36 ^s .69
Montreal — Greenwich			
19. Greenwich Obs'y. — McGill University	1892	4 ^h 54 ^m 18 ^s .62 ±0 ^s .024	36 ^s .70

In reducing these longitudes to Washington-Paris, the following adjusted differences of longitude have been used.

20. Greenwich — Paris	0 ^h 9 ^m 20 ^s .932
21. Washington — Cambridge	0 23 41.107
22. Montreal — Cambridge	0 9 47.588
23. Washington, New Obs'y.—Old Obs'y.	0 0 3.631

The Washington-Paris longitude given in the *American Ephemeris and Nautical Almanac* for 1901 to 1915, 5^h 17^m 36^s.75, is correlated with the Paris-Greenwich longitude, 9^m 20^s.97, which has been superseded by the value 9^m 20^s.93, corresponding to a Washington-Paris

longitude of 5^h 17^m 36^s.71. This value depends on the Washington-Greenwich longitude resulting from the adjustment made by Schott, (1) which is

$$5^h 8^m 15^s.78 \pm 0^s.05$$

This longitude depends largely upon the result of the 1892 cable determination of the longitude Montreal-Greenwich. At the time of the adjustment only the preliminary value of this result was available. As the definitive value gave a correction of $-0^s.05$ to the preliminary value, it is evident that Schott's value of the longitude Washington-Greenwich is too large.

Assuming the Greenwich-Paris longitude (2) to be

$$0^h 9^m 20^s.932 \pm 0^s.014$$

the Washington-Greenwich longitude resulting from the present determination is

$$5^h 8^m 15^s.726 \pm 0^s.014.$$

1-15. *U. S. Coast Survey Report*, 1867, pp. 59, 60.

16, 17, 18. *U. S. Coast and Geodetic Survey Report*, 1897, pp. 247, 218, see also p. 241 and 1872, p. 234 and 1884, p. 429.

19. Royal Observatory Greenwich. *Determination of Longitude*, 1888-1902, p. 124.

20. *Astronomische Nachrichten*, No. 3993, p. 157.

21, 22, 23. *U. S. Coast and Geodetic Survey Report*, 1897, pp. 253, 260.

(1) *U. S. Coast and Geodetic Survey Report*, 1897, pp. 254, 256, 260.

(2) *Astronomische Nachrichten*, No. 3993, p. 157.

OBSERVATIONS OF THE SIXTH SATELLITE OF JUPITER,

MADE WITH THE 26-INCH EQUATORIAL OF THE U. S. NAVAL OBSERVATORY.

By J. B. EPPES.

[Communicated by Captain J. A. HOOGWERFF, U. S. Navy, Superintendent.]

Date Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	$\log p \frac{\Delta}{\delta}$	Red. to App. Pl.
1910								
Apr. 8 11 37 15	1	18.6	+0 27.67	- 0 21.1	12 31 39.90	- 1 51 19.5	8.372 0.757	+1.10 - 7.8
10 13 52 13	1	18.6	-0 18.27	+ 5 56.6	12 30 53.96	- 1 45 1.1	9.459 0.753	+1.10 - 7.7
13 12 24 53	2	20.7	+4 8.17	- 9 32.1	12 29 51.88	- 1 36 20.2	9.200 0.751	+1.11 - 7.6
1911								
Mar. 21 15 1 10	3	25.5	+2 55.46	- 7 17.6	11 19 19.00	-11 15 43.3	8.637 0.848	+0.91 - 9.9
31 11 18 30	1	18.6	+1 11.76	+ 1 6.2	11 17 21.93	-11 38 41.1	8.861 0.847	+1.06 -10.5
Apr. 24 12 12 40	5	30.6	-1 29.31	+ 6 18.0	11 35 8.36	-13 58 17.1	8.518 0.841	+1.47 -12.1
26 13 19 39	6	30.6	-0 52.63	-11 33.1	11 33 56.46	-13 53 58.9	9.101 0.840	+1.48 -12.1
June 1 10 10 35	7	29.6	-2 41.22	+ 0 59.5	11 14 18.76	-12 37 36.0	8.813 0.836	+1.66 -12.5

Mean Places of the Comparison-Stars for the Beginning of the Year.

*	α	δ	Authority	*	α	δ	Authority
1	^h 12 ^m 31 ^s 11.13	— 1° 50' 50.3"	A.G. Nicolajew 3431	5	^h 14 ^m 36 ^s 36.20	— 14° 4' 53.0"	A.G. Camb., U.S. II 5153
2	12 25 42.30	— 1 26 40.2	A.G. Nicolajew 3424	6	14 34 47.61	— 13 42 13.4	A.G. Camb., U.S. II 5141
3	14 46 52.60	— 14 38 15.8	A.G. Washington 5492	7	14 17 1.32	— 12 38 23.0	A.G. Camb., U.S. II 5047
4	14 46 9.14	— 14 39 36.8	A.G. Washington 5178				

OCCULTATIONS OF STARS BY THE MOON,

OBSERVED WITH THE 26-INCH AND 12-INCH EQUATORIALS OF THE U. S. NAVAL OBSERVATORY,

[Communicated by Captain J. A. HOOGEWERFF, U. S. Navy, Superintendent].

Date	Object	Phen.	Wash. Sid. T.	Wash. M.T.	See'g	Instrum't	Pow'r	Obs.	Remarks
1913									
June 12	<i>f Virginis</i>	DD	^h 13 ^m 28 ^s 7.1	^h 8 ^m 5 ^s 7.6	f	26-inch	388	B	
12	<i>f Virginis</i>	RB	14 38 46.1	9 15 35.0	f	26-inch	388	B	Observation late.
24	317 <i>B. Aquarii</i>	DB	19 16 39.9	13 5 32.3	g	26-inch	388	B	Trifle hazy.
24	317 <i>B. Aquarii</i>	RD	20 3 24.5	13 52 9.2	g	26-inch	388	B	
24	317 <i>B. Aquarii</i>	DB	19 16 36.3	13 5 28.7	p	12-inch	160	W	
24	317 <i>B. Aquarii</i>	RD	20 3 24.7	13 52 9.4	g	12-inch	115	W	
July 13	π <i>Scorpii</i>	DD	16 25 9.1	8 59 47.2	p	26-inch	183	H	Unsteady.
13	π <i>Scorpii</i>	RB	17 50 16.0	10 24 40.2	p	26-inch	183	H	Very unsteady. Late prob. 1 st .
13	π <i>Scorpii</i>	DD	16 25 9.2	8 59 47.3	p	12-inch	235	B	
13	π <i>Scorpii</i>	RB	17 50 17.8	10 24 42.0	f	12-inch	160	B	
21	<i>h Aquarii</i>	DB	20 1 49.9	12 4 25.3	f	26-inch	183	H	Brt. limb, very unsteady.
21	<i>h Aquarii</i>	RD	20 59 43.6	13 2 9.5	f	26-inch	183	H	
21	<i>h Aquarii</i>	DB	20 1 51.8	12 4 27.2	f	12-inch	235	B	
21	<i>h Aquarii</i>	RD	20 59 43.7	13 2 9.6	f	12-inch	235	B	
Aug. 8	17 <i>G. Librae</i>	DD	17 3 6.8	7 55 25.1	g	12-inch	115	W	
Sept. 24	4 <i>Canceri</i>	DB	4 36 5.0	16 21 42.0	g	26-inch	388	B	Good disappearance.
24	4 <i>Canceri</i>	RD	5 31 6.7	17 16 34.7	g	26-inch	388	B	Good reappearance.
24	4 <i>Canceri</i>	DB	4 36 4.6	16 21 41.6	g	12-inch	160	W	Good disappearance.
24	4 <i>Canceri</i>	RD	5 31 6.8	17 16 34.8	g	12-inch	115	W	Dark limb visible.
Oct. 20	49 <i>Aurigæ</i>	DB	1 52 55.5	11 56 45.6	p	26-inch	388	H	Unsteady.
20	49 <i>Aurigæ</i>	DB	1 52 55.3	11 56 45.5	f	12-inch	160	W	
Noy. 17	134 <i>B. Geminorum</i>	DB	2 4 56.3	10 18 39.0	f	26-inch	250	B	Observation doubtful.
17	134 <i>B. Geminorum</i>	RD	3 8 50.5	11 22 22.7	f	26-inch	388	B	Good reappearance.
17	134 <i>B. Geminorum</i>	DB	2 4 58.8	10 18 41.5	f	12-inch	160	W	
17	134 <i>B. Geminorum</i>	RD	3 8 50.7	11 22 22.9	f	12-inch	115	W	
20	34 <i>Leonis</i>	DB	5 58 17.3	13 59 34.0	f	26-inch	250	B	Eye and ear.
20	34 <i>Leonis</i>	RD	6 23 25.2	14 24 37.8	f	26-inch	183	B	Eye and ear. Good reapp.
20	34 <i>Leonis</i>	DB	5 58 9.2	13 59 26.0	p	12-inch	160	W	
20	34 <i>Leonis</i>	BD	6 23 25.3	14 24 37.9	f	12-inch	115	W	Dark limb visible.
Dec. 5	96 <i>Aquarii</i>	DD	0 26 24.9	7 29 37.3	f	26-inch	495	B	Good disappearance.
5	96 <i>Aquarii</i>	RB	1 46 29.3	8 49 28.6	p	26-inch	250	B	Unsteady. Prob. a trifle late.
5	96 <i>Aquarii</i>	DD	0 26 25.1	7 29 37.5	g	12-inch	115	W	Dark limb visible.
12	38 <i>B. Aurigæ</i>	DD	7 29 33.5	14 4 5.2	p	26-inch	388	B	Disappearance gradual.
12	38 <i>B. Aurigæ</i>	RB	8 42 15.3	15 16 35.1	p	26-inch	183	B	Late. Poor observation.
12	38 <i>B. Aurigæ</i>	DD	7 29 33.9	14 4 5.6	p	12-inch	160	W	Uncertain.
12	38 <i>B. Aurigæ</i>	RB	8 42 7.7	15 16 27.5	p	12-inch	160	W	
13	406 <i>B. Tauri</i>	DB	0 1 34.2	6 33 23.4	f	26-inch	250	B	Disappearance gradual.
13	406 <i>B. Tauri</i>	RD	1 0 11.7	7 31 51.3	f	26-inch	250	B	Good reappearance.
13	406 <i>B. Tauri</i>	DB	0 1 31.0	6 33 20.2	p	12-inch	160	W	Rather uncertain.
13	406 <i>B. Tauri</i>	RD	1 0 12.1	7 31 51.7	f	12-inch	160	W	Late.
15	35 <i>B. Canceri</i>	DB	11 56 3.4	18 18 3.7	p	26-inch	250	B	Disappearance gradual. Haze.
15	35 <i>B. Canceri</i>	DB	11 56 0.6	18 18 0.9	f	12-inch	160	W	Hazy.
21	<i>i Virginis</i>	DB	10 34 15.3	16 32 53.5	g	12-inch	160	W	
21	<i>i Virginis</i>	RD	11 44 25.3	17 42 52.0	g	12-inch	115	W	Dark limb visible.

Date	Object	Phen.	Wash. Sid. T.	Wash. M.T.	See'g	Instrum't	Pow'r	Obs.	Remarks
1911			^h ^m ^s	^h ^m ^s					
Jan. 6	μ Arietis...	DD	2 18 8.6	7 15 13.5	g	26-inch	495	B	Good disappearance.
6	μ Arietis...	RB	3 40 56.2	8 37 47.5		26-inch	250	B	
6	μ Arietis...	RB	3 41 0.9	8 37 52.2	g g	12-inch	160	W	Eye and ear.
7	16 Tauri...	DD	9 2 59.5	13 55 2.1	f	26-inch	183	B	Haze. Good disappearance.
7	16 Tauri...	DD	9 2 59.6	13 55 2.2	g	12-inch	115	W	Hazy.
7	η Tauri...	DD	9 11 2.1	14 3 3.4	f	26-inch	183	B	Haze. Good disappearance.
7	η Tauri...	DD	9 11 2.2	14 3 3.5	f	12-inch	115	W	Hazy.
7	20 Tauri...	DD	9 28 48.8	14 20 47.2	p	26-inch	183	B	Haze. Fair disappearance.
7	20 Tauri...	DD	9 28 49.0	14 20 47.4	f	12-inch	115	W	Hazy. Star very faint.
10	49 Aurigæ	DD	5 19 20.6	10 0 12.2	p	26-inch	250	B	Good disappearance.
10	49 Aurigæ	RB	6 10 31.3	10 51 14.5	p	26-inch	250	B	Thin clouds.
10	49 Aurigæ	DD	5 19 20.7	10 0 12.3	g	12-inch	160	W	
10	49 Aurigæ	RB	6 10 33.1	10 51 16.3	p	12-inch	160	W	Thin clouds.
12	γ Cancri...	RD	9 36 20.7	14 8 38.3	p	12-inch	160	W	Probably late.
13	8 Leonis...	DB	6 46 58.3	11 15 47.8	p	26-inch	250	B	Disappearance gradual.
13	8 Leonis...	DB	6 46 53.7	11 15 43.2	p	12-inch	160	W	
13	8 Leonis...	RD	8 0 32.2	12 29 9.6	p	12-inch	160	W	
15	83 Leonis...	DB	9 11 8.2	13 31 42.3	f	26-inch	250	B	Disappearance gradual.
15	83 Leonis...	RD	10 4 42.6	14 25 7.9	f	26-inch	250	B	Late.
15	83 Leonis...	DB	9 11 2.8	13 31 36.9	f	12-inch	160	W	
15	83 Leonis...	RD	10 4 41.3	14 25 6.6	f	12-inch	115	W	
15	τ Leonis...	DB	10 2 25.4	14 22 51.1	g	26-inch	250	B	Good disappearance.
15	τ Leonis...	RD	10 35 50.5	14 56 10.7	f	26-inch	250	B	Haze. A trifle late.
15	τ Leonis...	DB	10 2 24.9	14 22 50.6	f	12-inch	160	W	Probably late.
15	τ Leonis...	RD	10 35 50.4	14 56 10.6	f	12-inch	115	W	
17	49 Virginis...	DB	9 3 41.5	13 16 24.9	p	26-inch	250	B	Disappearance gradual.
17	49 Virginis...	RD	9 57 28.2	14 10 2.8	f	26-inch	183	B	Good reappearance.
17	49 Virginis...	DB	9 3 36.3	13 16 19.7	p	12-inch	160	W	Very uncertain.
17	49 Virginis...	RD	9 57 28.3	14 10 2.9	p	12-inch	115	W	Dark limb visible.
Feb. 7	47 Geminorum...	DD	8 11 48.6	11 2 6.3	f	26-inch	388	B	
7	47 Geminorum...	DD	8 11 48.8	11 2 6.5	f	12-inch	115	W	
8	35 B. Cancri...	DD	9 33 35.8	12 19 44.3	f	26-inch	250	B	Good disappearance.
8	35 B. Cancri...	RB	10 13 20.0	12 59 22.0	p	26-inch	250	B	Unsteady.
8	35 B. Cancri...	DD	9 33 35.8	12 19 44.3	p	12-inch	115	W	
8	35 B. Cancri...	RB	10 13 23.2	12 59 25.2	p	12-inch	160	W	
11	c Leonis...	DB	7 29 3.0	10 3 44.1	p	26-inch	250	B	Disappearance gradual.
11	c Leonis...	RD	8 23 55.0	10 58 27.1	f	26-inch	250	B	Good reappearance.
11	c Leonis...	DB	7 28 56.5	10 3 37.6	p	12-inch	160	W	
11	c Leonis...	RD	8 23 55.1	10 58 27.2	p	12-inch	160	W	
17	135 B. Scorpii...	DB	13 56 10.9	16 6 13.2	b	26-inch	250	B	Disap. gradual. Uncertain, clouds.
17	135 B. Scorpii...	RD	15 8 50.5	17 18 40.9	b	26-inch	183	B	Uncertain, clouds.
17	135 B. Scorpii...	DB	13 56 15.0	16 6 17.3	p	12-inch	160	W	Very uncertain.
Mar. 3	18 Tauri...	RB	6 13 20.8	7 29 36.1	p	26-inch	250	B	
3	18 Tauri...	DD	4 41 49.6	6 1 19.4	p	12-inch	115	W	
3	18 Tauri...	RB	6 13 24.7	7 29 40.0	p	12-inch	160	W	
8	γ Cancri...	DD	9 48 56.4	10 44 56.9	f	26-inch	388	B	Good disappearance.
8	γ Cancri...	RB	10 11 13.6	11 7 10.4	f	26-inch	495	B	
8	γ Cancri...	DD	9 48 56.9	10 44 57.4	f	12-inch	115	W	A little late.
8	γ Cancri...	RB	10 11 21.2	11 7 18.0	f	12-inch	160	W	Late.
9	8 Leonis...	DD	7 18 53.6	8 11 22.8	p	26-inch	495	B	Thin clouds.
9	8 Leonis...	RB	8 29 41.9	9 21 59.5	p	26-inch	495	B	Thin clouds.
14	231 G. Virginis	RD	11 29 7.3	12 1 15.9	p	26-inch	183	B	Thin clouds. Haze.
14	231 G. Virginis	RD	11 29 8.0	12 1 16.6	p	12-inch	115	W	Late.
14	236 G. Virginis	DB	11 15 49.8	11 48 0.6	p	26-inch	250	B	Disap. gradual. Uncertain, clouds.
14	236 G. Virginis	RD	12 10 18.6	12 42 20.5	p	26-inch	183	B	Haze.
14	236 G. Virginis	DB	11 15 42.8	11 17 53.6	p	12-inch	160	W	Very uncertain.
14	236 G. Virginis	RD	12 10 19.5	12 12 21.4	p	12-inch	115	W	
Apr. 7	c Leonis...	RB	10 30 0.3	9 27 56.8	f	26-inch	250	B	Thin clouds.
9	ψ Virginis...	DD	13 8 42.9	11 58 21.6	p	26-inch	495	B	Disap. gradual. Very unsteady.
9	ψ Virginis...	RB	14 20 52.9	13 10 19.8	p	26-inch	250	B	Unsteady.
9	ψ Virginis...	DD	13 8 43.6	11 58 22.3	p	12-inch	160	W	

Date	Object	Phen.	Wash. Sid. T.	Wash. M.T.	See'g	Instrum't	Pow'r	Obs.	Remarks
1914			^h ^m ^s	^h ^m ^s					
Apr. 9	ψ <i>Virginis</i>	RB	14 20 54.7	13 10 21.6	f	12-inch	160	W	
12	<i>b</i> <i>Scorpii</i>	DB	11 42 34.1	10 20 39.2	p	26-inch	250	B	Disap.grad. Uncertain. Unsteady.
12	<i>b</i> <i>Scorpii</i>	RD	12 38 6.6	11 16 2.6	p	26-inch	495	B	Probably late.
12	<i>b</i> <i>Scorpii</i>	DB	11 42 33.2	10 20 38.3	p	12-inch	160	W	Very uncertain.
12	<i>b</i> <i>Scorpii</i>	RD	12 38 6.8	11 16 2.8	p	12-inch	115	W	Probably a little late.
12	4 <i>Scorpii</i>	DB	13 58 21.9	12 36 4.8	p	26-inch	250	B	Disappearance gradual.
12	4 <i>Scorpii</i>	RD	15 10 55.9	13 48 26.9	f	26-inch	183	B	Good reappearance.
12	4 <i>Scorpii</i>	DB	13 58 18.8	12 36 1.7	f	12-inch	160	W	
12	4 <i>Scorpii</i>	RD	15 10 56.0	13 48 27.0	f	12-inch	115	W	
18	δ <i>Capricorni</i>	DB	17 11 10.9	15 24 46.7	p	26-inch	250	B	Disappearance gradual.
18	δ <i>Capricorni</i>	RD	18 0 20.1	16 13 47.8	f	26-inch	495	B	Haze.
18	δ <i>Capricorni</i>	DB	17 11 7.7	15 24 43.5	p	12-inch	115	W	
18	δ <i>Capricorni</i>	RD	18 0 20.1	16 13 47.8	p	12-inch	115	W	
May 1	ζ <i>B. Cancri</i>	DD	12 22 31.3	9 45 47.6	g	12-inch	115	W	Haze.
8	43 <i>H. Virginis</i>	DD	10 59 14.9	7 55 13.4	f	12-inch	160	B	Thin clouds.
8	236 <i>G. Virginis</i>	RB	13 26 29.7	10 22 4.1	p	12-inch	160	B	Late. Thin clouds.
10	τ <i>Scorpii</i>	DB	18 56 24.2	15 43 12.8	f	12-inch	160	W	
10	τ <i>Scorpii</i>	RD	19 51 55.0	16 38 34.5	p	12-inch	115	W	A little late.
16	42 <i>Aquarii</i>	DB	17 20 1.4	13 43 30.3	p	26-inch	250	B	Disap. gradual. Uncertain.
16	42 <i>Aquarii</i>	RD	18 22 57.4	14 46 16.0	f	26-inch	183	B	
17	81 <i>Aquarii</i>	DB	17 55 54.8	14 15 21.9	p	26-inch	250	B	Disap.gradual. Uncertain.
17	81 <i>Aquarii</i>	RD	19 5 23.1	15 24 38.8	f	26-inch	388	B	
17	81 <i>Aquarii</i>	DB	17 55 52.2	14 15 19.3	p	12-inch	160	W	Extremely uncertain.
17	81 <i>Aquarii</i>	RD	19 5 23.2	15 24 38.9	f	12-inch	115	W	Dark limb visible.
28	κ <i>Geminorum</i>	DD	12 26 54.4	8 4 0.4	g	26-inch	388	B	Good disappearance.
28	κ <i>Geminorum</i>	RB	13 24 48.3	9 1 44.8	f	26-inch	250	B	
June 6	4 <i>Scorpii</i>	DD	14 34 54.5	9 36 16.3	f	26-inch	250	B	Good disappearance.
6	4 <i>Scorpii</i>	RB	15 52 9.0	10 53 18.2	p	26-inch	250	B	Haze.
6	4 <i>Scorpii</i>	DD	14 34 54.7	9 36 16.5	f	12-inch	160	W	
6	4 <i>Scorpii</i>	RB	15 52 11.9	10 53 21.1	p	12-inch	160	W	

NOTES.

DD signifies star disappeared at dark limb of *Moon*; DB, disappeared at bright limb; RD, reappeared at dark limb; RB, reappeared at bright limb. Under Seeing, g = good; f = fair; p = poor; b = bad. Observers: H = A. HALL; B = H. E. BURTON; W = C. B. WATTS.

All observations were recorded on chronograph, except as noted under Remarks.

Beginning June 24, 1913 with the 12-inch, and Nov. 17, 1913 with the 26-inch, a form of occulting bar devised by C. B. WATTS has generally been used for observations at the bright limb and sometimes at the dark limb when the moon was nearly full. The bars are shaped so as to conceal all of that part of the moon in the field, except a small section at the limb, the object being to reduce the glare of light which makes it difficult to see a star near the *Moon's* bright limb. They are attached to the eyepiece of power 160 on the 12-inch, and to that of power 250 on the 26-inch, and wherever these powers are mentioned, it is to be understood that the bars were used.

TRANSIT OF MERCURY.

Observer	Aperture	Power	Third Contact	Fourth Contact
SCHLESINGER	13 inch	180	9 ^h 7 ^m 28 ^s	9 ^h 9 ^m 12 ^s
HUDSON	3½ inch	80	7 25	9 9

Eastern Standard Time.

The times of the fourth contact should give the position of the planet much more accurately than those of the third, since the latter are much affected by irradiation.

Allegheny Observatory.

SUNSPOT OBSERVATIONS.

MADE AT BERWYN, PENN., WITH A 4½-INCH REFRACTOR,
By A. W. QUIMBY.

1914	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.	1914	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.	1914	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.		
July	1	7	—	—	—	fair	Aug.	28	10	—	—	—	—	poor	Oct.	27	7	—	1	2	—	fair
	2	7	—	—	—	1		29	4	1	1	3	—	fair		28	7	—	1	2	—	fair
	3	4	1	1	4	1		30	7	—	1	2	—	fair		29	7	1	2	8	1	fair
	4	7	—	1	4	1		31	7	—	—	—	—	fair		30	7	—	1	2	1	fair
	5	4	—	1	2	—		poor	Sept. 1	7	—	—	—	1		fair	31	9	—	1	1	—
	6	7	—	1	3	—	poor	2	7	—	—	—	—	fair	Nov.	1	7	—	1	6	1	fair
	7	4	—	1	2	1	poor	3	7	—	—	—	—	fair		2	8	—	1	2	1	fair
	8	9	—	1	2	—	fair	4	7	—	—	—	—	fair		3	8	1	2	8	2	fair
	9	12	—	1	1	—	poor	5	7	—	—	—	—	fair		4	8	—	1	6	1	fair
	10	7	—	1	1	—	poor	6	7	—	—	—	—	fair		5	8	—	1	2	1	fair
	11	7	—	1	1	1	poor	7	7	—	—	—	—	fair		6	8	—	1	4	—	fair
	12	7	—	1	1	1	fair	8	5	1	1	1	1	fair		7	8	2	2	30	1	good
	13	7	—	—	—	1	poor	9	7	—	1	1	1	fair		8	8	—	2	13	—	fair
	15	11	—	—	—	—	poor	10	7	1	2	2	1	fair		9	8	—	1	8	—	fair
	16	7	—	—	—	—	fair	11	7	—	2	2	1	fair		10	8	—	1	8	—	fair
	17	7	1	1	7	—	fair	12	5	1	3	5	1	fair	11	8	—	1	5	—	fair	
	18	7	—	1	4	—	fair	13	7	—	3	5	1	fair	12	8	—	1	6	1	fair	
	19	7	—	—	—	—	fair	14	7	—	3	5	—	fair	13	8	—	1	1	1	fair	
	20	7	—	—	—	—	fair	15	7	—	3	9	—	fair	14	8	1	2	4	1	fair	
	21	7	—	—	—	—	fair	16	7	—	3	6	—	fair	16	8	—	—	—	—	poor	
	22	7	—	—	—	1	fair	17	7	—	3	5	—	fair	17	8	—	—	—	—	fair	
	23	7	—	—	—	—	fair	18	7	—	2	2	1	fair	18	8	1	1	2	1	fair	
	24	7	—	—	—	—	poor	19	7	—	2	2	2	fair	20	11	—	1	2	1	poor	
	25	4	—	—	—	—	poor	20	7	1	3	5	3	fair	21	8	1	2	3	—	fair	
	26	7	—	—	—	—	fair	21	7	—	1	1	2	fair	22	8	—	2	4	—	fair	
	27	7	—	—	—	—	fair	22	7	—	1	1	2	fair	23	8	—	2	3	—	fair	
	28	7	—	—	—	—	fair	23	7	—	—	—	1	fair	24	8	1	3	5	1	fair	
	29	7	1	1	1	—	fair	24	7	—	1	1	1	fair	25	8	—	2	3	2	fair	
	30	7	—	—	—	1	fair	25	12	—	1	1	—	poor	26	8	—	2	3	1	poor	
	31	7	—	—	—	1	fair	26	7	—	1	1	—	fair	27	8	—	1	1	—	fair	
	Aug.	1	7	—	—	—	1	fair	27	7	—	1	1	—	fair	28	8	1	1	4	2	fair
2		7	—	—	—	1	fair	28	7	—	1	1	—	fair	29	8	1	2	5	3	fair	
3		7	—	—	—	—	fair	29	7	—	1	1	—	fair	30	3	—	2	7	1	poor	
4		7	—	—	—	—	fair	30	7	—	1	1	—	fair	Dec.	1	12	1	3	15	2	fair
*5		7	—	—	—	—	fair	Oct. 1	7	—	1	1	—	fair		3	9	1	4	20	1	fair
*6		7	—	—	—	—	fair	2	7	—	—	—	—	good		4	8	—	4	15	1	poor
*7		7	—	—	—	—	fair	3	7	—	—	—	—	poor		10	9	—	3	15	2	fair
*8		7	—	—	—	—	fair	4	7	—	—	—	—	fair		11	10	—	2	10	1	fair
*9		7	—	—	—	—	fair	5	7	—	—	—	—	fair		12	9	—	2	10	1	fair
*10		7	—	—	—	—	fair	6	7	—	—	—	—	fair		13	8	—	1	10	2	fair
*11		7	—	—	—	—	fair	8	11	—	—	—	1	fair		14	8	—	1	6	2	fair
*12		7	—	—	—	—	fair	9	7	—	—	—	1	fair		15	8	—	1	8	2	fair
*13		7	1	1	1	1	fair	10	7	1	1	4	1	fair		16	8	—	1	3	1	fair
*14		7	—	1	1	1	fair	11	7	—	1	1	—	fair		17	8	—	1	2	2	fair
15		5	—	1	1	1	fair	12	7	—	—	—	1	fair		18	8	1	1	3	2	fair
16		7	1	2	12	—	fair	13	7	—	—	—	—	fair		19	8	—	1	1	—	v. poor
17		7	—	1	8	1	good	14	7	—	—	—	—	fair		20	7	—	1	1	—	fair
18		7	—	1	15	1	good	17	7	—	—	—	—	fair		21	2	—	1	1	—	poor
19		7	—	1	21	—	good	18	7	—	—	—	—	fair		22	8	—	1	2	—	fair
20		7	—	1	26	—	good	19	7	1	1	3	—	fair		23	8	—	1	2	—	fair
21		7	—	1	16	—	fair	20	7	—	1	3	—	fair		25	8	1	1	1	1	poor
22		7	—	1	15	—	fair	21	7	—	—	—	—	fair		26	8	—	1	1	2	poor
23		7	1	2	4	1	fair	22	7	1	1	2	—	fair		27	8	—	1	1	1	poor
24		7	—	2	1	—	fair	23	7	—	1	2	—	fair		28	8	—	1	1	—	fair
25		7	—	1	1	—	fair	24	7	—	1	2	—	fair		30	8	1	2	4	—	fair
26		12	—	—	—	—	fair	26	12	1	1	2	—	fair		31	9	—	2	5	—	poor
27		12	—	—	—	—	fair															

* With 1½" refractor

OBSERVATIONS OF COMETS,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

By F. P. LEAVENWORTH.

1914* Central S. T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	$\log p\Delta$	Red. to App. Pl.
Comet c 1914 (NEUJMIN).								
July 17 10 37 26	1	5 6	$-1^m 23.07^s$	$+0^m 17.8^s$	17 47 4.43	$-8^{\circ} 50' 17.6''$	8.804 0.875	$+3.46^s - 6.5''$
22 11 28 14	2	3	$+0^m 55.10^s$		17 42 49.90		9.114	$+3.44^s$
11 4 20	2	3		$+6^m 54.7^s$		$-7^{\circ} 50' 5.7''$	 0.866	 $-6.7''$
26 11 54 24	3	4	$+0^m 33.66^s$		17 39 45.51		9.333	$+3.40^s$
• 11 28 37	3	3		$-1^m 53.0^s$		$-7^{\circ} 4' 25.6''$	 0.856	 $-6.6''$
29 9 12 57	4	7 4	$-2^m 11.50^s$	$+2^m 39.6^s$	17 37 44.48	$-6^{\circ} 33' 1.6''$	8.179 0.861	$+3.36^s - 6.2''$
Aug. 25 8 21 26	5	8 6	$+0^m 57.05^s$	$-3^m 10.9^s$	17 27 11.23	$-2^{\circ} 40' 53.3''$	9.099 0.832	$+2.97^s - 4.6''$
Comet a 1914 (KRITZINGER).								
July 29 12 20 47	6	8 3	$-0^m 45.04^s$	$-0^m 52.3^s$	22 44 21.58	$+39^{\circ} 36' 36.3''$	n9.374 9.990	$+2.90^s + 7.0''$
31 13 59 47	7	4 6	$+1^m 45.08^s$	$-1^m 31.8^s$	22 44 50.12	$+38^{\circ} 56' 21.8''$	n7.808 9.763	$+2.96^s + 7.9''$
Aug. 25 10 40 8	8	6 5	$+0^m 1.65^s$	$-0^m 35.8^s$	22 41 11.78	$+28^{\circ} 7' 34.4''$	n9.281 0.430	$+3.40^s + 16.9''$

Mean Places of Comparison Stars.

*	α	δ	Authority	*	α	δ	Authority
1	17 48 24.04	$-8^{\circ} 50' 28.9''$	A.G. Wien-Ottaking 6022	6	22 45 3.72	$+39^{\circ} 37' 21.6''$	A.G. Lund 10898
2	17 41 51.36	$-7^{\circ} 56' 53.7''$	" " 5992	7	22 43 2.08	$+38^{\circ} 57' 45.7''$	A.G. Lund 10881
3	17 39 8.45	$-7^{\circ} 2' 26.0''$	" " 5979	8	22 41 6.73	$+28^{\circ} 7' 53.3''$	Astrographie Cat. Oxford
4	17 39 52.62	$-6^{\circ} 35' 35.0''$	" " 5986				$\frac{1}{3}[28^{\circ}70445 + 29^{\circ}66112]$.
5	17 26 11.19	$-2^{\circ} 37' 37.8''$	A.G. Straszburg 5892				

* Six hours slow on Greenwich Mean Time.

Yerkes Observatory, 1914, October 10.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 26-INCH EQUATORIAL OF THE U.S. NAVAL OBSERVATORY.

[Communicated by Captain J. A. HOOGEWERFF, U.S. Navy, Superintendent.]

Date	Wash. M. T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	$\frac{\log. p \Delta}{\frac{a}{\delta}}$		Red. to App. Pl.
(433) <i>Eros</i>										
1912	^b ^{m.} ^s			^m ^s	["]	^h ^{m.} ^s	[°] ['] ["]			^s ["]
July	8 14 57 49	1	25, 5	-1 25.60	-1 47.7	20 54 30.42	-20 17 23.7	9.171	0.870	+2.88 + 5.6*
	12 14 54 22	2	25, 5	-1 5.08	-0 30.8	20 48 2.32	-20 7 45.2	9.269	0.866	+3.00 + 5.4*
	13 14 20 25	3	25, 5	-2 15.31	+0 31.0	20 46 22.16	-20 5 21.4	9.109	0.871	+3.03 + 5.4*
	26 12 45 55	4	30, 10	+0 48.67	-2 14.5	20 22 9.91	-19 28 34.7	8.946	0.871	+3.25 + 3.6†
Aug.	3 11 31 35	5	15, 3	+3 55.17	+3 44.2	20 7 6.42	-18 59 46.2	8.470	0.871	+3.28 + 1.9†
	5 11 1 31	6	25, 5	+2 32.14	+3 46.3	20 3 33.10	-18 53 32.8	7.998 _n	0.870	+3.28 + 1.8†
	6 11 40 28	6	30, 6	+0 43.97	+9 37.3	20 1 44.93	-18 47 41.7	8.924	0.868	+3.28 + 1.9†
(349) <i>Dembowska</i>										
1914										
Mar.	18 10 20 44	7	30, 6	+1 11.23	-1 30.6	10 55 15.75	+16 33 1.2	9.024 _n	0.527	+2.42 - 9.8†
	23 8 44 29	8	30, 6	+2 6.64	+0 34.4	10 51 31.97	+16 41 17.0	9.391 _n	0.554	+2.43 - 9.2†

Date	Wash. M. T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	$\frac{\log p \Delta}{\alpha \delta}$	Red. to App. Pl.
(718) <i>Erida</i> ?									
Mar. 20	13 6 59	9	30, 6	-1 36.69	+3 33.2	9 8 2.85	+26 12 11.2	9.634	0.520 +2.36 - 0.1*
23	10 31 26	10	30, 6	+2 59.17	-6 19.4	9 7 21.07	+26 4 41.4	9.276	0.330 +2.31 + 0.6*
[1914 <i>U M</i>]									
Mar. 26	11 44 1	11	30, 6	-1 3.34	+5 28.1	12 20 36.31	+ 2 36 17.1	8.631 _n	0.714 +2.27 -15.4†
Apr. 3	13 42 52	12	27, 9	-1 0.81	+3 24.9	12 13 8.03	+ 2 43 8.2	9.410	0.718 +2.32 -15.2†
	13 10 16 25	13	30, 6	-1 46.81	-4 24.2	12 4 38.68	+ 2 44 1.6	8.650 _n	0.713 +2.31 -14.6†
	17 11 13 59	14	30, 10	+0 43.83	+1 50.0	12 1 32.89	+ 2 41 22.8	9.030	0.714 +2.31 -14.5†
	21 9 53 6	14	30, 6	-2 0.15	-2 34.6	11 58 48.89	+ 2 36 58.5	8.224 _n	0.714 +2.29 -14.2†
Estimated magnitude 12.5 on March 26.									
(537) <i>Pauly</i>									
Apr. 2	13 9 0	15	20, 4	+0 56.22	-0 5.0	13 20 25.46	+ 6 32 52.2	8.800	0.671 +2.35 -17.6*
	3 11 48 18	16	30, 6	+1 54.30	+1 24.0	13 19 43.81	+ 6 38 33.9	8.951 _n	0.670 +2.37 -17.6*
	6 12 13 32	17	25, 5	-4 46.62	+1 19.9	13 17 28.82	+ 6 56 17.6	8.034 _n	0.666 +2.40 -17.7*
	13 12 58 57	18	30, 6	-2 18.18	-4 10.6	13 12 7.93	+ 7 33 46.8	9.160	0.662 +2.45 -16.9*
	17 13 8 52	19	30, 6	+1 49.69	+0 53.6	13 9 5.53	+ 7 52 20.6	9.298	0.662 +2.49 -16.5*
	21 11 22 27	20	25, 5	+3 20.47	-4 36.0	13 6 11.43	+ 8 8 8.0	8.445	0.651 +2.49 -16.0*

Observers: * H. E. BURTON, † C. B. WATTS.

Mean Places of Comparison-Stars for the Beginning of the Year.

*	α	δ	Authority	*	α	δ	Authority
1	^{h m s} 20 55 53.14	^{° ' "} -20 15 41.6	A.G. Algiers 9011	11	^{h m s} 12 21 37.38	^{° ' "} +2 31 4.4	A.G. Albany 4485
2	20 49 4.40	-20 7 19.8	$\frac{1}{2}$ } A.G. Algiers 8959 + same star compared with A.G. Algiers 8957 $\Delta\alpha = +0m 28s.98$ June 16, 1914 $\Delta\delta = -1' 22''.1$	12	12 14 6.52	+2 39 58.5	B.D. + 202529 compared with A.G. Albany 4451 $\Delta\alpha = +1m 34s.11$ May 17, 1914 $\Delta\delta = -4' 22''.5$
3	20 48 34.44	-20 5 57.8	A.G. Algiers 8957	13	12 6 23.18	+2 48 40.4	A.G. Albany 4442
4	20 21 17.99	-19 26 23.8	A.G. Algiers 8761	14	12 0 46.75	+2 39 47.3	A.G. Albany 4419
5	20 3 7.97	-19 3 32.3	A.G. Algiers 8623	15	13 19 26.89	+6 33 14.8	A.G. Leipzig II 6393
6	20 0 57.68	-18 57 20.9	A.G. Algiers 8605	16	13 17 47.17	+6 37 27.5	A.G. Leipzig II 6390
7	10 54 2.10	+16 34 41.6	A.G. Berlin A. 4317	17	13 22 13.04	+6 52 15.4	A.G. Leipzig II 6401
8	10 49 22.90	+16 40 51.8	A.G. Berlin A. 4293	18	13 14 23.66	+7 38 14.3	A.G. Leipzig II 6372
9	9 9 37.18	+26 8 38.1	A.G. Cam. (Engl) 4881	19	13 7 13.35	+7 51 43.5	A.G. Leipzig II 6341
10	9 4 19.59	+26 11 0.2	A.G. Cam. (Engl) 4841	20	13 2 48.47	+8 13 0.0	A.G. Leipzig II 6326

NOTES.

(349) *Dembowska* was found photographically by G. H. PETERS, Assistant in the Observatory.

[1914 *U M*] and (537) *Pauly* were found photographically by J. H. METCALF.

(718) *Erida*? was found by means of an ephemeris given in *Circular No 17*, Königliches Astr. Rechen-Institut.

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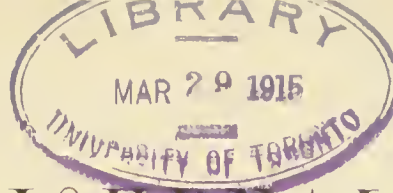
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MICROMETRICAL MEASURES OF DOUBLE STARS.

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY.

By F. P. LEAVENWORTH.

Through the kindness of Professor EDWIN B. FROST, Director of the Yerkes Observatory, I was given the privilege of using the 40-inch refractor in measuring a selected list of double stars. The telescope was at my disposal two to three nights a week, from July 16, to October 10, 1914.

The list was made up of stars needing measurement on account of motion, or because no recent measures had been made: close and faint stars being given the preference.

Each night the stars measured were the most difficult that the condition of the atmosphere would permit. During the entire time, there was no night when the full power of the telescope could be used. The highest power that the seeing would permit was always used; and this power was never greater than 1500. On several nights, however, the seeing was good, so that stars closer than 0".2 were separated.

Experiments were made with the iris diaphragm for diminishing the aperture of the object glass; and on a

number of nights a smaller aperture gave marked increase in defining power.

The effect of the smaller aperture usually was to diminish the "atmosphere" surrounding a brighter star, rendering visible a faint companion, before hidden in the glare. Diminishing the aperture frequently sharpened the images, but this was accompanied by increased dancing of the star. On some nights when the definition was poor, no good effect whatever could be noticed by reducing the aperture. This had to be determined each night by trial, except that on a few exceptional nights there was no need to diminish the aperture.

My impression of the instrument from this somewhat limited experience is that its defining power, up to the theoretical limit, is only limited by the atmospheric conditions; although it is possible that under poor atmospheric conditions the glare may be somewhat stronger than it should be.

The skill of Mr. F. R. SULLIVAN in managing the telescope very materially increased the efficiency of the instrument and the total output of measures.

Table with 3 main sections: 70 022, 150 beta 1015, and 156 beta 777. Each section contains rows of star measurements with columns for magnitude, position angle, and distance.

2266 α <i>Tauri</i>					<i>C</i> and <i>D</i>					8340 γ <i>Ophiuchi</i>				
4 ^h 31 ^m 16° 20'					.715 341°.2 6''.77 14 ^m .0 16 ^m .0					18 ^h 1 ^m 2° 33'				
<i>C</i> and <i>D</i>					.720 342°.9 6°.82 14°.5 16°.0					.602 142°.9 4''.23 4 ^m 6 ^m				
.720	276°.9	1''.63	12 ^m	15 ^m	1914.72	342°.0	6''.80	14 ^m .2	16 ^m .0	.613	141°.7	4°.35	...	
.739	275°.5	1°.49	...		Very faint and difficult.					.663	143°.1	4°.49	4	6°.5
.772	279°.0	1°.41	11	14°.5						1914.63	142°.6	4''.36	4 ^m	6 ^m .2
1914.74	277°.1	1''.51	1 ^m .5	14 ^m .8										
2395 β 1237					8133 <i>A</i> 233					<i>A</i> α				
4 ^h 48 ^m 23° 24'					17 ^h 40 ^m 24° 51'					.613 223°.6 40''.74 ... 13 ^m				
.709	59°.2	4''.25	8 ^m	11 ^m	.567	238°.8	3''.16	8 ^m	12 ^m .5	.663	223°.2	40°.46	...	11
.720	58°.8	4°.57	...		.674	237°.0	3°.40	8	13°.0	1914.63	223°.4	40''.60	...	12 ^m
1914.71	59°.0	4''.41	8 ^m	11 ^m	1914.62	237°.9	3''.28	8 ^m	12 ^m .8					
2523 <i>H</i> 446					8162 μ <i>Herculis</i>					8367 β 636				
5 ^h 2 ^m 22° 37'					17 ^h 43 ^m 27° 48'					18 ^h 4 ^m 2° 12'				
<i>A</i> and <i>B</i> , <i>C</i>					<i>A</i> and <i>B</i> , <i>C</i>					1914.748 126°.5 4''.34 7 ^m .5 13 ^m				
.715	182°.6	0''.93	9 ^m .0	9 ^m .2	.567	245°.6	32''.28	...		<i>A</i> and <i>C</i>				
.720	183°.2	0°.91	9°.0	9°.2	.580	245°.5	32°.72	...		1914.748	103°.6	14''.34	...	14°.5
1914.72	182°.9	0''.92	9 ^m .0	9 ^m .2	.638	246°.1	32°.53	...		8545 <i>Ho</i> 84				
No motion.					1914.60	245°.7	32''.51	...		18 ^h 21 ^m 27° 20'				
2544 β 1047					<i>B</i> and <i>C</i>					.638 320°.7 4''.34 9 ^m .5 12 ^m .0				
5 ^h 4 ^m 27° 56'					.567	156°.4	0''.39	10 ^m .0	10 ^m .5	.643	321°.7	3°.90	10°.0	13°.0
<i>A</i> and <i>B</i> , <i>C</i>					.580	155°.2	0°.44	...		.671	322°.2	4°.58	11°.0	13°.0
1914.720	26°.6	...	...		.638	152°.9	0°.48	10°.0	10°.8	.682	320°.1	3°.90	10°.0	13°.0
<i>B</i> and <i>C</i>					1914.60	154°.8	0''.44	10 ^m .0	10 ^m .7	1914.66	321°.2	4''.18	10 ^m .1	12 ^m .8
.720	83°.6	0''.35	8 ^m	9 ^m	Comparison with ephemeris.					Angle and distance probably in-				
.739	88°.7	0°.35	...		LOHSE	<i>O</i> - <i>C</i>	+38°.9	-0''.22	creasing, but the measures are very					
1914.73	86°.2	0''.35	8 ^m	9 ^m	AITKEN	<i>O</i> - <i>C</i>	+13°.9	-0''.14	discordant.					
Measures of angle very discrepant.					8235 β 130					8548 Σ 2315				
2690 β 888					17 ^h 50 ^m 40° 2'					18 ^h 22 ^m 27° 20'				
5 ^h 19 ^m 37° 18'					.556	119°.4	1''.82	6 ^m	9 ^m	1914.638	181°.9	0''.24	7 ^m .0	8 ^m .5
<i>A</i> and <i>B</i>					.575	121°.2	1°.89	5	9	8663 <i>O</i> Σ 358				
.715	167°.8	8''.22	...		.580	123°.6	1°.93	6	9°.5	18 ^h 32 ^m 16° 53'				
.720	165°.8	8°.71	...		1914.57	121°.3	1''.88	5 ^m .7	9 ^m .2	.548	189°.0	2''.01	7 ^m .0	7 ^m .3
1914.72	166°.8	8''.46	...		No motion.					.575	188°.8	1°.98	7°.0	7°.2
Probably no motion.					8303 τ <i>Ophiuchi</i>					.663	187°.6	1°.96	7°.0	7°.2
<i>A</i> and <i>C</i>					17 ^h 58 ^m -8° 11'					1914.60	188°.5	1''.98	7 ^m .0	7 ^m .2
.715	331°.4	...	...		.548	260°.7	2''.27	5 ^m .0	5 ^m .6	8770 <i>A</i> 253				
.720	330°.5	27''.35	...		.556	262°.5	2°.11	5°.0	5°.3	18 ^h 40 ^m 31° 36'				
1914.72	331°.0	27''.35	...		.643	259°.1	2°.19	5°.0	6°.0	.575	138°.4	0''.63	...	
No motion.					.649	261°.4	2°.02	...		.638	134°.2	0°.70	9 ^m .0	10 ^m .0
					1914.60	260°.9	2''.15	5 ^m .0	5 ^m .6	1914.61	136°.3	0''.66	9 ^m .0	10 ^m .0

8788 β 968 18 ^h 42 ^m 37° 31'					10040 β 983 20 ^h 12 ^m 25° 20'					<i>A, B and C</i>				
<i>A and B</i>					.638	160°.5	1''.14	5 ^m	9 ^m .5	.638	118°.9	23''.79	...	13 ^m .5
1914.748	50°.1		...	15 ^m .5	.663	159°.4	1°.21	6	10°.0	.739	118°.7	23°.97	...	12°.5
<i>A and C</i>					1914.65	160°.0	1''.18	5 ^m .5	9 ^m .8	.748	119°.6	23°.67	...	13°.0
1914.748	272°.4	45''.46	4 ^m .5	15 ^m	10266 β 63 20 ^h 26 ^m 10° 36'					<i>A, B and D</i>				
9319 Σ 2525 19 ^h 23 ^m 27° 9'					<i>A and B</i>					.638	329°.3	38''.36	...	11 ^m
.548	307°.9	0''.92	7 ^m .0	7 ^m .3	.671	346°.1	0''.74	6 ^m	8 ^m	.739	329°.7	38°.33	...	10°.5
.580	308°.6	0°.82	7°.2	7°.4	.682	347°.9	0°.81	6	8	.748	328°.4	38°.71	...	
.690	309°.7	0°.83	7°.7	8°.0	.690	345°.3	0°.68	6	8°.5	1914.71	329°.1	38''.47	...	10 ^m .8
1914.61	308°.7	0''.86	7 ^m .3	7 ^m .6	1914.68	346°.4	0''.74	6 ^m	8 ^m .2	10385 β 435 20 ^h 35 ^m 14° 42'				
9500 Σ 2556 19 ^h 36 ^m 22° 2'					<i>A, B and C</i>					.613	112°.7	3''.14	8 ^m .5	11 ^m
.638	129°.8	0''.43	7 ^m .5	8 ^m .0	.682	347°.8	16''.82	...	14 ^m	.622	113°.2		8°.0	11
.663	129°.9	0°.42	...		.690	347°.2		...	15	.638	115°.4	3°.05	8°.5	10
.693	132°.0	0°.45	7°.5	7°.8	1914.69	347°.5	16''.82	...	14 ^m .5	.671	117°.9	3°.09	8°.0	11
1914.66	130°.6	0''.43	7 ^m .5	7 ^m .9	No motion.					1914.64	114°.8	3''.09	8 ^m .2	10 ^m .8
9659 β 361 19 ^h 46 ^m 22° 27'					10281 DA 1 20 ^h 27 ^m 10° 58'					10520 β 65 20 ^h 44 ^m 5° 42'				
.608	348°.6	3''.60	9 ^m .0	9 ^m .5	<i>A and B, C</i>					.608	191°.0	1''.65	5 ^m	9 ^m .5
.663	349°.7	3°.90	9°.0	9°.5	.542	255°.5	16''.20	...		.682	192°.5	1°.78	5°.5	9°.0
1914.64	349°.2	3''.75	9 ^m .0	9 ^m .5	.575	255°.7	16°.04	...		1914.64	191°.8	1''.72	5 ^m .2	9 ^m .2
Probably fixed.					.580	255°.7	15°.95	...		10533 0 Σ 413 20 ^h 44 ^m 36° 11'				
9719 Σ 2597 19 ^h 51 ^m -6° 58'					.583	256°.2	15°.95	...		<i>A and B</i>				
.682	91°.4	1''.25	7 ^m	8 ^m	.602	256°.5	15°.84	...		.715	57°.6	0''.73	5 ^m	7 ^m
.690	89°.0	1°.27	7	8	1914.58	255°.9	15''.97	...		.734	57°.1	0°.72	5	6°.5
1914.69	90°.2	1''.26	7 ^m	8 ^m	<i>A and D</i>					1914.72	57°.4	0''.72	5 ^m	6 ^m .8
Probably no motion.					.542	104°.8	23''.87	...		Comparison with HUSSEY's ephemeris from rectilinear motion.				
9833 0 Σ 395 19 ^h 58 ^m 24° 42'					.575	105°.7	23°.30	...		<i>O - C</i> +1°.3 -0''.06				
.583	106°.3	0''.77	6 ^m .0	6 ^m .4	.580	106°.3	23°.75	...		10559 Σ 2729 20 ^h 46 ^m -5° 56'				
.586	105°.3	0°.78	...		.583	106°.3	23°.63	...		.608	332°.4	0''.42	6 ^m	7 ^m
.602	108°.4	0°.71	6°.0	6°.5	1914.57	105°.8	23''.64	...		.638	332°.0	0°.46	6	7
.605	104°.3	0°.80	6°.0	6°.5	<i>B and C</i>					1914.62	332°.2	0''.44	6 ^m	7 ^m
1914.59	106°.1	0''.76	6 ^m .0	6 ^m .5	.580	210°.1	0''.43	...		Comparison with AITKEN's ephemeris, <i>Lick</i> , Vol. XII.				
9908 β 428 20 ^h 3 ^m 12° 42'					.602	209°.5	0°.39	7°.5	7°.5	<i>O - C</i> +3°.5 -0''.01				
.583	353°.4	0''.72	7 ^m .0	8 ^m .5	1914.59	212°.1	0''.42	7 ^m .5	7 ^m .5	10643 Σ 2737 20 ^h 55 ^m 3° 58'				
.602	355°.7	0°.74	7°.0	8°.5	10363 β Delphini 20 ^h 34 ^m 14° 18'					1914.663	307°.0	0''.19	...	
1914.59	354°.6	0''.73	7 ^m .0	8 ^m .5	<i>A and B</i>					Plane of orbit very nearly in the line of sight. Distance diminishing about 0''.04 a year. Very close and difficult for the next few years.				
Fixed.					.638	291°.2	0''.27	4 ^m	5 ^m					
					.739	292°.7	0°.24	...						
					.748	288°.4	0°.29	4	5					
					1914.71	290°.8	0''.27	4 ^m	5 ^m					

10685 Σ 2744					11125 β 122					A, B and C				
20 ^h 58 ^m 1° 12'					21 ^h 35 ^m -0° 27'					.715 297°.6 12''.65				
.548 158°.6 6 ^m .0 6 ^m .5					A and B					.729 297°.1 				
.567 157°.4 1''.79 6°.0 7°.5					1914.663 293°.5 0''.51 7 ^m .0 7 ^m .5					1914.72 297°.4 12''.65 				
.580 157°.9 1°.50 6°.0 7°.0					The drawing in BURNHAM's General Catalogue has the coördinates 180°					11329 β 1213				
.583 157°.5 1°.35 					wrong.					21 ^h 50 ^m 13° 10'				
1914.57 157°.6 1''.55 6 ^m .0 7 ^m .0					A, B and C					1914.739 311°.8 0''.72 9 ^m .0 9 ^m .5				
10689 β 69					1914.663 146°.7 41''.44 ... 11 ^m .0					No motion.				
20 ^h 59 ^m 21° 21'					11190 Lx 10					11443 Ho 610				
A and B					21 ^h 39 ^m -11° 32'					21 ^h 58 ^m 26° 26'				
.586 319°.1 0''.87 8 ^m .0 9 ^m .0					.567 284°.7 1''.21 8 ^m .5 10 ^m					A and B				
.605 322°.0 0°.88 7°.5 9°.0					.575 283°.0 1°.21 8°.3 9					.682 237°.8 0''.72 9 ^m .0 9 ^m .6				
.663 319°.9 0°.92 8°.0 8°.7					1914.57 283°.8 1''.21 8 ^m .4 9 ^m .5					.709 239°.9 0°.70 9°.0 9°.7				
1914.62 320°.3 0''.89 7 ^m .8 8 ^m .9					Slow increase in angle and decrease in distance.					1914.70 238°.8 0''.71 9 ^m .0 9 ^m .6				
A, B and C					Discovered with the 26-inch equatorial of the McCormick Observatory, and not at the Haverford College Observatory, as stated in BURNHAM's General Catalogue.					No motion.				
1914.663 239°.3 77''.49 					11214 μ Cygni					A, B and C				
C and D					21 ^h 40 ^m 28° 21'					.682 329°.1 32''.62 ... 10 ^m .5				
.586 157°.0 18''.75 7 ^m .5 13 ^m .0					A and B					.709 329°.3 32°.76 ... 11°.0				
.605 157°.7 18°.90 7°.0 12°.5					.586 131°.9 1''.89 4 ^m .0 5 ^m .5					1914.70 [329°.2 32''.69 ... 10 ^m .8				
1914.60 157°.3 18''.82 7 ^m .2 12 ^m .8					.605 131°.0 1°.79 					Not measured before.				
10726 A 178					.608 130°.8 1°.82 4°.0 5°.0					A, B and D				
21 ^h 2 ^m 20° 57'					.622 132°.4 2°.07 4°.0 5°.5					.682 282°.7 34''.97 ... 10 ^m .7				
1914.720 73°.0 0''.89 8 ^m 10 ^m .5					1914.61 131°.5 1''.89 4 ^m .0 5 ^m .3					.709 283°.1 34°.74 ... 11°.5				
10829 δ Equulei					61 Cygni type.					1914.70 [282°.9 34''.86 ... 11 ^m .1				
21 ^h 10 ^m 9° 39'					A and C					Not measured before.				
.709 206°.5 0''.34 5 ^m .0 5 ^m .2					.586 274°.5 45''.28 					A, B and E				
.739 209°.1 0°.30 					.605 274°.3 45°.62 ... 11 ^m .0					1914.709 238°.9 57''.71 ... 10 ^m .7				
1914.72 207°.8 0''.32 5 ^m .0 5 ^m .2					.622 274°.4 45°.81 ... 11°.5					Not measured before.				
10846 τ Cygni					1914.60 274°.4 45''.57 ... 11 ^m .2					11646 H 962				
21 ^h 11 ^m 37° 40'					11222 κ Pegasi					22 ^h 16 ^m 5° 21'				
A and B					21 ^h 41 ^m 25° 16'					A and B				
.663 212°.7 0''.80 5 ^m 7 ^m					A and B					.602 18°.7 6''.28 6 ^m 12 ^m .5				
.715 209°.6 1''.11 5 9					.715 158°.7 0''.16 					.605 19°.5 6°.14 6 12°.5				
.720 213°.5 0''.93 5 					.734 146°.3 0°.17 					.613 18°.4 5°.90 6 12°.4				
1911.70 211°.9 0''.95 5 ^m 8 ^m					1914.73 152°.5 0''.16 					1914.61 18°.9 6''.11 6 ^m 12 ^m .5				
Comparison with ATKIN's ephemeris, <i>Lick</i> , Vol. XII.					Comparison with LOUSE's ephemeris, <i>Potsdam</i> , Vol. 20.					No motion.				
O - C +2°.7 +0''.09					O - C +14°.9 -0''.06 or 0''.02					A and C				
A and C					1914.61 224°.4 12''.20 ... 12 ^m .1					.602 225°.2 12''.23 ... 12 ^m .0				
1911.720 227°.7 20''.15 ... 13 ^m					.605 224°.4 12°.23 ... 12°.2					.613 223°.5 12°.13 ... 12°.0				

11691 β 172						A, B and C						12205 β 852					
22 ^h 19 ^m $-5^{\circ} 17'$						.720 198 ^o .3						23 ^h 6 ^m $26^{\circ} 3'$					
.608 353 ^o .8 0 ^{''} .56						.734 199 .0						A and B					
.663 355 .4 0 .63						1914.73 198 ^o .6						.734 282 ^o .5 58 ^{''} .40 7 ^m					
1914.64 354 ^o .6 0 ^{''} .59						12046 β 178						.756 282 .6 58 .35 7					
11793 Ho 475						22 ^h 50 ^m $-5^{\circ} 27'$						1914.74 282 ^o .6 58 ^{''} .38 7 ^m					
22 ^h 28 ^m $26^{\circ} 0'$						1914.605 327 ^o .7 0 ^{''} .63 8 ^m .0 9 ^m .5						No motion.					
A and B						Slow increase in angle.						B and C					
.682 142 ^o .2 0 ^{''} .96 8 ^m .5 8 ^m .5						12058 β 383						.709 358 ^o .6 1 ^{''} .61 10 ^m .5 11 ^m .0					
.709 139 .8 0 .72 8 .5 8 .6						22 ^h 52 ^m $9^{\circ} 0'$						.715 360 .2 1 .55 10 .5 11 .0					
.715 141 .8 0 .83 8 .0 8 .1						A and B						.739 358 .9 1 .46 10 .7 11 .5					
1914.70 141 ^o .3 0 ^{''} .84 8 ^m .3 8 ^m .4						.602 118 ^o .4 2 ^{''} .36 8 ^m 13 ^m .5						.756 361 .8 1 .31 11 .0 12 .0					
A, B and C						.613 116 .5 8 13 .5						1914.73 359 ^o .9 1 ^{''} .48 10 ^m .7 11 ^m .4					
.682 224 ^o .4 8 ^{''} .08 . . . 11 ^m .0						.671 118 .4 2 .61 6 .5 13 .5						Slow decrease in angle.					
.709 223 .1 8 .19 . . . 12 .0						1914.63 117 ^o .8 2 ^{''} .48 7 ^m .5 13 ^m .5						B and D					
.715 224 .2 8 .15 . . . 11 .0						No motion.						.709 210 ^o .4 19 ^{''} .25 . . . 15 ^m					
1914.70 223 ^o .9 8 ^{''} .14 . . . 11 ^m .3						A and C						.715 210 .2 19 .83 . . . 15					
11803 β 77						.602 241 ^o .2 16 ^{''} .38 . . . 12 ^m						.739 208 .8 18 .84 . . . 14					
22 ^h 30 ^m $-2^{\circ} 14'$						.613 241 .1 15 .66 . . . 11						1914.72 209 ^o .8 19 ^{''} .31 . . . 14 ^m .7					
A and B						.671 242 .2 15 .64 . . . 11 .5						12290 β 80					
.603 218 ^o .9 2 ^{''} .71 9 ^m .2 9 ^m .6						1914.63 241 ^o .5 15 ^{''} .89 . . . 11 ^m .5						23 ^h 14 ^m $4^{\circ} 56'$					
.613 215 .3 2 .66 9 .0 9 .8						No motion.						1914.739 240 ^o .0 0 ^{''} .63 8 ^m .3 9 ^m .8					
.671 214 .1 2 .72 8 .8 9 .3						12069 Σ 2959						12311 Ho 488					
1914.63 216 ^o .1 2 ^{''} .70 9 ^m .0 9 ^m .6						22 ^h 53 ^m $-3^{\circ} 42'$						23 ^h 16 ^m $2^{\circ} 0'$					
A and C						A and B						A and B, C					
.603 225 ^o .5 27 ^{''} .98 . . . 11 ^m .5						.542 107 ^o .4 12 ^{''} .42 6 ^m .5 10 ^m						.575 201 ^o .5 4 ^{''} .40					
.613 226 .2 28 .03 . . . 11 .5						.548 107 .3 12 .61 6 .5 10						.613 200 .7 4 .35					
.671 226 .2 27 .63 . . . 11 .0						.575 106 .6 12 .52						.638 199 .0 4 .06					
1914.63 226 ^o .0 27 ^{''} .88 . . . 11 ^m .3						.671 107 .3 12 .70 6 .8 10						.649 203 .4 4 .22					
Distance diminishing.						1914.58 107 ^o .2 12 ^{''} .56 6 ^m .6 10 ^m						1914.62 201 ^o .1 4 ^{''} .26					
11895 Ho 296						Proper motion in B.						Fixed.					
22 ^h 37 ^m $14^{\circ} 6'$						A and C						B and C					
.734 42 ^o .0 0 ^{''} .36 5 ^m .0 6 ^m .0						1914.542 102 ^o .2 22 ^{''} .47 . . . 13 ^m						.575 215 ^o .5 0 ^{''} .97					
.748 39 .2 0 .43 5 .5 6 .0						B and C						.638 213 .3 0 .84 10 ^m 10 ^m .7					
1914.73 40 ^o .6 0 ^{''} .40 5 ^m .2 6 ^m .0						.548 95 ^o .7 10 ^{''} .88 10 ^m 13 ^m						.649 218 .4 0 .73					
Angle decreasing.						.575 94 .7						1914.62 215 ^o .7 0 ^{''} .85 10 ^m 10 ^m .7					
12008 Ho 482						.671 95 .7 10 .87 10 13 .5						No change.					
22 ^h 47 ^m $25^{\circ} 56'$						1914.60 95 ^o .4 10 ^{''} .88 10 ^m 13 ^m .2						12363 Ho 489					
A and B						12094 0Σ 483						23 ^h 22 ^m $27^{\circ} 14'$					
.720 207 ^o .1 0 ^{''} .21 7 ^m .5 7 ^m .7						22 ^h 55 ^m $11^{\circ} 16'$						A and B					
.734 203 .0 0 .22 7 .0 8 .0						.567 233 ^o .2 0 ^{''} .92 6 ^m 8 ^m						.682 243 ^o .3 0 ^{''} .53 7 ^m .5 8 ^m .0					
1914.73 205 ^o .0 0 ^{''} .22 7 ^m .2 7 ^m .8						.575 229 .9 0 .97 7 8						.715 242 .9 0 .59 8 .5 9 .0					
Rapid increase in angle probable.						.602 232 .6 1 .06 6 .5 8						.720 242 .1					
						1914.58 231 ^o .9 0 ^{''} .98 6 ^m .5 8 ^m .0						.734 241 .8 0 .52 8 .0 8 .0					
												1914.71 242 ^o .5 0 ^{''} .55 8 ^m .0 8 ^m .3					
												Probably no change.					

12432 β 720					12701 β 733					A and C				
23 ^h 30 ^m 30° 51'					23 ^h 58 ^m 26° 38'					1914.758 333° 9 34'' 40 ... 12 ^m .5				
					A and B									
.567 182° 4 0'' 44 6 ^m .0 6 ^m .1					.638 120° 0 ... 6 ^m 10 ^m									
.682 185 .9 0 .42 					.715 122 .1 									
.709 186 .5 0 .41 6 .0 6 .0					.720 118 .2 0'' 81 ... 12					12723 A 429				
					.734 121 .6 0 .90 6 11					23 ^h 59 ^m 27° 30'				
1914.65 184° 9 0'' 42 6 ^m .0 6 ^m .0										A and B				
					1914.70 120° 5 0'' 86 6 ^m 11 ^m					.734 342° 4 0'' 51 9 ^m .0 9 ^m .3				
12501 Ho 303					Comparison with LOHSE's ephemeris, <i>Potsdam</i> , Vol. 20.					.739 345 .0 0 .58 8 .8 9 .2				
23 ^h 36 ^m 19° 53'					O - C - 11° 2 + 0'' 06					.748 339 .2 0 .51 				
.720 192° 2 ... 8 ^m .0 12 ^m .0										1914.74 342° 2 0'' 53 8 ^m .9 9 ^m .2				
.734 191 .6 0'' 65 8 .0 12 .0										Angle reversed from AITKEN's measure; magnitudes estimated with care.				
.739 192 .9 														
1914.73 192° 2 0'' 65 8 ^m .0 12 ^m .0					A and C					A, B and C				
					1914.682 335° 6 52'' 08 					.734 288° 8 5'' 16 ... 9 ^m .2				
12609 β 859					12709 β 281					.739 289 .0 5 .02 ... 9 .3				
23 ^h 48 ^m 22° 29'					23 ^h 58 ^m 1° 39'					.748 289 .3 5 .29 				
.567 212° 2 0'' 58 8 ^m .5 8 ^m .6										1914.74 289° 0 5'' 16 ... 9 ^m .2				
.580 211 .3 0 .68 					A and B					Minneapolis, Minn., November 30, 1914.				
1914.57 211° 8 0'' 63 8 ^m .5 8 ^m .6					1914.758 193° 0 1'' 41 7 ^m .5 11 ^m									

ASTEROID *JY*, 776,

By F. E. SEAGRAVE.

$M = 105^{\circ} 7' 27''.72$	} = March 29.5000 = 1914. G.M.T.	$q = \frac{p}{(1+e)} = a(1-e) = (a \cos \varphi) \tan (45^{\circ} - \frac{1}{2}\varphi)$		
$\omega = 305^{\circ} 33' 20''.69$				
$\pi = 25^{\circ} 9' 11''.71$		0.4578170	9.9364938	0.4618734
$\Omega = 79^{\circ} 35' 51''.02$		0.0553938	0.4659298	9.9405498
$i = 18^{\circ} 0' 55''.91$		Log $q = 0.4024232 = 0.4024236 = 0.4024232$		
Log $e = 9.1336686$				
Log $a = 0.4659298$				
Log $q = 0.4024232$				
$\mu = 709''.7606$				

$$\begin{aligned} x &= r(9.9789156) \sin (169^{\circ} 4' 25''.71 + u.) \\ y &= r(9.9556718) \sin (87^{\circ} 48' 52''.10 + u.) \\ z &= r(9.7213838) \sin (48^{\circ} 1' 36''.59 + u.) \end{aligned}$$

Boston, September 3, 1914.

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PREFERENTIAL MOTION ACCORDING TO TYPE,

By H. RAYMOND.

The present paper is a determination of the vertex of preferential motion by SCHWARZSCHILD's* method, based on 6033 proper-motions from Boss's Preliminary General Catalogue. The rejections are practically the same as those made by Prof. LEWIS BOSS in his investigations on solar motion (*A.J.*, 612, 614 and 623-624). The proper-motions were first divided into those less than and those greater than 20'' per century. The stars of small motions were further grouped according to type. The sky was divided into 108 approximately equal areas by circles of galactic† latitude and longitude, the pole of the galaxy being taken at R.A. 193°.33, Decl. +27°.35, and its ascending node on the equator as the origin of longitudes.

All the work to this point, together with the computation of the position-angles of the proper-motions, had been done by Prof. BOSS, in connection with his investigations above cited.

SCHWARZSCHILD's method rests on the hypothesis that the distribution of peculiar motions is "ellipsoidal," i.e., that the number of stars having the rectangular components of their peculiar motions between u and $u + du$, v and $v + dv$, w and $w + dw$, is

$$\frac{N}{\pi^{3/2} AB^2} e^{-u^2/A^2 - v^2/B^2 - w^2/B^2} du dv dw$$

where the axis of u is directed toward the vertex, and N is the number of stars. A and B are proportional to the mean velocity in the direction of preferential motion and at right angles to that motion, respectively. α and β are corresponding quantities for the ellipse into which the ellipsoidal velocity-figure projects against the celestial sphere; and if $\beta/\alpha = \cos \psi$, $B/A = \cos \Psi$, and χ is the angle between the line of sight and the

direction of preferential motion, we have

$$\tan \psi = \tan \Psi \sin \chi.$$

The center of the projected ellipse is shifted from the origin by solar motion. The position-angle θ_1 of the solar apex divides the proper-motions into two numerically equal groups. To find the direction θ_0 of the vertex, turn a pair of rectangular axes about the origin, letting the number of proper-motions in each quadrant as so divided, be n_1, n_2, n_3, n_4 , in order; the n 's changing with the position-angle θ of the rotating x -axis. When $\log n_1 n_3 / n_2 n_4 = 0$, and is decreasing as θ increases, the turning axes are parallel to the principal axes of the ellipse, and $\theta = \theta_0$.

For $\theta = \theta_0$ compute $v(\mu) = \frac{(n_3 - n_1) - (n_4 - n_2)}{N}$ and $v(v) = \frac{(n_3 - n_1) + (n_4 - n_2)}{N}$, where $v(\mu)$, $v(v)$ represent

the probability integral for the arguments μ, v . If h is the tangential component of solar motion, $h/\alpha = \mu/\cos(\theta_1 - \theta_0)$, $h/\beta = v/\sin(\theta_1 - \theta_0)$; from which we find β/α and $\tan \psi$.

If h is small (near apex or antapex), or if the sine or cosine of $\theta_1 - \theta_0$ is small, $\tan \psi$ will not be well determined. In these cases it will be necessary to find another pair of conjugate diameters of the ellipse. If θ' and θ'' be the position-angles of lines through the origin parallel to such a pair, and the number of proper-motions in the parts (not in general quadrants) into which they divide the ellipse, beginning from θ' , be l_1, l_2, l_3, l_4 , then $l_1 l_3 / l_2 l_4 = 1$; whence

$$\frac{l_1 + l_2}{l_3 + l_4} = \frac{l_1}{l_3} = \frac{l_2}{l_4}$$

But θ' may be chosen arbitrarily, so that the first ratio is known, and the second or third may be made equal to it by trial, varying θ'' . Then $\beta^2/\alpha^2 = -\tan(\theta' - \theta_0) \tan(\theta'' - \theta_0)$. In practice θ' should be so chosen that $\tan(\theta' - \theta_0)$ will be near the expected value of β/α .

**Nachr. v. d. Kgl. Gesell. d. Wissenschaften zu Göttingen, Math.-Phys. Kl.*, 1907, p. 614; 1908, p. 191; from which the following summary is adapted. See also BELIAWSKI, *A.N.*, 4291; RUDOLPH, *A.N.*, 4369.

† As no other latitudes or longitudes will be referred to, the adjective will be hereafter omitted.

The position-angles were in two forms; first, referred to the pole in the usual way, and second, referred to the apex (R.A. 270°.5, Decl. +34°.3). Clearly, the nearer the pole of reference is to the vertex (or antivertex) to be determined, the less, other things being equal, will be the effects due to the fact that the areas are extended regions and not points. The North Pole is about 80° from the vertex, the apex about 45° from the antivertex. The second system of position-angles was therefore used, except for a few areas near the apex and antapex.

The numbers of position-angles in each area were tabulated for each 10° of position-angle. These tables were smoothed only in a few cases where the differences were so irregular that the necessary interpolations could not otherwise be made. The values of θ_0 when found were referred to the direction of the north galactic pole as zero line by adding the appropriate corrections.

Each area gives two equations for finding the vertex and the ratio of the axes in the velocity-ellipsoid by least squares:

$$\begin{aligned} -\tan \psi \sin \theta_0 &= \sin l \cdot X - \cos l \cdot Y \\ -\tan \psi \cos \theta_0 &= \cos l \sin b \cdot X + \sin l b \cdot Y - \cos b \cdot Z \end{aligned}$$

where

$$\begin{aligned} X &= \cos L \cos B \tan \Psi \\ Y &= \sin L \cos B \tan \Psi \\ Z &= \sin B \tan \Psi \\ B/A &= \cos \Psi \end{aligned}$$

L and B are respectively the longitude and latitude of the vertex, l and b those of the center of the area. Substituting h/β for $\tan \psi$ and θ_1 for θ_0 , we get V/B and the coördinates of the apex by a similar process. The results are given in Table I.

TABLE I.

Group	No. of Stars	Vertex			Apex					
		L_0	B_0	B/A	A_0	D_0	A_1	D_1	B/V	A/V
A (BS - .14)	1647	168.9	+ 1.4	.49	96.5	+10.6	271.5	+26.6	.74	1.5
F (.45 - F8)	656	173.0	- 1.3	.56	96.0	+ 5.6	262.0	+26.0	1.26	2.2
G	444	148.9	- 3.4	.52	81.6	+25.5	251.6	+41.0	1.28	2.5
K	1227	162.0	+ 3.8	.54	95.4	+17.7	266.2	+38.9	1.07	2.0
M	222	148.9	+10.3	.61	95.3	+32.2	271.6	+27.2	1.22	2.0
All types	4686	169.5	+ 0.9	.52	96.3	+ 9.7	264.0	+33.7	.92	1.8
Large P.M.	652	158.4	0.0	.39	90.0	+19.1	270.2	+30.7	.72	1.8
Complete	6033	164.9	+ 0.1	.52	93.4	+13.0	268.5	+35.5	.97	1.9

The group "all types," consisting of stars of types from 0e5 through M with proper-motions under 20" per century, was solved by hemispheres, dividing the sky into halves in three different ways. The results are shown in Table II. The differences give an idea of the

TABLE II.

"All Types," by Hemispheres.

Hemisphere	L_0	B_0	B/A
Lat. 0 to + 90	169.1	+0.6	.52
0 to - 90	169.6	+0.2	.51
Long. 0 to 180	167.4	-0.2	.54
180 to 360	172.0	+1.3	.56
90 to 270	170.7	-0.1	.52
270 to 90	168.1	+0.9	.51

errors to be expected in this amount of material. For the solutions by separate types some combination of areas was necessary, because of the scantiness of the data. As no prohibitive discrepancies appeared between opposite hemispheres, the material of antipodal areas was combined for each of the other groups, reducing the number of areas from 108 to 54. The position-angles in the area thus transferred to its opposite were of course subtracted from

360°. For the small types F , G and M , adjoining areas were further combined into 15 large areas. The original 108 areas were given equal weights. Unequal weights would have complicated the solution without corresponding gain in accuracy.

In each group, separate solutions were made of the "galactic" areas, between +30° and -30° of latitude, and the "non-galactic" areas, outside those limits. The results are given in Table III. The galactic areas determine L with about one-third, and B with about two-thirds, the weight of the whole material.

TABLE III.

Group	Galactic			Non-Galactic		
	L_0	B_0	B/A	L_0	B_0	B/A
BS to .14	177.9	- 0.8	.50	163.2	+ 5.9	.47
.15 to F9	166.7	+ 3.2	.62	176.2	-10.9	.52
G	156.1	+12.0	.60	145.7	-30.0	.42
K	151.1	+ 1.1	.53	167.5	+ 2.7	.54
M	159.1	+ 0.8	.61	142.6	+34.8	.54
All types	170.7	+ 2.5	.50	168.7	- 3.2	.53
Large P.M.	172.0	- 1.3	.40	149.4	+ 2.8	.37
Complete	165.3	- 0.3	.48	164.6	+ 1.2	.55

The group "all types" contains not only the groups which precede it in Tables I and III, but 490 stars of type *B* (0e5 to *B*5). Type *B* could not be treated by this method, since its peculiar motions are so small relative to the effect of solar motion that the position-angles in an area often have a range of less than 90° . Such areas cannot be solved for θ_0 , and other areas so far approximate to this condition that their solution has no value. The final group contains the groups "all types" and "large p.m." and 695 stars of undetermined type with proper-motions less than $20''$. It thus nearly corresponds to EDDINGTON's* solution (*M.N.*, LXXI, 21) by an altogether different method, from which it differs less than 2° in the position of the vertex.

Some of the more important results exhibited in the tables are :

1. The vertices for the various groups lie in a band making a small but well marked angle with the galaxy: —practically in the direction of solar motion. The vertices for the early types *A* and *F* and those for the late types *K* and *M* lie somewhat separated from each other near the two ends of this band. *G* is anomalous. As Mr. BENJAMIN BOSS finds† a similar phenomenon in the determinations of the apex of solar motion according to type (roughly confirmed, at least in part, in the latter part of Table I), the question arises whether the determination of apex and vertex are completely independent of each other. Mathematically the recognized methods seem to be sound; but they rest upon assumptions in regard to the areal and radial distribution of the stars, the similarity of motions in different regions, and the laws of distribution of those motions, which can hardly be true save in the roughest way.

The distribution of vertices here found is nearly along a meridian. If, however, it were due to errors in the proper-motions in declination, it should also appear in Table II. It is difficult to believe that errors of sufficient size could exist, varying with type but not with position in the sky.

2. Examination of Table III shows that the range of "galactic" and "non-galactic" solutions, for each group, falls, when at all large, into the same belt. Type *G* is again anomalous, in regard to both the amount and the direction of the range. Anomalies seem to be more frequent for non-galactic than for galactic stars. These results can claim no great accuracy, especially for the smaller types.

* EDDINGTON omitted stars known to be of type *B*, and was more drastic in excluding members of stellar streams. His vertex is at $94^\circ.2$, $+11^\circ.9$.

† *A.J.*, 649-650, p. 15; 668, p. 193.

3. The ratios B/A show, rather uncertainly, that the the preferential element of motion is stronger in the earlier types, and within the earlier types, for non-galactic stars‡; and decisively, that it is stronger for large proper-motions. The last result has been found†† by DYSON and others, and accords with the underlying theory of this method. It should be noted, however, that when the the small and moderate proper-motions have been removed, the method no longer strictly applies, and will give too small a value of B ; on the other hand, a weak group is likely to make B/A large, because of the greater element of chance.

4. Attention has been called§ to the fact that the proper-motions of the earlier type stars are greater in the non-galactic than in the galactic regions; PLUMMER¶ has explained this, and the converse relation found by CAMPBELL in the radial velocities, on the basis of parallelism of peculiar motions to the galactic plane. It has been suggested that the velocity-figure for these stars might be an ellipsoid of three unequal axes, thin in the direction of the galactic poles. If such were the case, the ratio of axes in the projected ellipse should be least in the galactic region, giving, when solved separately, a smaller B/A ; which is contrary to the testimony of these results. The evidence, such as it is, tends to the conclusion that the proper-motions within and those without the galaxy are differently distributed, the non-galactic ellipsoid being for early types more prolate. This favors the idea that the non-galactic stars are outside the galaxy as the result of a selective process.||

5. The ratios B/V and A/V show a marked increase from type *A* to later types, corresponding to the well-known increase of velocities with advancing type. If account be taken of the increase of V with advance of type, from about 16 km./sec. for *A* and *F* to about 21 km./sec. for *K* and *M***, the increase in the semi-axes A and B , and consequently in the peculiar motions, is still more marked, and is progressive through type *F*.

‡ See B. BOSS, "Systematic Motions of the Stars Arranged According to Type," *A.J.*, 635-636. Especially sections 4, 7 and 8 at the end.

†† Proc. R. S. Edinburgh, XXIX, 376-392. Also BELJAWSKI, *A.N.* 4291.

§ L. BOSS, in *A.J.*, 614, p. 122; 623-624, pp. 193, 194.

¶ *M.N.*, LXXII, 170 ff.

|| L. BOSS in *A. J.* 623-624, p. 194.

** CAMPBELL, *L. O. B.* 196.

ON RING-MICROMETER OBSERVATIONS OF COMETS.

By WILLIAM DOBERCK.

The following observations were made at Kowloon, Sutton, Surrey in $51^{\circ}22'19''.8$ N., $0^{\text{h}}0^{\text{m}}44^{\text{s}}.53$ W.Gr., 167 feet above M.S.L., with the double ring micrometer (described in *V.J.* Vol. 48, p. 153) acquired by aid of a grant from the trustees of the ELIZABETH THOMPSON Science Fund. It is used on a $7\frac{1}{4}$ -in. COOKE refractor. The magnifying power is 53.

The diameters of the rings were measured by central transits of stars of different magnitudes, and also by pairs of stars of known differences of declination. The results agree, but the latter method appears to have been devised for use before equatorials were furnished with efficient driving clocks. PETERS's method is needed only in case the telescope is not equatorially mounted. There is an error introduced, as the star cannot be placed exactly in the center of the ring, but it amounts to only a very small fraction of a second of arc and is eliminated by neglecting refraction. The latter must otherwise be taken into account in the same way as it should be allowed for in case of the transit instrument if the wires are not observed symmetrically on both sides of the middle wire.

Corrections have been applied for refraction, proper motion of the comet, and curvature, when necessary, but not for retardation. It is on account of the latter that the diameters of the rings depend upon the magnitude of the star observed. The outer diameters increase and the inner decrease with the magnitude, but the mean diameter is constant. It is the same when observations are made in the daytime as at night. In consequence, declinations are not much affected by retardation.

The retardation of fixed stars observed with bar micrometers has been investigated by PIHL (on occulting micrometers, Christiania, 1893) and his results are confirmed here, although his telescope was much smaller than mine. As regards the comets observed here, the retardation was determined by comparing the time it took the comet to pass behind the ring with the time calculated from the diameters of the ring and the shortest distance of the comet from the center.

The correction for retardation to be added to the observed R.A. was found to amount to $\div 3^{\text{s}}.28$ in case of METCALF's comet (this comet may have been lost to sight before it reached the ring), $\div 1^{\text{s}}.21$ for KRITZINGER's comet, $\div 0^{\text{s}}.86$ for WESTPHAL's comet, and $-0^{\text{s}}.28$ for ZLATINSKY's comet. There was no correction in case of DELAVAN's comet.

METCALF's comet was seen as an extremely faint and ill defined nebula, several minutes in diameter, a little brighter near the center. The nucleus was about the 13th magnitude, but it could not be seen when close to a ring. There was much haze in the air and cirro-stratus.

WESTPHAL's comet, seen through fog on the 6th of October, was very faint.

KRITZINGER's comet was very diffused and no nucleus was seen, but the existence of a nucleus of about the 13th magnitude was suspected.

The observations of METCALF's comet on the 28th of September, and the 18th and 22d of October, and the observation of WESTPHAL's comet on the 6th of October, were made with the ELIZABETH THOMPSON wire micrometer described in A.N. 4343 and 4432.

Date	Sutton M.T	Δ R. A.	Δ Decl.	Cp.	App. R.A	$\log p \downarrow$	App. Decl.	$\log p \uparrow$	Red. l.	Ad. App.	*
Comet 1913 <i>b</i> (METCALF)											
Sept. 21	^{h m s} 9 40 2	^{m s} + 3 0.96	^{" "} + 2 11.1	5, 3	^{h m s} 4 13 1.49	0.178	^{° ' "} +75 55 49.6	0.331 <i>n</i>	^s +7.26	^{" "} + 1.4	1
26	11 20 35	- 5 56.71	- 4 35.0	4, 3	3 3 24.81	0.112	+77 20 33.1	0.283	+7.89	+ 7.1	2
28	8 15 28	- 4 4.10	- 7 23.8	5, 9	1 44 34.89	0.210	+77 22 42.3	9.100 <i>n</i>	+7.26	+13.1	3
Oct. 18	6 36 35	+ 0 1.08	- 5 30.9	6, 8	20 59 57.75	8.823	+26 23 31.9	0.568 <i>n</i>	+2.36	+19.6	4
22	6 18 8	+ 2 6.15		7, 1	20 53 18.43	8.741			+2.41		5
22	6 40 35		- 0 49.2	, 4			+17 25 8.5	0.689 <i>n</i>		+16.9	5
Comet 1913 <i>d</i> (WESTPHAL)											
Oct. 3	8 12 32	+ 3 6.36	- 4 14.6	6, 6	21 33 9.05	8.734	+ 3 11 50.4	0.815 <i>n</i>	+3.19	+15.7	6
3	8 12 32	- 2 46.73	- 8 55.6	6, 6	21 33 9.19	8.734	+ 3 11 51.5	0.815 <i>n</i>	+3.19	+15.7	7
3	8 40 0	- 3 22.02	-17 14.0	3, 3	21 33 6.37	7.932	+ 3 13 7.9	0.814 <i>n</i>	+3.19	+15.7	8
4	11 7 53	+ 1 0.74	- 1 37.0	8, 6	21 29 52.46	9.350 <i>n</i>	+ 4 9 18.2	0.813 <i>n</i>	+3.15	+15.7	9
6	9 19 0	- 0 26.23	+ 1 14.6	16, 8	21 24 27.12	8.937 <i>n</i>	+ 5 47 13.0	0.797 <i>n</i>	+3.08	+15.8	10

Date	Sutton M.T.	Δ R. A.	Δ Decl.	Cp.	App. R.A.	log $p\Delta$	App. Decl.	log $p\Delta$	Red. l.	Ad. App.	*
Comet 1914 <i>a</i> (KRITZINGER)											
May ¹⁹¹⁴ 18	^h 12 ^m 22 ^s 5	— 0 30.85	+ 1 21.9	6, 6	^h 19 ^m 37 ^s 53.00	9.543	+32 53 55.4	0.597 _n	+1.84	—16.3	11
21	11 47 10	— 1 13.88	+ 4 48.1	6, 3	19 51 52.84	9.598	+34 55 48.7	0.618 _n	+1.80	—15.7	12
25	11 52 42	— 3 9.33	— 3 10.5	8, 8	20 10 12.88	9.608	+37 19 10.1	0.585 _n	+1.76	—14.7	13
June 30	11 43 47	+ 1 28.03	—15 9.7	4, 4	22 11 46.81	9.644	+44 45 33.9	0.451 _n	+2.16	— 3.6	14
Comet 1914 <i>b</i> (ZLATINSKY)											
May 25	9 27 17	— 4 21.25	— 9 25.4	4, 4	6 39 33.15	9.651 _n	+37 27 19.5	0.938 _n	+0.69	+10.5	15
Comet 1913 <i>f</i> (DELANVAN)											
Sept. 24	7 8 14	— 1 21.79	— 8 29.4	8, 8	11 9 18.49	9.663 _n	+47 48 12.3	0.832 _n	+1.77	—13.0	16
25	7 0 32	— 8 59.56	+17 56.2	4, 4	11 16 46.00	9.674 _n	+47 25 33.2	0.821 _n	+1.74	—13.1	17
27	6 49 5	— 1 11.76	— 1 32.8	5, 5	11 31 27.08	9.681 _n	+46 35 24.7	0.810 _n	+1.67	—13.4	18
29	7 7 31	+10 39.63	+ 0 11.6	8, 8	11 45 55.18	9.655 _n	+45 37 55.8	0.832 _n	+1.68	—13.7	19
30	6 44 39	— 3 10.53	+ 1 12.8	8, 8	11 52 50.35	9.686 _n	+45 7 31.2	0.796 _n	+1.57	—13.8	20
Oct. 1	6 50 39	— 4 14.48	+12 6.0	8, 8	11 59 47.01	9.679 _n	+44 34 55.0	0.870 _n	+1.53	—13.9	21
8	6 39 54	—10 23.44	— 5 14.9	6, 4	12 44 34.47	9.666 _n	+40 13 0.8	0.790 _n	+1.41	—14.4	22

Mean Places of Comparison-Stars.

*	R. A. 1913.0	Decl. 1913.0	A. G. Cat.	*	R. A. 1914.0	Decl. 1914.0	A. G. Cat.
1	^h 4 ^m 9 ^s 53.29	+75 53 36.9	Kasan 664	11	^h 19 ^m 38 ^s 22.01	+32 52 49.8	Leiden 7578
2	3 9 13.57	+77 25 1.8	Kasan 473	12	19 53 4.93	+34 51 16.3	Leiden 7786
3	1 48 31.67	+77 29 52.9	Kasan 283	13	20 13 20.45	+37 22 35.3	Lund 9189
4	20 59 54.31	+26 28 43.2	Camb., E. 12062	14	22 10 16.62	+45 0 47.2	Bonn 16491
5	20 51 9.84	+17 25 40.8	Berl. A. 8480	15	6 43 53.71	+37 36 34.4	Lund 3534
6	21 29 59.51	+ 3 15 49.6	Albany 7544	16	11 10 38.51	+47 56 54.7	Bonn 7947
7	21 35 52.72	+ 3 20 31.2	Albany 7571	17	11 25 43.87	+47 7 50.0	Bonn 8075
8	21 36 25.19	+ 3 30 5.9	Albany 7576	18	11 32 37.17	+46 37 11.0	Bonn 8120
9	21 28 48.58	+ 4 10 38.8	Albany 7535	19	11 35 13.87	+45 37 57.9	Bonn 8147
10	21 24 50.27	+ 5 45 42.6	Leipzig II 10765	20	11 55 59.31	+45 6 32.2	Bonn 8299
				21	12 3 59.96	+44 23 2.9	Bonn 8359
				22	12 54 56.55	+40 18 28.9	Bonn 8745

POSITIONS OF HALLEY'S COMET.

Derived from photographs taken with the Astrographic telescope of the Observatorio Nacional Argentino, Cordoba.

1910 Greenwich M. T.		1910.0		log $p\Delta$	
		α	δ	α	δ
Apr. 18	21 ^h 49 ^m 31 ^s .7	23 ^h 52 ^m 31 ^s .16	+7° 47' 23".4	9.672 _n	0.690 _n
19	21 57 33.9	23 51 55.91	7 46 55.3	9.662 _n	0.694 _n
24	21 27 56.8	23 50 19.57	7 49 10.3	9.669 _n	0.691 _n
25	21 53 26.3	23 50 22.31	7 51 02.1	9.641 _n	0.702 _n
25	22 01 25.0	23 50 21.64	7 51 02.2	9.632 _n	0.704 _n
30	21 19 48.8	23 53 07.45	8 10 41.3	9.658 _n	0.698 _n
May 5	20 36 43.0	0 02 41.69	9 00 23.5	9.684 _n	0.687 _n
6	20 36 20.2	0 05 58.91	9 16 25.1	9.685 _n	0.688 _n
7	20 39 48.8	0 09 57.12	9 35 25.2	9.683 _n	0.690 _n
7	21 55 51.0	0 10 10.76	9 36 31.6	9.605 _n	0.721 _n
8	21 26 28.2	0 14 54.50	9 58 00.3	9.644 _n	0.711 _n

1910 Greenwich M. T.		1910.0		log $p \Delta$	
		α	δ	α	δ
May 11	21 ^h 01 ^m 32 ^s .5	0 ^h 36 ^m 33 ^s .47	+11° 35' 33".3	9.678 n	0.700 n
12	21 02 20.3	0 47 35.89	12 22 11.3	9.685 n	0.698 n
13	21 19 13.3	1 01 48.37	13 19 08.6	9.681 n	0.705 n
21	11 10 23.9	6 33 22.24	16 43 30.5	9.672	0.726 n
22	10 55 25.1	7 21 52.08	14 07 04.5	9.590	0.750 n
23	10 51 05.3	7 59 20.39	11 37 56.4	9.512	0.751 n
24	11 19 06.1	8 28 5.25	9 29 9.1	9.516	0.736 n
25	11 24 04.1	8 49 27.99	7 46 22.7	9.484	0.727 n
26	12 02 06.0	9 06 17.09	6 22 02.8	9.540	0.711 n
27	10 35 22.3	9 18 28.30	5 19 24.7	9.228	0.716 n
28	10 57 05.1	9 29 06.43	4 24 39.3	9.300	0.706 n
29	10 51 47.4	9 37 38.12	3 38 40.7	9.254	0.699 n
29	12 40 07.8	9 38 12.63	3 35 35.5	9.572	0.688 n
30	10 33 32.2	9 44 41.47	3 01 05.3	9.129	0.693 n
31	10 25 37.7	9 50 42.21	2 28 54.6	9.055	0.688 n
June 1	10 30 26.8	9 55 53.95	2 00 58.8	9.083	0.683 n
3	10 40 30.0	10 04 20.65	1 15 17.8	9.149	0.674 n
4	11 04 09.1	10 07 52.65	0 56 10.9	9.280	0.670 n
5	10 42 41.5	10 10 55.88	0 39 36.5	9.171	0.667 n
6	11 11 16.5	10 13 46.57	0 24 09.7	9.320	0.664 n
7	11 20 28.7	10 16 19.41	+0 10 18.1	9.362	0.661 n
8	10 59 36.8	10 18 36.20	-0 02 07.4	9.283	0.659 n
9	10 48 45.0	10 20 42.87	0 13 38.0	9.239	0.656 n
11	11 45 23.0	10 24 34.55	0 34 53.3	9.243	0.652 n
12	11 12 13.3	10 26 14.57	0 44 04.7	9.367	0.651 n
13	11 40 34.5	10 27 52.80	0 53 06.5	9.463	0.650 n
15	11 05 56.7	10 30 46.71	1 09 16.5	9.371	0.646 n
16	10 44 33.4	10 32 07.17	1 16 46.7	9.296	0.644 n
17	11 04 16.5	10 33 25.89	1 24 12.6	9.383	0.643 n
18	11 14 11.8	10 34 41.10	1 31 08.0	9.424	0.643 n
20	11 03 00.6	10 37 00.87	1 44 20.5	9.406	0.640 n
23	10 55 01.1	10 40 13.45	2 02 49.8	9.409	0.636 n
24	10 50 40.2	10 41 13.91	2 08 41.1	9.404	0.635 n
26	12 43 53.6	10 43 14.71	2 20 30.2	9.639	0.644 n
28	10 56 41.6	10 45 02.07	2 31 06.8	9.458	0.632 n
29	11 10 22.0	10 45 56.77	2 36 33.6	9.500	0.633 n
30	10 54 29.9	10 46 49.31	2 41 50.2	9.468	0.630 n
July 1	11 30 11.1	10 47 43.05	2 47 15.5	9.554	0.634 n
3	11 22 57.1	10 49 24.79	2 57 37.6	9.552	0.632 n
4	11 07 37.7	10 50 14.30	3 02 43.8	9.528	0.630 n
6	11 13 49.9	10 51 52.89	3 12 56.2	9.553	0.629 n
8	11 53 27.3	10 53 30.40	3 23 12.9	9.624	0.634 n
10	12 55 24.1	10 55 06.95	3 33 29.7	9.684	0.646 n
18	11 04 16.4	11 01 07.83	4 13 04.6	9.600	0.624 n
19	11 43 22.4	11 01 53.15	4 18 12.6	9.651	0.633 n
25	11 47 14.6	11 05 30.96	4 43 18.2	9.672	0.636 n
26	11 46 28.1	11 06 57.75	4 53 26.3	9.673	0.636 n
27	11 59 26.2	11 07 41.06	-4 58 33.2	9.683	0.639 n

NOTES

The negatives were secured by Messrs. WINTER and SYMONDS. The measurements of the plates were made by Mr. SYMONDS and the reductions by Dr. GLANCY.

Each position is the mean of two or three images of comet and stars on the same plate.

The comparison stars average about four per plate.

Cordoba, November 30, 1914.

OBSERVATIONS OF COMETS.

MADE WITH THE 26-INCH EQUATORIAL OF THE U. S. NAVAL OBSERVATORY,
[Communicated by Captain J. A. HOOGEWERFF, U. S. Navy, Superintendent.]

Date Wash. M.T.	*	Comp.	λa	$\lambda \delta$	App. α	App. δ	$\log p$ a	δ	Red. to App. Pl.	
Comet 1913 <i>f</i> (DELAN)										
1914 Mar. 3	8 ^h 34 ^m 57 ^s	1	25, 5	-3 25.30	+ 3 46.3	2 43 17.11	+ 4 9 1.5	9.631	0.725	+0.18 + 3.5*
14	7 46 20	2	30, 6	+1 48.90	+ 1 38.1	2 48 9.96	+ 6 11 1.6	9.624	0.714	+0.04 + 4.1*
20	7 27 9	3	30, 6	-1 1.58	+ 0 0.3	2 51 28.73	+ 7 18 31.3	9.626	0.708	+0.03 + 4.5*
24	7 29 43	4	30, 6	+2 18.60	+ 1 46.5	2 53 56.13	+ 8 4 4.2	9.639	0.710	-0.02 + 4.7*
Comet 1914 <i>a</i> (KRITZINGER)										
Mar. 30	14 47 8	5	30, 6	+1 9.02	- 1 11.8	16 15 22.88	- 8 53 7.8	9.055 <i>n</i>	0.810	+1.79 -16.8†
Apr. 3	14 46 45	6	30, 6	-1 39.43	+ 3 8.4	16 28 6.89	- 6 36 8.3	9.033 <i>n</i>	0.794	+1.85 -17.2†
17	15 19 27	7	30, 6	+1 36.20	- 7 58.9	17 18 48.11	+ 4 7 16.4	8.529 <i>n</i>	0.698	+1.97 -18.6†
21	12 50 29	8	30, 6	-1 10.42	- 3 12.8	17 34 39.13	+ 7 47 20.0	9.486 <i>n</i>	0.677	+1.95 -18.8†
May 17	12 53 11	9	30, 6	-1 25.62	- 0 52.3	19 34 16.48	+32 20 52.6	9.584 <i>n</i>	0.314	+1.82 - 8.7†
June 29	12 40 41	10	30, 6	-2 15.17	+ 7 27.8	22 10 14.26	+44 47 0.0	9.658 <i>n</i>	9.685	+2.12 - 3.8†
Comet 1914 <i>b</i> (ZLATINSKY)										
May 21	9 2 40	11	25, 5	-4 8.20	- 6 52.8	5 19 22.95	+46 23 34.1	9.780	0.768	+0.17 +11.7*
23	8 49 3	12	19, 4	+1 4.22	+ 2 54.9	6 4 21.61	+42 13 17.8	9.781	0.702	+0.45 +11.4*
26	8 54 59	13	30, 6	+1 17.32	+ 2 36.3	6 59 22.47	+34 3 58.8	9.741	0.675	+0.75 + 9.5*
31	8 53 16	14	30, 6	-0 54.05	- 3 45.0	7 58 19.98	+20 16 45.7	9.684	0.685	+0.94 + 4.7*
June 2	9 6 29	15	25, 5	+1 45.30	+ 1 0.3	8 13 59.95	+15 36 1.0	9.674	0.707	+0.93 + 3.0*

Observers: * H. E. BURTON, † C. B. WATTS.

Mean Places of Comparison-Stars for 1914.0.

*	α	δ	Authority	*	α	δ	Authority
1	2 46 42.53	+4 5 11.7	A.G. Albany 792	9	19 35 40.28	+32 21 53.6	A.G. Leiden 7531
2	2 46 21.02	+6 9 19.4	A.G. Leipzig II 1050	10	22 12 27.31	+44 39 36.0	A.G. Bonn 16527
3	2 52 30.28	+7 18 26.5	A.G. Leipzig II 1101	11	5 23 30.98	+46 30 15.2	A.G. Bonn 4473
4	2 51 37.55	+8 2 13.0	A.G. Leipzig II 1092	12	6 5 25.38	+42 10 11.5	A.G. Bonn 5062
5	16 14 12.07	-8 51 39.2	A.G. Wien-Ottakring 5660	13	6 58 4.40	+34 1 13.0	A.G. Leiden 2944
6	16 29 44.47	-6 38 59.5	A.G. Wien-Ottakring 5733	14	7 59 13.09	+20 20 26.0	A.G. Berlin B. 3240
7	17 17 9.94	+4 15 33.9	A.G. Albany 5744	15	8 12 13.72	+15 34 57.7	A.G. Berlin A. 3264
8	17 35 47.60	+7 50 51.6	A.G. Leipzig II 7972				

NOTES

1913 *f*

- Mar. 3. Comet faint. Could not see it in 5-inch finder. Moonlight. Seeing poor.
 Mar. 14. Comet very faint at times. Thin clouds and haze. Seeing poor.
 Mar. 20. Comet barely visible in 2-inch finder. Fairly bright in 26-inch telescope. Perhaps trace of tail. Seeing poor. Windy.
 Mar. 24. Comet visible in 5-inch finder. Perhaps barely visible in 2-inch finder. Fairly bright nucleus. Twilight at beginning. Seeing fair.

1914 *a*

- Mar. 30. Comet invisible in 5-inch finder. Faint nucleus. Apparently a short tail preceding. Probably some haze.
 Apr. 3. Faint nucleus. Seeing fair.
 Apr. 17. Seeing fair.

- Apr. 21. Comet visible in 5-inch finder. Faint nucleus and nebulosity. Slightly hazy.
 May 17. Faint nucleus and nebulosity. Seeing good.
 June 29. Comet faint at times. Faint nucleus and nebulosity.

1914 b

- May 20. Comet picked up with field glasses, and rough position obtained with the 26-inch. Apparently neither tail nor nucleus.
 May 21. Comet visible in 2-inch finder. Faint nucleus and nebulosity. Apparently no tail.
 May 23. Comet visible in 2-inch finder. Appearance same as on May 21. Comet and star very faint at times on account of clouds. Seeing fair.
 May 26. Comet visible in 2-inch finder. Faint nucleus surrounded by nebulosity. Seeing good.
 May 31. Comet visible in 5-inch finder. Appearance same as on May 26. Moonlight. Seeing good.
 June 2. Comet visible in 5-inch finder. Moonlight and haze. Seeing fair.

TRANSIT OF MERCURY.

OBSERVED AT THE OBSERVATORIO NACIONAL ARGENTINO, CORDOBA, 1914 NOVEMBER 6-7.

Observations of the Contacts by C. D. Perrine.

Ingress, external contact $21^h 57^m 36^s$ Greenwich M.T.
 internal contact $21^h 59^m 16^s$ Greenwich M.T.

Egress, internal contact $2^h 7^m 33^s$ Greenwich M.T.
 external contact $2^h 9^m 36^s$ Greenwich M.T..

The observations were made with the 12-inch equatorial, the aperture being reduced to $2\frac{1}{2}$ inches.

Ingress. The contacts at ingress were observed by projection on a screen. The images were very unsteady. The external contact is certainly 10^s late and may be as much as 20^s estimated from the diameter of the planet. The internal contact is thought to be subject to an uncertainty of not more than $\pm 5^s$.

Egress. At egress the contacts were observed directly with a power of about 125, with a colored glass over the eye-piece.

The image of the planet was generally round and sharp. The limb of the *Sun* was also well defined and sufficiently regular to permit of good observation. It is believed that the uncertainty of the times at egress are in error by little more than $\pm 2^s$.

The internal contacts are geometrical, the "black-drop" effect being scarcely noticeable.*

No halo was detected, and no trace of the planet was seen when off the solar disc.

Following is a comparison with the times as computed from the American Ephemeris in the sense observed minus ephemeris.

Contact	I	+29 ^s
	II	- 4
	III	-10
	IV	-19

Observations of the Contacts by M. L. Zimmer.

Egress, internal contact $2^h 7^m 19^s$ Greenwich M.T.
 external contact $2^h 9^m 5^s$ Greenwich M.T.

By comet seeker on roof of optical shop.

Black drop distinctly seen at 3d contact. Wind shaking telescope somewhat. Images good.

Observations of the Contacts by A. E. Glaney.

Egress, internal contact $2^h 06^m 59^s$ Greenwich M.T.
 external contact $2^h 09^m 8^s$ Greenwich M.T.

Geometrical contacts. The observations were made with the 12-inch finder.

Observations of the Contacts by E. Chaudet.

Egress, internal contact $9^h 50^m 37^s$ Cordoba M.T.
 external contact $52^m 20^s$ Cordoba M.T.

The observations were made with the 5-inch equatorial.

Cordoba, November 30, 1914.

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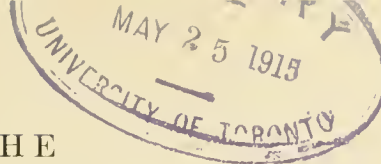
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NO. 5

OBSERVATIONS OF THE SATELLITES OF SATURN,

By E. E. BARNARD.

Besides the following measures of the nearer satellites of Saturn, three nights' measures of *Phoebe*, the ninth satellite, were obtained. They were printed in *A.N.* 4740. The present measures are a continuation of those published in *A. J.* 649-50, and previous numbers. They are in Central Standard Time (6^h 0^m slow of Greenwich Mean Time) and are not corrected for refraction.

Tethys AND *Mimas*.

1913-14	C. S. Time	P. A.	Dist.	Cmps.	P.A. of Wires	Remarks
Oct. 25	13 ^h 33 ^m 42 ^s	344°.66		5		Seeing bad.
	13 40 10		19".70	8	74°.9	
	13 45 13	343 .03		2		Stopped by clouds.

Tethys AND *Enceladus*

Oct. 14	15 24 7	272 .49		5		
	15 28 49		65 .76	5	2 .5	Stopped by clouds.
Nov. 11	11 23 29	275 .26		5		Faint in thick sky and bad seeing.
	11 28 23		60 .82	8	5 .1	
16	13 4 45	57 .96		5		
	13 8 0		44 .6	1	147 .4	Stopped by clouds.
22	12 34 12	320 .90		5		? if <i>Enceladus</i> .
	12 37 59		8 .47	8	50 .5	Faint in clouds. Seen only by glimpses.
Dec. 21	13 37 20	31 .96		5		
	13 42 43		31 .46	8	122 .0	

Dione AND *Mimas*

Dec. 16	9 25 40	278 .09		5		Mimas very faint.
	9 30 50		34 .49	8	7 .8	

Dione AND *Enceladus*.

Sept. 27	13 43 47	331 .79		5		
	13 48 0		10 .72	8	63 .1	
Oct. 30	13 21 35	318 .14		5		Seeing very bad.
	13 26 54		13 .70	8	47 .5	
Nov. 15	12 55 15	10 .62		5		Seeing very bad.
	12 59 41		16 .87	6	100 .6	
23	12 48 8	21 .92		5		Seeing very bad.
	12 52 54		34 .43	8	111 .3	
Dec. 7	11 41 18	359 .81		5		Faint and difficult.
	11 46 22		7 .39	8	90 .2	Seeing very bad.
13	11 27 34	296 .43		5		
	11 31 48		18 .93	8	26 .4	Seeing excessively bad.
14	10 21 11	201 .15		5		
	10 25 40		46 .83	9	110 .7	

Dione AND Enceladus. (Continued.)

1913-14	C. S. Time	P. A.	Dist.	Cmps.	P.A. of Wires	Remarks
Dec. 16	9 ^h 36 ^m 8 ^s	289°.19		5		
	9 40 42		32".12	8	18°.8	
Jan. 24	10 41 43	217°.70		5		Seeing very bad.
	10 47 25		48 .55	8	127°.0	Very difficult.

Dione AND Tethys.

Aug. 26	16 11 15	239°.28		5		
	16 16 15		8".25	8	149°.4	
Oct. 14	15 13 47	281°.82		5		
	15 18 5		19 .12	8	11°.5	
26	13 54 20	111°.28		5		Seeing excessively bad.
	13 59 8		10 .68	8	21°.1	
30	13 45 39	276°.61		5		Seeing very bad.
	13 51 14		90 .64	8	6°.5	
Nov. 15	13 14 17	64°.22		5		
	13 19 21		74 .33	8	154°.2	Seeing very bad.
22	11 56 49	203°.22		5		
	12 1 8		55 .50	8	113°.2	Clouds.
Dec. 7	11 11 30	293°.80		5		Seeing very bad.
	11 16 10		27 .98	8	23°.2	
14	10 30 26	232°.72		5		
	10 34 8		12 .86	8	142°.7	
20	13 27 8	107°.61		5		Seeing bad.
	13 31 18		18 .13	8	17°.8	
21	13 48 16	353°.03		5		
	13 52 46		32 .92	8	83°.0	
Feb. 8	9 57 15	64°.67		5		Seeing very bad.
	10 3 30		60 .01	9	154°.6	

Rhea AND Mimas.

Dec. 14	10 8 59	256°.46		5		<i>Mimas</i> very faint.
	10 14 58		60 .76	9	166°.7	

Rhea AND Enceladus.

Oct. 11	13 57 25	315°.71		5		Very difficult from bad seeing
	14 2 55		39 .36	9	46°.0	and clouds.
Nov. 16	13 38 21	280°.12		5		Clouds.
Feb. 8	9 36 36	111°.46		5		
	9 41 16		40 .61	8	21°.6	

Rhea AND Tethys.

Sept. 2	15 49 55	144°.22		5		
	15 54 58		61 .33	8	54°.1	
9	15 42 47	284°.77		5		Seeing very bad.
	15 47 52		77 .88	8	14°.5	
27	13 52 15	30°.18		5		
	13 57 0		52 .38	8	120°.1	
Oct. 11	13 43 21	281°.74		5		Seeing excessively bad.
	13 50 5		112 .69	8	11°.8	
Nov. 1	13 23 13	92°.76		5		
	13 26 28		44 .36	3	2°.9	Stopped by clouds.
11	11 4 41	60°.48		5		Seeing excessively bad.
	11 9 0		44 .63	8	150°.4	
23	12 26 23	87°.44		5		Excessively bad seeing.
	12 31 38		80 .23	8	177°.3	
Dec. 7	11 0 10	107°.20		5		Seeing excessively bad. Wind
	11 5 29		53 .84	8	17°.2	shaking telescope.
13	10 20 8	297°.49		5		
	10 24 1		27 .85	5	27°.4	

Rhea AND Tethys. (Continued.)

1913-1914	C.S. Time	P. A.	Dist.	Cmps.	P.A. of Wires	Remarks
Dec. 16	9 ^h 45 ^m 24 ^s	76°.95		5		
	9 50 15		59".43	8	166°.8	
	13 58 21	67.19		5		
21	14 4 21		17.42	8	157.0	
	9 33 23	91.2		1		Stopped by clouds.
	9 49 53	90.38		5		Faint in clouds.
30	9 56 33		87.60	8	0.4	
	10 49 38	54.90		5		Faint in clouds.
	10 56 58		66.40	8	145.0	
Jan. 17	10 18 20	220.69		5		
	10 24 22		27.66	8	130.0	
	9 42 54	244.51		5		Seeing very bad.
Feb. 7	9 48 32		73.11	8	154.6	

Rhea AND Dione.

Sept. 9	15 32 34	328.23		5		Seeing very bad.
	15 38 3		66.69	8	58.2	
Oct. 11	14 9 15	321.63		5		Through clouds.
	14 14 5		16.00	8	52.0	
14	15 3 21	78.20		5		
	15 8 30		142.69	8	168.0	
25	14 14 47	289.58		5		Through clouds.
	14 21 18		29.27	8	19.4	
26	13 40 14	251.16		5		Seeing excessively bad.
	13 45 21		81.36	8	161.1	
30	13 58 1	215.37		5		Seeing bad.
	14 3 50		53.06	8	125.5	
Nov. 1	12 35 31	101.51		5		Very faint in clouds.
	12 40 34		32.39	8	11.4	
11	11 14 2	21.47		5		Seeing bad.
	11 18 26		73.24	8	111.4	
15	12 33 37	85.00		5		Seeing fair.
	12 39 16		42.81	8	174.9	
22	11 47 19	258.75		5		Faint in clouds.
	11 52 2		43.08	8	168.5	
23	12 36 39	148.92		5		Seeing bad.
	12 42 4		50.17	10	58.3	
Dec. 13	11 19 15	37.80		5		
	11 23 26		29.16	8	127.4	
20	13 17 18	171.41		5		Seeing excessively bad.
	13 22 0		27.43	8	81.4	
30	9 0 36	73.97		5		Faint in clouds.
	9 6 34		139.94	8	163.8	
Jan. 24	10 29 55	269.06		5		Seeing very bad.
	10 35 15		67.63	8	179.0	
Feb. 1	10 3 20	324.75		5		Very faint and difficult from haze and bad seeing.
	10 7 38		59.35	7	54.4	
3	7 59 50	125.99		5		Seeing very bad.
	8 5 54		88.58	8	216.2	
7	10 9 38	252.41		5		Seeing very bad.
	10 14 52		64.10	8	162.6	
8	9 45 55	120.41		5		
	9 51 6		77.64	8	30.6	

Titan AND Tethys.

Nov. 15	13 25 38	239.90		5		Seeing fair.
	13 30 44		126.59	8	149.6	

Titan AND Dione.

1913-1914	C. S. Time	P. A.	Dist.	Cmps.	P. A. of Wires	Remarks
Sept. 25	14 ^h 18 ^m 53 ^s	293°.53		5		Seeing very bad.
	14 22 19		127".26	4	23°.1	Stopped by clouds.
Dec. 16	9 58 59	253°.88		5		Seeing very bad.
	10 4 56		140.28	8	163.8	
30	10 16 29	287°.09		5		Very faint in thick haze and
	10 22 34		155.02	8	16.8	bad seeing.
Feb. 7	10 21 31	102°.97		5		Seeing very bad.
	10 26 54		111.51	8	12.6	

Titan AND Rhea.

Sept. 9	15 54 16	270.77		5		Seeing very bad.
	15 59 28		130.10	8	0.51	
Oct. 11	14 19 1	276.57		5		Seeing poor.
	14 24 34		110.73	8	6.5	
30	14 8 49	246.14		5		Seeing bad.
	14 14 23		96.02	8	156.0	
Nov. 11	11 35 8	278.79		5		Seeing very bad.
	11 40 25		148.71	8	8.4	Faint in haze.
23	12 58 8	36.51		5		Seeing excessively bad.
	13 4 31		117.55	8	126.3	
Dec. 7	11 31 3	83.50		5		Seeing excessively bad.
	11 35 43		115.80	8	173.2	
13	11 36 32	281.84		5		Seeing excessively bad.
	11 40 50		115.00	8	11.4	
14	10 39 19	290.66		5		A 12½ mag. star n. p. <i>Titan</i> .
	10 43 42		113.94	8	20.7	
20	13 35 37	162.42		5		
	13 40 4		56.22	8	71.8	Seeing very bad.
21	14 10 15	131.79		5		Seeing very poor.
	14 15 48		118.17	8	42.0	Images breaking.
Feb. 8	10 19 1	89.35		5		Temperature — 12°.5 (F).
	10 24 31		107.37	8	179.4	

Titan AND Hyperion (?)

Nov. 23	13 39 14	265.40		5		Seeing very bad.
	13 44 57		122.84	9	175.3	

Titan AND Iapetus (?)

Nov. 23	13 16 27	153.19		5		Very bad seeing.
	13 21 38		111.57	8	63.3	
Dec. 14	10 50 50	90.61		5		
	10 56 4		191.67	8	0.7	

Tethys AND a Star (?)

Oct. 30	14 20 24	208.12		5		Star = 11½ mag.
	14 24 34		47.21	8	117.5	

Dione AND a Star (?)

Nov. 15	12 44 46	317.70		5		Star = 13 mag.
	12 49 56		27.43	8	47.6	Seeing excessively bad.

ESTIMATED MAGNITUDES OF THE SATELLITES.

1913-14	<i>Mimas</i>	<i>Enceladus</i>	<i>Tethys</i>	<i>Dione</i>	<i>Rhea</i>	Remarks
Aug. 26	.	13	12	12	10	
Sept. 2	.	13	12	12	12	
9	.		12½	13	12	

ESTIMATED MAGNITUDES OF THE SATELLITES. (Continued.)

1913-14	Mimas	Enceladus	Tethys	Dione	Rhea	Remarks
25	..	..	13	13	12	
27	..	14	12.2	13	12	
Oct. 11	..	13	12	12.1	12	
14	..	14	12.2	12.0	11½	
25	14	..	12½	12	12	
26	..	..	12½	12½	12	
30	..	13	12½	12	11½	
Nov. 1	..	14	12½	12½	12	
11	..	13	12½	12½	12	
15	..	13	12	12.2	11	
16	..	14	12.1	12.3	12	
22	..	14?	12.8	12.7	11.8	
23	..	13	12½	12½	11.9	{ <i>Iapetus</i> 10 ^m
Dec. 7	..	13	12½	12½	12	{ <i>Hyperion</i> 14.5
13	..	13	12½	12½	12	
14	14-15	13½	12¾	12	11.5	{ <i>Titan</i> orange
16	15	13	12.3	12	11.9	{ <i>Iapetus</i> 13 ^m
20	..	..	12½	12½	12	
21	..	13½	..	..	11½	
30	..	..	13	12.2	12	
Jan. 17	..	14	13	13	12	
24	..	13	12½	12¾	12	
Feb. 3	..	..	13	12½	12	
7	..	..	13	13	12	
8	..	13	12	12.2	11.8	

Yerkes Observatory, Williams Bay, Wisconsin, 1914, July 13.

OBSERVED MAXIMA AND MINIMA OF VARIABLE STARS,

By PAUL S. YENDELL.

103 *T Andromedæ*.

From 1902 July 30 to Sept. 23, I observed this star ten times. These observations indicate a maximum of 7^m.3 on 1902, Aug. 22. The star was 8^m.7 on the first date and 7^m.8 when last seen. During the present year it has been looked for on fourteen evenings, from Aug. 23 to Nov. 21, inclusive, but not seen. On the last occasion, the limit of vision was estimated at 11^m±.

906 *R Trianguli*.

From 1913, Sept. 5 to 1914, Feb. 2, I made twenty-seven observations of *R Trianguli*. The star was first certainly seen on 1913, Oct. 5, when it was 11^m.5± by eye-estimation. It increased steadily to a maximum of 7^m.4, which is indicated by the observations on 1913, Dec. 14. When last observed, it had decreased to 8^m.7. During the year 1914, I observed the star from Aug. 22 to Nov. 12, thirteen times. At the first observation it was already at or past the maximum, and its light was estimated at 7^m.0; no maximum is indicated by these observations, but a slow decrease to 8^m.7.

3493 *R Leonis*.

Eighteen observations of *R Leonis*, from 1912, Feb. 20 to June 8, show a maximum of about 6^m.2 on April 23. Its brightness at the first observation was 8^m.7, and at the last, 7^m.8.

4511 *T Ursæ Majoris*.

Fifteen observations of this star, from 1913, July 3 to Aug. 24, show a maximum of 7^m.7 on July 27; when first seen, the star's brightness was 8^m.9, and at the last observation, 8^m.0.

5601 *S Ursæ Minoris*.

Observations of *S Ursæ Minoris* were secured on seventeen occasions, from 1913, July 6, when it was estimated at 10^m.4, to Nov. 2, the last observed brightness being 9^m.7. A maximum of 8^m.5 is indicated on Sept. 21, by these observations.

6005 *R Draconis*.

Forty observations, from 1913, May 25 to Nov. 5. The star was first seen at 10th magnitude on the evening of June 2, from which time it increased rapidly in brightness until July 15, when it had risen to 7^m.6, at which

brightness it passed a maximum on July 24, then decreasing slowly until on Sept. 23 it had fallen to 8^m.7. On Nov. 5 it had disappeared, the limit of vision at the time being estimated as below 10^m.

6624 *T Serpentis*.

T Serpentis was looked for during the autumn of 1913, from Sept. 23 to Oct. 21, on eight evenings. It was seen only on the evenings of Sept. 28, 29 and 30, and Oct. 5. On all these occasions its brightness was estimated at about the 10th magnitude. During the current year the star has been watched for from July 19 until Oct. 23, on nine evenings in all, but not seen, the limit of visibility being estimated at from 10^m.5 to 12^m.

7085 *Rt Cygni*.

Fifteen observations of *Rt Cygni*, made in the year 1913, from July 1 to Nov. 2, show a minimum on July 20, of 10^m.7, and a maximum of 7^m.4 on Sept. 27. At the last observation, the star's brightness was estimated at 9^m.0.

Ta Cygni.

Seven observations of this star, on 1914, Sept. 19, indicate a maximum at 8^h 0^m, Local M.T.

7792 *SS Cygni*.

From 1914, June 12, as close a watch as possible has been kept on *SS Cygni*, no evening on which it was practicable for me to observe having been missed up to the time of writing; there has been much cloudy weather and interference by moonlight, but the observations number in all forty-five.

They are given below:

1914 June	12.427	normal
	16.417	not seen < 10 ^m
	17.417	not seen < 11 ^m
	20.345	not seen < 11 ^m .5
	22.424	normal
	25.434	not seen < 11 ^m

1914 July	3.414	8 ^m .47
	4.382	8 ^m .30
	18.417	seen — normal
	19.399	held — normal
	21.382	held — normal
	22.414	10 ^m .40
Aug.	1.373	not seen, moon bright
	22.372	7 ^m .66
	23.385	8 ^m .25
	24.375	8 ^m .25 hazy
	25.257	8 ^m .25
	26.340	8 ^m .44 moon 5 d.
	30.330	10 ^m .41 moon bright
Sept.	12.312	not seen < 10 ^m .9
	13.361	not seen < 10 ^m .9
	14.351	not seen < 10 ^m .9
	17.395	not seen < 10 ^m .9
	19.309	not seen < 10 ^m .9
	20.367	not seen < 10 ^m .9
	21.333	seen — normal
	22.339	seen — normal
	26.330	10 ^m .35 moon 1 qr.
	27.333	10 ^m .51 moon bright
	.407	9 ^m .90 moon bright
Oct.	1.312	8 ^m .32 moon bright
	2.306	8 ^m .32 moon bright
	3.299	8 ^m .32 moon bright
	3.299	8 ^m .40 moon bright
	5.306	9 ^m .21 moon bright
	6.337	10 ^m .41 moon bright
	7.302	10 ^m .92 moon bright
Nov.	15.319	normal
	22.344	not seen < 10 ^m .9
	12.330	not seen < 10 ^m .9
	14.315	11 ^m .3
	.433	11 ^m .5
	16.340	not seen < 10 ^m .9
	17.368	11 ^m .6
	21.267	normal

Time is Boston M.T., = Greenwich M.T. - 4^h 44^m.5.

Dorchester, Mass., U.S.A., 1914 December 10.

OBSERVATIONS OF THE COMPANION OF SIRIUS.

By E. E. BARNARD.

These measures are a continuation of those in *A.J.* 648 and earlier numbers.

Date	P. A.	Dist.	Remarks
1914.123 Feb. 14	79°.68	10".21	Faint. Seeing poor.
.137 19	79°.99	10".04	Seeing poor.
.151 24	77°.84	10".53	Very difficult.
.164 Mar. 1	78°.69	10".40	Very difficult.
.184 8	79°.18	10".06	Well seen, but images jumping badly.
1914.152	79°.08	10".25	

Yerkes Observatory, Williams Bay, Wis., 1914 July 13.

OBSERVATIONS OF THE SATELLITE OF NEPTUNE,

By E. E. BARNARD.

The following observations of the satellite of *Neptune* are a continuation of those printed in *A.J.* 649-650, and previous numbers. The observations are in Central Standard Time (6^h 0^m slow Greenwich Mean Time). They are not corrected for refraction.

The following observations of the satellite of <i>Neptune</i> are a continuation of those printed in <i>A.J.</i> 649-650, and previous numbers. The observations are in Central Standard Time (6 ^h 0 ^m slow Greenwich Mean Time). They are not corrected for refraction.						1913-14	C. S. Time	P. A.	Dist.	Cps.	Remarks
						Dec. 16	^h ^m ^s 12 0 4 12 5 11	[°] 218.43	["] 10.73	5 8	V. fnt. Only seen for mo- ments. See- ing exces. bad
						Jan. 24	12 11 51 12 21 12	333.99	 13.90	5 10	See'g v. bad. Meas. diffie.
1913-14	C. S. Time	P. A.	Dist.	Cps.	Remarks						
	^h ^m ^s	[°]	["]								
Oct. 26	14 18 28 14 24 2	99.90	 16.17	6 8	See'g excess. bad.	29	13 38 0 13 45 42	42.80	 11.28		See'g v. bad. V. high wind.
Nov. 11	15 43 57	180.12		5	Seen only momentarily.	31	12 13 47 12 19 58	281.53	 16.40	5 10	See'g v. bad. V. high wind.
	15 51 35		11.05	8	See'g excess. bad.	Feb. 3	11 11 20 11 16 28	102.27	 16.37	5 8	V. diffie. See- ing exces. bad.
15	14 25 53 14 31 12	303.91	 16.10	5 8	Seeing good.	24	11 7 28 11 13 36	261.18	.. 14.32	6 8	Fairly well seen, but see- ing poor.
22	13 33 40 13 42 6	254.79	 13.22	5 9	See'g v. bad. V. fnt. and dif. in clouds.	26	10 47 19 10 52 52 11 9 5 11 13 42	121.54 121.48	 16.75 .. 16.48	6 8 5 8	V. fnt. in very thick sky. In clearer sky.
	13 51 33 13 59 28	253.84	 13.25	6 8	Excess. bad seeing. Faint in clouds	Mar. 1	10 59 22 11 5 45	299.76	 16.55	6 8	See'g v. bad. Planet and sat. badly blurred.
23	14 11 31 14 16 17	164.77	 12.01	6 8	Faint and blurred. See'g excess. bad.	8	10 16 56 10 23 12	247.62 ..	 12.89	6 8	Fnt. from moon light and thick sky. Seeing fair
Dec. 7	13 1 28 13 11 58	52.94	 12.09	5 9	Seen with dif. See'g excess. bad.						

Yerkes Observatory, Williams Bay, Wisconsin, 1914, July 13.

OBSERVATIONS OF THE SATELLITES OF URANUS,

By E. E. BARNARD.

The low southern altitude of *Uranus*, and the poor atmospheric conditions, prevented any measures of the satellites but the few given here. The two inner ones were difficult and were only observed on one night. The observations are in Central Standard Time (6^h 0^m slow of Greenwich Mean Time) They are not corrected for refraction. These are a continuation of the measures printed in *A.J.*, 637, and previous numbers.

Uranus AND Ariel.

1913	C. S. Time	P. A.	Dist.	Cps.	Remarks
	^h ^m ^s	[°]	"		
July 27	10 59 5	326.91		5	Very faint.
	11 6 56		16.67	10	Seeing poor.

Uranus AND Umbriel.

1913	C. S. Time	P. A.	Dist.	Cps.	Remarks
	^h ^m ^s	[°]	"		
July 27	11 13 52	3.40		..	
	11 20 21		14.15	10	

Uranus AND Titania.

1913	C. S. Time	P. A.	Dist.	Cps.	Remarks
	^h ^m ^s	[°]	"		
June 10	14 54 3	358.80		5	Very faint in
	15 0 28		31.24	8	moonlight.
July 1	12 43 20	155.08		5	The disc of
	12 51 40		29.43	8	the planet is
6	13 34 59	354.77		6	elongated in
	13 41 46		31.77	9	the direction
8	13 2 30	67.36		6	of this sat-
	13 11 23		21.01	10	ellite.
29	12 5 50	209.67		6	Excess. diffie.
	12 10 27		26.52	8	fr. bad see'g.
Aug. 26	9 35 15	307.01		5	Definition gd.
	9 41 7		23.96	10	Satellite faint.

Uranus AND Oberon.						Uranus AND A STAR.					
1913	C. S. Time	P. A.	Dist.	Cps.	Remarks	1913	C. S. Time	P. A.	Dist.	Cps.	Remarks
	^h ^m ^s	[°]	"				^h ^m ^s	[°]	"		
June 10	15 6 42	159.14		5	Lost in dawn and bad see- ing.	July 1	13 17 51	333.78		5	10 mag.
	15 11 39		40.13	6			13 23 38		23.45	8	
July 1	12 58 54	353.03		5		Uranus and the following star of a small double.					
	13 7 36		42.54	10		1913 July 6 183°.09 (2) 115''.52 (2)					
6	13 16 46	137.39		6	Seeing excess. bad.	These give					
	13 24 9		34.27	9		$\Delta\alpha = -0^m 0^s.44$ $\Delta\delta = -1' 55''.3$ at 13 ^h 49 ^m C.S.T.					
8	13 20 14	177.85		6	Very bad see'g.	The following position of the star results:					
	13 28 22		42.79	9		1913.0 α 20 ^h 35 ^m 51 ^s .1 $\delta = 19^\circ 19'.7$					
29	11 55 14	11.14		5		The double itself.					
	12 0 5		40.90	8		1913.513 July 6 272°.08 (3) 4''.34 (6) 13 ^m .0-13 ^m .6					
Aug. 26	9 47 15	32.85		6	Seeing poor.	Yerkes Observatory, Williams Bay, Wisconsin, 1914, July 13.					
	9 53 12		35.80	10							

MELLISH'S COMET (1915 α),

On the Condensations or Secondary Comets of May 12.

By E. E. BARNARD.

On May 12 Mr. SULLIVAN and I examined MELLISH'S comet with the 40-inch telescope. The sky was misting over, with a storm approaching from the west. The comet was already dim to the eye from the dense haze.

There were two round condensations or secondary comets in a line with the head and preceding it. Calling the main comet *A*, these would be *B* and *C*, in the order of distance. *B* was very faint and nearer *C*. *C* was quite bright and was perhaps 1½ or 2 times less bright than *A*. Except for brightness it resembled *A* very much and was a little smaller. They were all involved in the nebosity of *A*, and were in a brighter region, perhaps the axis of the tail. The following measures were made. Central Standard Time (6^h0^m slow of G.M.T.)

POSITION ANGLE AC

1915 May 12 ^d	13 ^h 30 ^m 28 ^s	286°.11	(5)
	13 39 13	284°.32	(4)

DISTANCE AC

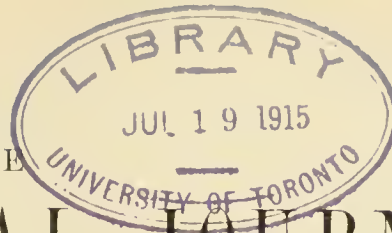
1915 May 12 ^d	13 ^h 34 ^m 39 ^s	26''.53	(4)
	13 36 30	27''.84	(4)
	13 41 29	28''.62	(3)

Hastening to the Bruce Observatory, I made an exposure of ten minutes on the comet with the 6-inch and 10-inch telescopes. The comet was disappearing in the haze when the exposure began and was completely blotted out at the end of the ten minutes. The sky remained cloudy. The plates show *A* and *C*, but with insufficient exposure to produce a good picture.

Yerkes Observatory, Williams Bay, Wisconsin, May 13, 1915.

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DOUBLE-STAR MEASURES,

MADE WITH THE 10½-INCH EQUATORIAL OF THE UNIVERSITY OF MINNESOTA,

By F. P. LEAVENWORTH.

The reference number for each star is taken from BURNHAM'S *General Catalogue of Double Stars*. The right ascensions and declinations are for 1900.

156 β 777 α 0 ^h 16 ^m $\delta - 0^{\circ} 48'$ 1914.788 167°.6 4".24 8 ^m .7 10 ^m .5 .791 165°.8 3".94 8".8 11".0 .805 167°.2 4".27 8".8 11".0 1914.795 166°.7 4".15 8 ^m .8 10 ^m .8 Probably fixed.	Comparison with RABE'S ephemeris, A.N. 4735. $O - C \quad - 3^{\circ}.6 \quad - 0''.04$ 612 β 303 α 1 ^h 4 ^m $\delta + 23^{\circ} 15'$ 1914.920 285°.6 0''.59 7 ^m .0 7 ^m .2 .925 287°.1 0".68 7".0 7".6 1914.922 286°.4 0''.64 7 ^m .0 7 ^m .4 No change. 1015 Σ 186 α 1 ^h 51 ^m $\delta + 1^{\circ} 26'$ 1907.944 32°.8 0''.71 7 ^m .0 7 ^m .0 1914.920 37°.1 0''.95 925 38°.6 0".82 7 ^m .1 7 ^m .0 .928 39°.1 958 37°.4 0".99 1914.933 38°.0 0''.92 7 ^m .1 7 ^m .0 1427 Σ 305 α 2 ^h 42 ^m $\delta + 18^{\circ} 57'$ 1914.920 314°.9 3''.32 7 ^m .5 8 ^m .0 .958 316°.7 3".07 6".5 7".0 1914.939 315°.8 3''.20 7 ^m .0 7 ^m .5 3701 Σ 984 α 6 ^h 50 ^m $\delta + 32^{\circ} 35'$ 1913.288 161°.2 4''.88 291 162°.2 4".98 294 161°.0 4".99 1913.291 161°.5 4''.95 479 0Σ 20 α 0 ^h 49 ^m $\delta + 18^{\circ} 38'$ 1906.865 313°.0 0''.43 6 ^m .0 7 ^m .0 1914.813 302°.6 0".55 6".0 7".0 .958 304°.6 0".46 6".0 7".0 1914.886 303°.6 0''.50 6 ^m .0 7 ^m .0 Motion of 130° in 70 years. 482 Σ 73 α 0 ^h 50 ^m $\delta + 23^{\circ} 6'$ 1914.920 38°.3 0''.86 6 ^m .5 6 ^m .6 .925 38°.2 0".82 6".2 6".4 .928 39°.9 0".64 6".3 6".2 .958 38°.5 0".70 1914.933 38°.7 0''.76 6 ^m .3 6 ^m .4	4122 <i>Castor</i> α 7 ^h 29 ^m $\delta + 32^{\circ} 7'$ 1913.329 220°.2 5''.70 343 220°.1 5".54 362 220°.9 5".35 1913.345 220°.4 5''.53 A and C 1913.343 164°.4 73''.48 5388 γ <i>Leonis</i> α 10 ^h 14 ^m $\delta + 20^{\circ} 44'$ 1914.342 117°.7 3''.97 350 118°.1 3".98 2 ^m .5 4 ^m .0 .389 118°.8 3".90 2".5 5".5 1914.360 118°.2 3''.95 2 ^m .5 4 ^m .8 5765 Σ 1536 α 11 ^h 19 ^m $\delta + 11^{\circ} 6'$ 1914.350 42°.5 2''.06 4 ^m .0 7 ^m .0 .370 42°.7 1".98 375 41°.6 2".21 4".0 7".0 .389 41°.0 2".15 4".0 7".0 1914.371 42°.0 2''.10 4 ^m .0 7 ^m .0 5766 β 26 α 11 ^h 19 ^m $\delta - 9^{\circ} 52'$ 1907.328 66°.2 2''.90 8 ^m .0 10 ^m .5 14.350 68°.5 2".76 8".0 10".0 1910.839 67°.4 2''.83 8 ^m .0 10 ^m .2 No motion.
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6243 γ Virginis					6641 Σ 1785					A, B and C				
α 12 ^h 37 ^m $\delta - 0^\circ 54'$					α 13 ^h 45 ^m $\delta + 27^\circ 29'$					1912.480	259 ^o .4	100 ^{''} .19		10 ^m .5
1913.442	325 ^o .7	6 ^{''} .02			1912.447	330 ^o .3	1 ^{''} .17			12.485	259 ^o .1			10 ^m .5
.444	325 ^o .4	5 ^{''} .93			.466	329 ^o .3				13.403	258 ^o .6	99 ^{''} .89		
.447	325 ^o .1	6 ^{''} .00	3 ^m .5	3 ^m .5	.469	329 ^o .2	1 ^{''} .24	7 ^m .5	7 ^m .8	1912.790	259 ^o .0	100 ^{''} .04		10 ^m .5
.452	324 ^o .9	6 ^{''} .06	3 ^m .5	3 ^m .5	.480	329 ^o .6	1 ^{''} .21	7 ^m .5	7 ^m .8	7034 ξ Bootis				
1913.446	325 ^o .3	6 ^{''} .00	3 ^m .5	3 ^m .5	1912.465	329 ^o .6	1 ^{''} .21	7 ^m .5	7 ^m .8	α 14 ^h 47 ^m $\delta + 19^\circ 31'$				
					Near minimum distance.					1912.437	126 ^o .6	2 ^{''} .30	6 ^m .0	7 ^m .5
1914.397	326 ^o .2	5 ^{''} .98	3 ^m .7	3 ^m .5	6780 Σ 1819					.470	126 ^o .0	2 ^{''} .09	5 ^m .0	7 ^m .0
.402	325 ^o .5	6 ^{''} .11	3 ^m .6	3 ^m .5	α 14 ^h 10 ^m $\delta + 3^\circ 36'$					.480	124 ^o .0	2 ^{''} .10	5 ^m .0	7 ^m .5
.408	325 ^o .4	6 ^{''} .10			1913.442	347 ^o .1	1 ^{''} .08			.486	126 ^o .1	2 ^{''} .20	5 ^m .0	8 ^m .0
1914.402	325 ^o .7	6 ^{''} .06	3 ^m .6	3 ^m .5	.444	346 ^o .7	1 ^{''} .11	8 ^m .1	8 ^m .0	1912.468	125 ^o .7	2 ^{''} .17	5 ^m .2	7 ^m .5
6312 0Σ 256					.447	346 ^o .0	1 ^{''} .19	8 ^m .0	8 ^m .1	1913.444	118 ^o .0	2 ^{''} .33	5 ^m .0	7 ^m .0
α 12 ^h 51 ^m $\delta - 0^\circ 25'$					.452	347 ^o .9	1 ^{''} .15	8 ^m .0	7 ^m .9	.447	118 ^o .4	2 ^{''} .27		
1914.350	79 ^o .3	0 ^{''} .75	7 ^m .2	7 ^m .0	1913.446	346 ^o .9	1 ^{''} .13	8 ^m .0	8 ^m .0	.452	119 ^o .0	2 ^{''} .27	5 ^m .0	7 ^m .5
.370	81 ^o .1	0 ^{''} .71	7 ^m .2	7 ^m .0	1914.370	344 ^o .9	1 ^{''} .34	8 ^m .1	8 ^m .0	1913.448	118 ^o .5	2 ^{''} .29	5 ^m .0	7 ^m .2
1914.360	80 ^o .2	0 ^{''} .73	7 ^m .2	7 ^m .0	.408	346 ^o .8	1 ^{''} .16	8 ^m .4	8 ^m .0	1914.438	111 ^o .0	2 ^{''} .04	4 ^m .0	7 ^m .0
6367 β 929					.454	348 ^o .6	1 ^{''} .23	8 ^m .0	8 ^m .1	.454	110 ^o .0	2 ^{''} .04	4 ^m .0	7 ^m .0
α 12 ^h 59 ^m $\delta - 3^\circ 7'$					1914.411	346 ^o .8	1 ^{''} .24	8 ^m .2	8 ^m .0	1914.446	110 ^o .5	2 ^{''} .04	4 ^m .0	7 ^m .0
1911.410	219 ^o .4	0 ^{''} .63	6 ^m .0	6 ^m .7	6811 β 116					7049 0Σ 288				
.487	212 ^o .5	0 ^{''} .58	6 ^m .0	6 ^m .5	α 14 ^h 14 ^m $\delta - 13^\circ 14'$					α 14 ^h 49 ^m $\delta + 16^\circ 7'$				
1911.448	216 ^o .0	0 ^{''} .60	6 ^m .0	6 ^m .6	1910.476	274 ^o .9	3 ^{''} .11	8 ^m .0	8 ^m .1	1912.437	187 ^o .6	1 ^{''} .58	6 ^m .0	6 ^m .7
					11.426	276 ^o .2	3 ^{''} .30	7 ^m .8	8 ^m .2	.470	187 ^o .1	1 ^{''} .43	7 ^m .0	7 ^m .8
1913.324	210 ^o .3	0 ^{''} .70	6 ^m .0	6 ^m .3	13.403	276 ^o .7	3 ^{''} .24	8 ^m .3	9 ^m .0	.480	187 ^o .3	1 ^{''} .68	7 ^m .0	8 ^m .0
14.350	212 ^o .3	0 ^{''} .67	6 ^m .0	6 ^m .4	13.452	275 ^o .5	3 ^{''} .26	8 ^m .0	8 ^m .4	1912.462	187 ^o .3	1 ^{''} .56	6 ^m .7	7 ^m .5
14.370	212 ^o .1	0 ^{''} .55	6 ^m .3	6 ^m .6	1912.189	275 ^o .8	3 ^{''} .23	8 ^m .0	8 ^m .4	1914.408	187 ^o .5	1 ^{''} .81	7 ^m .7	8 ^m .5
1914.015	211 ^o .6	0 ^{''} .64	6 ^m .1	6 ^m .4	Change doubtful.					.416	185 ^o .9	1 ^{''} .72	7 ^m .2	7 ^m .8
Slow retrograde motion.					6955 Σ 1865					.454	187 ^o .8	1 ^{''} .66	7 ^m .0	8 ^m .0
					α 14 ^h 36 ^m $\delta + 14^\circ 10'$					1914.426	187 ^o .1	1 ^{''} .73	7 ^m .3	8 ^m .1
					A and B					7132 Σ 3090				
6406 Σ 1728					1912.480	141 ^o .9	0 ^{''} .57	3 ^m .8	3 ^m .5	α 15 ^h 4 ^m $\delta - 0^\circ 36'$				
α 13 ^h 5 ^m $\delta + 18^\circ 4'$					.485	142 ^o .9	0 ^{''} .63			1912.470	275 ^o .8	1 ^{''} .36	9 ^m .2	8 ^m .8
1914.350	188 ^o .0	0 ^{''} .57	5 ^m .0	5 ^m .2	.488	142 ^o .9	0 ^{''} .57	4 ^m .2	4 ^m .0	.491	278 ^o .1	1 ^{''} .21	8 ^m .3	8 ^m .5
.370	190 ^o .4	0 ^{''} .57	5 ^m .0	5 ^m .3	1912.484	142 ^o .5	0 ^{''} .59	4 ^m .0	3 ^m .8	.494	276 ^o .1	1 ^{''} .34	8 ^m .5	8 ^m .8
1914.360	189 ^o .2	0 ^{''} .57	5 ^m .0	5 ^m .2	1913.403	141 ^o .2	0 ^{''} .69	4 ^m .3	4 ^m .0	1912.485	276 ^o .7	1 ^{''} .30	8 ^m .7	8 ^m .7
6530 Σ 1757					.411	143 ^o .1	0 ^{''} .63	4 ^m .0	4 ^m .0	Very slow decrease in distance probable.				
α 13 ^h 29 ^m $\delta + 0^\circ 12'$					.428	141 ^o .8	0 ^{''} .68			7214 Σ 1932				
1914.370	85 ^o .7	2 ^{''} .72	8 ^m .0	8 ^m .5	.477	143 ^o .5	0 ^{''} .74	4 ^m .3	4 ^m .0	α 15 ^h 14 ^m $\delta + 27^\circ 12'$				
.386	85 ^o .1		8 ^m .0	8 ^m .5	1913.430	142 ^o .4	0 ^{''} .68	4 ^m .2	4 ^m .0	1914.416	358 ^o .4	0 ^{''} .59	6 ^m .5	6 ^m .8
.397	85 ^o .4	3 ^{''} .05			1914.370	139 ^o .7	0 ^{''} .67	4 ^m .2	4 ^m .0	.454	362 ^o .3	0 ^{''} .63	6 ^m .5	6 ^m .6
.454	87 ^o .8	2 ^{''} .68	8 ^m .0	8 ^m .3	.408	141 ^o .3	0 ^{''} .85			1914.435	360 ^o .4	0 ^{''} .61	6 ^m .5	6 ^m .7
1914.402	86 ^o .0	2 ^{''} .82	8 ^m .0	8 ^m .4	.416	140 ^o .0	0 ^{''} .69	4 ^m .1	4 ^m .0					
					1914.398	140 ^o .3	0 ^{''} .74	4 ^m .2	4 ^m .0					

7251 η <i>Corona Borealis</i>					7717 ζ <i>Herculis</i>					10281 DA 1				
α 15 ^h 19 ^m δ + 30° 39'					α 16 ^h 38 ^m δ + 31° 47'					α 20 ^h 26 ^m δ + 10° 55'				
1912.489 42°.1 1''.02 6 ^m .1 6 ^m .1					1913.463 124°.9 1''.22 3 ^m .0 6 ^m .0					A and B, C				
.491 42°.3 0''.92 6°.0 6°.2					13.474 121°.8 1''.09 3°.0 6°.0					1914.788 255°.3 15''.68				
1912.490 42°.2 0''.97 6 ^m .0 6 ^m .2					13.477 121°.8 1''.27 3°.0 7°.0					.791 256°.6 15''.97				
					14.791 117°.0 1''.10 3°.0 7°.0					.813 256°.6 16''.35 6 ^m .5 7 ^m .0				
1913.439 46°.6 0''.84 6 ^m .0 6 ^m .0					1913.801 121°.4 1''.17 3 ^m .0 6 ^m .5					1914.797 256°.2 16''.00 6 ^m .5 7 ^m .0				
.455 48°.8 0''.88														
.458 45°.7 0''.96														
1913.451 47°.0 0''.89 6 ^m .0 6 ^m .0					8303 τ <i>Ophiuchi</i>					B and C				
					α 17 ^h 58 ^m δ - 8° 11'					1914.788 207°.6 0''.43 7 ^m .0 7 ^m .3				
7340 β 122					1913.554 261°.0 1''.95 6 ^m .0 7 ^m .0					.791 217°.6 0''.44				
α 15 ^h 34 ^m δ - 19° 27'					.576 260°.1 1''.98					.813 220°.0 0''.42				
1914.416 31°.4 1''.94 7 ^m .0 7 ^m .2					.584 261°.3 2''.06					1914.797 215°.1 0''.43 7 ^m .0 7 ^m .3				
.454 32°.1 1''.92 7°.0 7°.3					1913.571 260°.8 2''.00 6 ^m .0 7 ^m .0									
1914.435 31°.8 1''.93 7 ^m .0 7 ^m .2														
Possibly slow increase in angle.					9319 Σ 2525					10533 0 Σ 413				
7487 ξ <i>Scorpii</i>					α 19 ^h 23 ^m δ + 27° 7'					α 20 ^h 44 ^m δ + 36° 7'				
α 15 ^h 59 ^m δ - 11° 6'					1909.733 313°.0 0''.75 8 ^m .0 8 ^m .1					1914.788 52°.0 0''.52 5 ^m .0 7 ^m .0				
A and B					09.747 308°.8 0''.96 8°.0 8°.2					.791 55°.1 0''.58 5°.0 7°.0				
1912.491 339°.6 0''.66 5 ^m .0 5 ^m .5					09.758 312°.8 0''.86					.813 50°.5 0''.63 5°.0 7°.0				
13.463 344°.0 0''.72					14.791 309°.5 0''.80 8°.0 8°.2					1914.797 52°.5 0''.58 5 ^m .0 7 ^m .0				
13.472 342°.9 0''.65					1911.007 311°.0 0''.84 8 ^m .0 8 ^m .2									
13.474 341°.7 0''.70 5°.0 5°.7					Comparison with DOBERCK'S					10538 β 66				
1913.227 342°.0 0''.68 5 ^m .0 5 ^m .6					ephemeris. A.N. 4515					α 20 ^h 44 ^m δ + 27° 5'				
					$O-C$ + 3°.0 + 0''.17					1911.599 161°.1 8 ^m .0 8 ^m .5				
A, B and C					9634 Σ 2583					13.715 162°.4 1''.01 9°.0 9°.5				
1912.491 64°.4 7''.39 8 ^m .0					α 19 ^h 44 ^m δ + 11° 34'					1912.657 161°.8 1''.01 8 ^m .5 9 ^m .0				
13.463 63°.7 7''.33					1913.699 113°.1 1''.39 6 ^m .0 6 ^m .5					1914.791 162°.6 1''.03 8 ^m .5 9 ^m .2				
13.474 64°.9 7''.23 8°.0					13.702 114°.5 1''.50 6°.0 6°.4					.813 163°.4 1''.19 8°.3 8°.8				
1913.143 64°.3 7''.32 8 ^m .0					13.715 116°.7 1''.34 6°.0 6°.5					1914.802 163°.0 1''.11 8 ^m .4 9 ^m .0				
					14.791 114°.6 1''.38 6°.5 6°.9					There appears to be a slow decrease				
					1913.977 114°.7 1''.40 6 ^m .1 6 ^m .6					in distance.				
7617 β 950					9755 A. CLARK 12					10732 61 <i>Cygni</i>				
α 16 ^h 20 ^m δ - 9° 38'					α 19 ^h 53 ^m δ - 2° 30'					α 21 ^h 2 ^m δ + 38° 13'				
1911.487 355°.4 0''.81					1914.788 316°.4 0''.94 8 ^m .0 8 ^m .7					1914.928 129°.0 23''.50 5 ^m .0 5 ^m .7				
12.491 351°.5 0''.88 8 ^m .0 9 ^m .5					.791 318°.2 1''.00 8°.0 9°.0					.997 130°.0 23''.64				
1911.989 353°.4 0''.84 8 ^m .0 9 ^m .5					1914.790 317°.3 0''.97 8 ^m .0 8 ^m .8					1914.962 129°.5 23''.57 5 ^m .0 5 ^m .7				
7649 λ <i>Ophiuchi</i>					Slow retrograde motion.					11125 β 1212				
α 16 ^h 26 ^m δ + 2° 12'					9908 β 428					α 21 ^h 34 ^m δ - 0° 31'				
1913.463 71°.1 1''.02 4 ^m .0 5 ^m .5					α 20 ^h 2 ^m δ + 12° 39'					A and B				
.472 71°.1 1''.05 4°.0 5°.8					1914.788 354°.5 0''.59 8 ^m .0 9 ^m .5					1914.786 300°.4 0''.38 6 ^m .5 7 ^m .0				
.554 69°.8 0''.90					.791 352°.6 8°.0 8°.5					.788 298°.7 0''.42 6°.0 6°.5				
.559 71°.0 0''.94					1914.790 353°.6 0''.59 8 ^m .0 9 ^m .0					.791 295°.2 0''.54 6°.5 7°.0				
.584 71°.3 0''.98										1914.788 298°.1 0''.45 6 ^m .3 6 ^m .8				
1913.526 70°.9 0''.98 4 ^m .0 5 ^m .6					Distance diminishing quite rapidly.					Slow increase in angle.				
					No motion.									

A, B and C					11743 ζ Aquarii					A and C				
1914.786	146 ^o .7	42".19		11 ^m .0	α 22 ^h 24 ^m	$\delta - 0^{\circ} 32'$				1914.920	318 ^o .5	79".76	..	10 ^m .3
.788	145 .5	41 .43		11 .5	1913.712	309 ^o .8	2".99	4 ^m .0	4 ^m .2	.925	318 .8	79 .80		10 .5
.791	146 .1	41 .63		11 .0	.715	311 .3	2 .91	4 .0	4 .3					
1914.788	146 ^o .1	41".75		11 ^m .2	1913.714	310 ^o .6	2".95	4 ^m .0	4 ^m .2	1914.922	318 ^o .6	79".78		10 ^m .4
11590 Σ 2878					12601 Σ 3042									
α 22 ^h 10 ^m	$\delta + 7^{\circ} 29'$				α 23 ^h 47 ^m	$\delta + 37^{\circ} 21'$				1914.838	86 ^o .1	4".91	7 ^m .0	7 ^m .1
A and B					.958	307 .1	2 .76			.909	85 .3	4 .85		
1913.712	126 ^o .9	1".22	7 ^m .2	8 ^m .2	.980	307 .5	2 .64							
13.715	125 .5	1 .25	8 .0	9 .5	.994	309 .6	2 .88			1914.874	85 ^o .7	4".88	7 ^m .0	7 ^m .1
14.797	128 .7	1 .20	7 .5	9 .0	.997	307 .8	2 .77							
14.813	127 .1	1 .40	6 .8	8 .0	1914.971	308 ^o .1	2".78	4 ^m .0	4 ^m .0	12675 Σ 3050				
1914.260	127 ^o .0	1".27	7 ^m .4	8 ^m .6	12008 Ho 482					α 23 ^h 54 ^m	$\delta + 33^{\circ} 11'$			
Apparently fixed.					α 22 ^h 47 ^m	$\delta + 25^{\circ} 51'$				1914.827	221 ^o .0	1".99	6 ^m .0	6 ^m .4
A and C					A, B, and C					.830	220 .8	2 .20	6 .0	6 .3
1914.797	122 ^o .7	65".72	...	9 ^m .5	1914.786	198 ^o .1	51".11	7 ^m .0	9 ^m .2	.838	222 .0			
.813	123 .3	65 .64	...	9 .5	.788	198 .5	51 .02	6 .0	9 .5	.909	222 .3	1 .96	6 .0	6 .4
1914.805	123 ^o .0	65".68		9 ^m .5	1914.787	198 ^o .3	51".06	6 ^m .5	9 ^m .4	1914.851	221 ^o .5	2".05	6 ^m .0	6 ^m .4
11592 Σ 2877					12046 β 178					12709 β 281				
α 22 ^h 10 ^m	$\delta + 16^{\circ} 42'$				α 22 ^h 51 ^m	$\delta - 5^{\circ} 32'$				α 23 ^h 58 ^m	$\delta + 1^{\circ} 35'$			
1914.788	5 ^o .9	13".06	6 ^m .0	9 ^m .5	1914.786	334 ^o .6	0".47	6 ^m .0	7 ^m .5	A and B				
.791	6 .0	12 .75	6 .0	8 .5	.788	335 .7	0 .44			1914.791	195 ^o .1	1".23	8 ^m .0	9 ^m .5
1914.790	6 ^o .0	12".90	6 ^m .0	9 ^m .0	1914.787	335 ^o .2	0".46	6 ^m .0	7 ^m .5	.802	195 .5	1 .06	7 .5	10 .0
11596 H δ 170					12332 Σ 3007					.813	189 .8	1 .02	8 .0	10 .5
α 22 ^h 10 ^m	$\delta + 16^{\circ} 44'$				α 23 ^h 18 ^m	$\delta + 20^{\circ} 0'$				1914.802	193 ^o .5	1".10	7 ^m .8	10 ^m .0
1914.788	242 ^o .3	7".44	10 ^m .2	10 ^m .5	A and B					A and C				
.791	242 .2	7 .37	10 .3	10 .7	1914.920	86 ^o .4	6".03	7 ^m .0	10 ^m .6	1914.791	333 ^o .6	34".29	...	12 ^m .0
1914.790	242 ^o .2	7".40	10 ^m .2	10 ^m .6	.925	85 .7	6 .28	7 .0	10 .5	.802	333 .2	35 .05	...	11 .5
Proper motion.					1914.922	86 ^o .0	6".16	7 ^m .0	10 ^m .5	.813	332 .1	34 .81	...	12 .0
										1914.802	333 ^o .0	34".72	...	11 ^m .8

OBSERVATIONS OF THE TRANSIT OF *MERCURY*, 1914, NOV. 6.

By DAVID TODD.

Transit of Mercury (Egress)

Contact III 21^h 17^m 37^s ± 3 Wt. 2 Chronom. W.
-18.3 = ΔW

21 17 18.7 Amherst M. T.

Bisection (dichotomy) of disk (estimated)

21^h 18^m 40^s Wt. 1
-18

21 18 22 Amherst M. T.

Contact IV 21^h 19^m 45^s Wt. 1

-18

21 19 27 Amherst M. T.

Atmosphere quite unsteady. Power 200.

Latitude of Observatory $+12^{\circ} 21' 56''$, long. $1^{\circ} 50' 54''$ W

Amherst College Observatory, 1914, November 8

SUNSPOT OBSERVATIONS.

MADE AT BERWYN, PENN., WITH A $4\frac{1}{2}$ -INCH REFRACTOR,

By A. W. QUIMBY.

1915	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.	1915	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.	1915	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.	
Jan. 1	8	1	3	5	1	fair	Mar. 10	8	—	5	9	3	fair	May 6	7	1	6	22	3	fair	
2	8	—	2	9	1	fair	11	8	—	3	14	3	fair	7	7	—	6	23	3	fair	
3	8	—	2	24	3	fair	12	8	—	2	8	3	fair	8	7	—	4	5	3	poor	
4	8	—	2	20	3	fair	13	7	—	2	4	2	fair	9	7	—	4	1	3	fair	
5	8	—	2	19	3	fair	14	7	1	2	8	2	fair	10	7	—	1	1	3	fair	
6	11	—	2	14	2	fair	15	7	—	2	8	2	fair	12	11	—	—	—	—	poor	
7	3	—	1	14	1	fair	16	7	—	2	9	2	fair	13	9	—	—	—	—	fair	
8	8	—	1	14	1	fair	17	5	1	3	14	3	fair	14	7	—	—	—	1	fair	
9	9	—	1	14	—	fair	18	7	—	3	20	3	fair	15	7	—	—	—	—	fair	
10	8	—	1	10	1	fair	19	7	1	3	10	2	fair	16	6	—	—	—	—	poor	
11	9	—	1	3	1	poor	20	7	—	2	8	2	fair	17	6	—	—	—	1	fair	
13	9	2	2	4	2	fair	21	7	—	2	7	2	poor	18	6	—	—	—	—	1	fair
14	8	1	3	18	2	fair	22	7	—	1	3	2	poor	19	6	—	—	—	—	0	fair
15	8	1	4	9	2	fair	23	7	1	1	2	2	poor	20	6	—	—	—	—	—	fair
16	8	—	4	9	2	fair	24	7	—	1	1	2	fair	22	6	2	2	10	1	fair	
19	10	—	2	3	—	poor	25	7	—	1	2	1	fair	23	6	—	2	16	1	fair	
20	10	—	1	2	—	poor	26	7	—	1	2	1	fair	24	10	1	3	6	1	poor	
21	8	—	1	2	—	poor	27	7	—	1	1	1	fair	25	6	—	3	13	1	fair	
22	9	—	1	2	1	fair	28	7	—	1	1	—	fair	26	6	—	3	8	—	poor	
25	1	—	—	—	1	fair	29	7	1	2	2	2	fair	27	6	—	3	10	—	fair	
26	8	—	—	—	2	fair	30	7	1	3	11	1	fair	28	6	1	4	14	2	fair	
28	11	2	2	7	2	fair	31	7	—	3	11	1	fair	29	11	—	2	5	—	poor	
29	8	—	—	—	—	fair	Apr. 1	5	1	3	26	4	good	30	4	—	2	5	—	poor	
30	8	—	—	—	—	fair	2	7	—	3	26	4	good	31	6	—	2	4	2	fair	
Feb. 4	10	3	3	20	—	fair	4	4	1	4	56	1	good	June 1	6	—	2	3	2	fair	
6	8	1	4	30	2	fair	5	7	—	3	40	—	fair	3	12	—	—	—	—	v. p.	
7	2	—	3	13	—	poor	6	8	—	3	50	—	fair	4	7	—	—	—	2	fair	
8	8	2	5	31	2	fair	7	4	—	3	50	1	fair	5	6	1	1	3	1	fair	
9	8	1	6	32	2	fair	8	7	—	3	40	1	fair	6	9	1	1	4	—	fair	
10	8	—	6	40	3	fair	9	7	—	3	32	2	fair	7	12	1	1	5	—	poor	
11	8	—	6	32	4	fair	10	7	—	3	13	2	fair	8	11	1	1	4	—	fair	
12	4	—	5	26	3	fair	11	4	1	3	8	4	fair	9	6	1	1	1	1	fair	
13	8	—	3	20	2	fair	12	5	—	3	5	4	fair	10	6	1	—	—	2	fair	
14	9	—	2	10	2	fair	13	7	1	1	3	1	fair	11	6	1	—	—	1	fair	
15	10	—	2	2	—	poor	14	7	—	1	1	—	fair	12	6	1	—	—	1	fair	
16	11	—	2	2	—	poor	15	5	1	1	1	2	good	13	6	2	2	10	2	fair	
17	10	—	2	5	3	fair	16	7	—	—	—	—	good	14	6	—	2	24	2	fair	
18	8	—	1	1	2	fair	17	7	—	—	—	—	good	15	6	2	4	35	2	fair	
19	8	—	1	1	2	fair	18	7	1	1	1	2	good	16	6	1	5	59	2	fair	
20	8	—	—	—	1	fair	19	7	1	1	2	5	good	17	6	—	5	59	2	fair	
21	8	2	2	5	2	fair	20	7	—	2	6	1	good	18	6	—	5	62	2	fair	
22	9	—	2	9	2	fair	21	7	—	2	15	1	fair	19	6	2	7	68	2	fair	
23	9	1	3	10	1	fair	22	7	—	1	13	1	poor	20	6	—	6	82	3	fair	
25	10	1	3	10	2	fair	23	5	—	1	16	1	fair	21	6	—	6	76	4	fair	
26	8	1	3	10	1	fair	24	7	—	1	15	—	fair	22	5	—	6	77	3	fair	
27	10	1	4	10	1	fair	25	7	—	1	6	1	fair	23	6	—	6	74	3	fair	
28	8	1	5	13	2	fair	26	5	2	3	11	3	fair	24	5	1	5	56	3	fair	
Mar. 1	8	—	5	10	1	fair	27	7	—	3	16	3	fair	25	6	—	4	50	2	fair	
2	9	1	6	9	2	fair	28	7	—	3	16	3	fair	26	6	—	3	33	2	fair	
3	9	—	6	9	3	fair	29	5	—	3	16	2	fair	27	6	—	2	30	1	fair	
4	4	1	5	9	3	fair	30	5	3	6	23	3	fair	28	5	1	3	24	3	fair	
5	9	—	5	8	1	poor	May 1	5	—	4	21	1	fair	29	5	1	4	24	3	fair	
7	9	1	3	4	—	poor	2	5	—	4	43	2	fair	30	6	—	4	24	3	poor	
8	9	2	6	10	2	poor	3	7	—	4	52	2	fair								
9	4	—	5	12	3	fair	5	5	1	5	22	2	fair								

A GRAPHICAL METHOD FOR DETERMINING STELLAR PARALLAX FACTORS IN RIGHT ASCENSION.

By FRANK SCHLESINGER.

The parallax factor in any direction is the quantity by which it is necessary to multiply the annual parallax in order to obtain the shift in that direction at any desired time. The expression for the parallax factor in right ascension is

$$R \cdot \sin \odot \cdot \cos \alpha \cdot \cos \epsilon = R \cdot \sin \alpha \cdot \cos \odot$$

Here R is the radius vector of the *Earth* in its orbit, never differing from unity by more than 0.017; $\odot$ is the longitude of the *Sun* at the date of observation; α is the right ascension of the star, and ϵ is the inclination of the equator to the ecliptic. It will be noticed that the declination of the star does not enter this expression.

We readily see that the parallax factor is also equal to

$$R \cdot \cos Q,$$

Q being an arc joining the position of the *Sun* (at the date of observation) with a point on the equator whose right ascension is 90° greater than that of the star. The

factors may therefore be obtained graphically by means of two great circles on a sphere, inclined $23^\circ.5$ to each other. One of them is to be graduated for the longitude of the *Sun*; the other is reserved for the right ascension of the star, but these graduations must begin with 18 hours at the intersection of the two circles. By measuring Q with a flexible scale of cosines, graduated from $+1$ to -1 , we obtain the parallax factors directly, except for the small correction due to R .

Small hemispheres of ground glass, excellently adapted to this and many other purposes, may be obtained for a nominal sum at Harvard University. A hemisphere serves the present application as well as a sphere, if only we have due regard to the matter of signs. In general, for parallax observations in or near the meridian, those made before midnight have negative factors, and those made after midnight have positive factors.

*Allegheny Observatory of the University of Pittsburgh,
26 May, 1915.*

NOTE ON VAN MAANEN'S LIST OF STARS WITH MOTION EXCEEDING HALF A SECOND ANNUALLY.

By J. G. PORTER.

In the *Astrophysical Journal* for April there is given a list of stars with proper-motion exceeding $0''.50$ annually. A cursory review of this list has shown that it contains a number of stars which do not belong in it. Work on our new proper-motion catalogue is not yet far enough advanced to permit an exhaustive examination.

I give two lists, first of those stars which have quite small motion, and second of those whose motion falls only a little below the limit of half a second.

LIST I

No.		
7	<i>A. G. Chris.</i> 15	Motion very small.
63	<i>A. G. Chris.</i> 416	Motion only $0''.13$
69	<i>A. W.</i> 1436	Motion only $0''.16$
167	<i>A. G. Harv.</i> 3181	Motion small. See <i>A.J.</i> 398.
196	<i>A. G. Harv.</i> 3591	Motion very small. See <i>A.J.</i> 398.
226	<i>A. G. Chris.</i> 1806	Motion small.
364	<i>A. G. Chris.</i> 2517	Motion small.
381	<i>A. G. Chris.</i> 2661	Motion small.
389	<i>A. G. Ber. A.</i> 6369	Motion very small.
390	<i>A. G. Chris.</i> 2703	Motion only $0''.16$
441)	<i>Groom.</i> 3142	Motion very small. See Boss.
442)		

No.

480	<i>Lal.</i> 43065	Motion small. See Boss.
514	<i>A. G. Harv.</i> 8232	Motion small.
526	<i>Pi.</i> 23 ^b , 242	Motion about $0''.26$

LIST II

10	<i>A. W.</i> 120	Motion about $0''.45$
25	<i>Groom.</i> 145	0 .41
29	<i>Lal.</i> 1799	0 .44
55	<i>Lal.</i> 3922	0 .47
128	<i>Rü.</i> 1696	0 .43
154	<i>Lal.</i> 15394	0 .47
156	<i>A. W.</i> 6270	0 .45
201	<i>W.</i> 10 ^b , 366	0 .47
240	<i>Lal.</i> 22901	0 .43
280	<i>Pi.</i> 13 ^b , 89	0 .46
333	<i>Lal. Bo.</i> 2265	0 .47
409	<i>Lal.</i> 34986	0 .47
416	<i>A. W.</i> 15076	0 .46
446	<i>Mü.</i> 25146	0 .38
468	<i>Lal.</i> 41318	0 .44
492	<i>Kli.</i> 6256	0 .43

Cincinnati Observatory, May 10, 1915

PHOTOGRAPHIC POSITIONS OF COMETS.

BY WILLIAM O. BEAL.

The plates were taken at the observatory of the University of Minnesota with the 10½-inch equatorial. The photographs were made for position only, the exposure times varying from two to five minutes. The scale of the plates is 1^{mm}=1'. The measurements were made with the REPSOLD measuring machine. The star places were taken from the A. G. Zones. In the table the following designations are used:

B = WILLIAM O. BEAL*M* = CHARLES A. MANEY*D* = ALFRED DAVIS*P* = HAROLD PETERSON*Lv* = F. P. LEAVENWORTH

Minneapolis M.T. 1911				α 1911.0		δ 1911.0		$\log p \Delta$		No. of Exposures	No. of Comp. Stars	Computer	Photo. by
								α	δ				
Comet 1911 f (QUÉNISSET).													
1	Oct.	3 ^d	8 ^h 59 ^m 1 ^s	15 ^h 18 ^m 53 ^s .94	+54° 11' 17".1	9.849	0.625	4	4	B	Lv		
2	Oct.	4	8 46 33	15 21 8.22	52 15 40 .0	9.831	0.616	4	4	B and Lv	D		
3	Oct.	7	8 4 36	15 26 40.14	46 44 42 .2	9.781	0.586	4	4	B	Lv		
4	Oct.	10	8 26 1	15 30 59.83	+41 30 50 .7	9.744	0.681	1	3	B	Lv		
Comet 1912 a (GALE).													
1912				α 1912.0		δ 1912.0							
1	Oct.	4 ^d	6 ^h 43 ^m 53 ^s	15 ^h 26 ^m 59 ^s .89	-1° 40' 16".6	9.567	0.797	1	4	Lv	Lv		
2	Oct.	7	6 46 26	15 33 20.97	+1 56 19 .3	9.575	0.785	2	4	M	M		
3	Oct.	9	7 3 25	15 37 5.28	4 14 23 .5	9.593	0.779	2	4	P	P		
4	Oct.	13	6 45 49	15 43 27.97	8 31 26 .5	9.590	0.761	2	4	Lv	Lv		
5	Oct.	14	6 37 50	15 44 52.65	9 32 9 .0	9.587	0.755	2	4	M	M		
6	Oct.	15	6 25 5	15 46 12.40	10 31 21 .0	9.579	0.747	2	4	Lv	Lv		
7	Nov.	1	6 11 6	16 2 59.01	24 41 32 .8	9.641	0.686	4	4	P	Lv		
8	Nov.	4	6 10 5	16 5 25.01	26 48 41 .7	9.653	0.681	4	2	B	M		
9	Nov.	5	6 11 33	16 6 13.63	27 30 15 .0	9.658	0.681	5	3	B	Lv		
10	Nov.	8	6 27 37	16 8 40.99	+29 33 3 .9	9.676	0.698	3	4	B	P		

Minneapolis, Minn., Dec. 15, 1914.

OBSERVATIONS OF LONG PERIOD VARIABLES.

BY CAROLINE E. FURNESS.

The predicted maxima and minima in this table are computed from the elements in HARTWIG's Ephemeris published in the *Vierteljahrsschrift* for 1914.

Star	Dates Included	No. Obs.	Maximum or Minimum		O—C	Mg.	J. D. Max. or Min.
			Observed	Computed			
<i>Y Cephei</i>	1913, Jan. 9— June 2	8	Max. 1913, Mar. 21	1913, Apr. 12	- 22	9.5	2419848
<i>RV Cassiopeiæ</i>	1913, Oct. 3—1914, Mar. 20	8	Max. 1913, Dec. 1	1913, Oct. 14	+ 48	8.5	2420103
<i>U Andromedæ</i>	1913, Jan. 14— Mar. 5	5	Max. 1913, Feb. 16	1913, Apr. 22	- 65	9.4	2419815
<i>S Arietis</i>	1913, Oct. 22—1914, Mar. 20	9	Max. 1913, Nov. 10	1913, Nov. 14	- 4	10.4	2420082
<i>RR Persei</i>	1911, Feb. 26—1913, Apr. 30	11	Min. 1913, Jan. 10	1912, Dec. 21	+ 20	14.4	2419778
<i>U Arietis</i>	1913, Jan. 28— Mar. 5	4	Max. 1913, Feb. 17	1913, Feb. 20	- 3	7.8	2419816
<i>R Tauri</i>	1913, Nov. 17—1914, Apr. 13	7	Min. 1914, Mar. 4	1914, June 13	-101	14.0	2420196
<i>V Tauri</i>	1913, Jan. 28— Apr. 30	7	Max. 1913, Mar. 19	1913, Apr. 1	- 14	9.4	2419846
<i>U Aurigæ</i>	1913, Jan. 28— May 7	7	Max. 1913, Apr. 23	1913, Apr. 5	+ 18	8.7	2419881

Star	Dates Included	No. Obs.	Maximum or Minimum Observed	Computed	O-C	Mg.	J. D. Max. or Min.
<i>V Camelopard</i>	1912, Jan. 20—1913, May 29	24	Min.	1912, Aug. 29	Unknown		2419644
		..	Max.	1913, Mar. 5	1913, Apr. 20	- 46	2419832
<i>RR Auriga</i>	1912, Nov. 30—1913, May 7	11	Min.	1913, Jan. 18	Unknown		2419786
<i>V Monocerotis</i>	1913, Jan. 30— Apr. 30	8	Max.	1913, Mar. 31	1913, Feb. 9	+ 49	2419858
<i>S Can. Min.</i>	1913, Jan. 30— May 20	8	Max.	1913, Mar. 26	1913, Jan. 20	+ 67	2419853
<i>T Can. Min.</i>	1913, Jan. 30— May 20	8	Max.	1913, Apr. 3	1913, Mar. 9	+ 25	2419861
<i>U Caneri</i>	1913, Jan. 30— May 29	8	Max.	1913, Apr. 29	1913, June 3	- 35	2419887
<i>RU Urs. Maj.</i> ..	1913, Jan. 8— June 4	12	Min.	1913, Mar. 14	Unknown		2419841
	1914, Jan. 22— May 27	7	Max.	1914, Feb. 22	1914, Mar. 5	- 11	2420186
<i>RV Draconis</i> ..	1912, Nov. 30—1913, June 2	9	Max.	1912, Dec. 29	1913, Jan. 4	- 6	2419766
		..	Min.	1913, May 13	Unknown		2419901
<i>RS Urs. Maj.</i> ..	1913, Jan. 8—1914, Oct. 1	23	Max.	1913, Feb. 17	1913, Jan. 17	+ 31	2419816
		..	Max.	1913, Oct. 20	1913, Sept. 30	+ 20	2420061
		..	Min.	1914, Mar. 24	Unknown		2420216
<i>RR Urs. Maj.</i> ...	1913, Oct. 3—1914, May 27	13	Max.	1913, Dec. 22	1913, Nov. 19	+ 33	2420124
<i>T Urs. Min.</i>	1913, Jan. 8— June 2	8	Max.	1913, Feb. 11	1913, Mar. 26	- 43	2419810
<i>Z Cygni</i>	1913, Jan. 9— Feb. 4	4	Max.	1913, Jan. 27	1913, Feb. 13	- 17	2419795
<i>R Delphini</i>	1913, Oct. 2— Dec. 18	7	Max.	1913, Nov. 8	1914, Jan. 6	- 59	2420080
<i>T Delphini</i>	1913, Oct. 2— Dec. 18	6	Max.	1913, Nov. 1	1913, Sept. 7	+ 55	2420073
<i>R Equulei</i>	1912, Oct. 16—1913, Jan. 14	8	Max.	1912, Dec. 18	1912, Dec. 16	+ 2	2419755
<i>Y Pegasi</i>	1913, Oct. 6— Nov. 25	6	Max.	1913, Nov. 7	1913, Dec. 29	- 52	2420079
<i>V Cassiopeia</i> ..	1913, Oct. 3—1914, Mar. 20	7	Max.	1913, Dec. 4	1913, Dec. 22	- 18	2420106

Vassar College Observatory. May 20, 1915.

OBSERVATIONS OF COMET α , 1915 (MELLISH),

MADE WITH MICROMETER OF 5-INCH EQUATORIAL TELESCOPE, UNIVERSITY OF IOWA OBSERVATORY, IOWA CITY, IOWA,
By O. H. TRUMAN.

1915 G. M. T.	$\Delta\alpha$	$\Delta\delta$	Comp.	α App.	δ App.	$\log p \Delta$ in α	$\log p \Delta$ in δ	Star
27 Apr. 21 ^h 45 ^m 04 ^s	+1 ^m 47 ^s .23	-9' 38".9	9, 8	18 ^h 40 ^m 13 ^s .64	- 9° 24' 59".9	9.685 _n	0.834	1
10 May 21 36 56	+0 9.73	+0' 25".34	4, 4	19 02 22.32	-18 51 50.0	8.585 _n	0.881	2
12 May 21 21 04	+1 53.78	+9' 11".1	8, 8	19 06 39.22	-21 00 26.5	8.789 _n	0.895	3
15 May 21 37 09	-1 13.40	+0' 37".8	4, 5	19 13 56.42	-24 46 42.8	8.401 _n	0.908	4

Star	Authority	α 1915.0	δ 1915.0	Red. to App. Pl.
1	Schj. γ_{6903}^{6903} Munch. Stern. Supp. Bd. 12 2487	18 ^h 38 ^m 24 ^s .12	- 9° 15' 11".45	+2 ^s .29 -9".51
2	Munch. Stern. Supp. Bd. 13 797. Yarnall 8290	19 02 9.83	-18 52 10.4	+2.76 -3.94
3	π <i>Sagittarii</i> N. Almanac	19 04 42.58	-21 09 34.72	+2.85 -2.9
4	Wash. Zones. 1846—1852	17799	19 15 06.83	-24 47 19.8 +2.99 -0.75

The observation No. 3 was corrected a whole revolution of the mic. screw, in $\Delta\delta$, amounting to 27".5. It is now believed to be correct.

ERRATA.

In the article by PAUL S. YENDELL, contained in No. 677, the following corrections should be made:

Page 38 For *Rt Cygni* read *RT Cygni*. For *Tx Cygni* read *TX Cygni*.

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NOS. 7-8

MEASURES OF THE EARLIER JONCKHEERE STARS,

By ERIC DOOLITTLE.

Of the 1034 Stars thus far added to our lists by ROBERT JONCKHEERE, the earlier ones have now been discovered for so long a time that it is not improbable that a re-measurement of these may show interesting changes in some cases. A few years ago the first 130 of them, — J 1 to J 130, — were added to the observing list of the Flower Observatory, and this series has now been practically finished, though there are still a few on which, either from errors in the identifications or from errors in the descriptions, measures have not been secured. Quite a considerable number of the later discoveries have also been measured here, but these measures are reserved for a later publication.

The first 130 of the JONCKHEERE pairs were announced in the *Journal Astronomique*, Vol. I, Nos. 1 to 8, and they were also published in the *A.N.*, No. 4406. Occasional

discrepancies occur in the two lists, which will be found referred to in the Notes.

The following measures have been made in the usual manner; at least four measures of double distance and four measures of angle being obtained on each night. The positions are for 1880.0. It is much to be desired that all published results should for the present be referred to this standard epoch, since any other course simply entails upon one who uses the measures the labor of reducing them. In time, a new epoch must doubtless be chosen for all double stars, but this need not be considered now, nor for many years to come.

Although but five years have elapsed since the pairs here measured were discovered, it will be seen that with several of them there is already decided evidence of an interesting change.

J.	Right Asc.	Decl.	Date	θ	ρ	Mags.	n	Notes
26	3 ^h 29 ^m 32 ^s	14°52'	1912.08	146°.6	2''.57	9.2 9.4	3	1
27	3 39 39	28 32	1912.10	55 .3	3 .24	9.8 12.2	4	2
28	3 43 44	28 25	1912.10	192 .5	2 .27	9.8 11.7	3	3
30	3 56 39	6 22	1912.17	226 .7	1 .36	9.2 10.7	3	4
11	4 46 2	43 32						5
31	4 49 22	37 40	1913.14	351 .0	3 .52	9.4 12.3	3	6
12	4 51 45	42 41						7
47	4 53 41	0 20	1913.06	310 .5	1 .70	9.6 10.4	4	8
13	4 55 32	43 30	1913.03	163 .2	0 .87	9.4 9.7	3	9
14	4 59 56	27 2	1913.05	228 .6	1 .87	9.6 10.3	3	10
33	5 1 32	6 26	1913.04	19 .9	1 .56	9.8 11.1	4	11
15	5 5 30	8 50	1913.06	240 .3	2 .07	9.3 12.0	3	12
48	5 9 30	1 5	1913.06	27 .9	2 .22	9.2 9.7	4	13
34	5 21 6	1 20	1913.05	349 .6	1 .80	9.9 10.6	3	14
7	5 24 16	-2 52	1913.09	211 .2	1 .09	9.6 10.3	4	15
8	5 36 59	25 53	1912.07	250 .5	1 .85	10.2 11.2	3	16
35	5 40 21	6 20	1912.07	357 .0	1 .57	9.5 9.9	3	17
36	5 41 56	3 51	1912.19	113 .5	1 .88	8.9 10.2	3	18
50	5 56 10	8 1	1913.10	63 .7	0 .48	8.7 8.8	4	19
17	5 56 33	43 2	1913.04	150 .8	2 .49	9.2 9.6	3	20

J.	Right Asc.	Decl	Date	θ	ρ	Mags.	n	Notes
16	5 ^h 56 ^m 34 ^s	6°30'	1913.05	180°.6	1''.93	9.5 12.1	3	21
51	5 56 51	8 11	1912.18	121.9	1.06	8.8 9.7	3	22
9	5 59 0	27 38	1912.18	147.5	1.89	10.8 11.4	3	23
10	5 59 37	27 32	1912.18	127.1	3.25	9.6 11.9	3	24
52	6 5 30	2 30	1913.04	279.9	2.04	8.2 9.4	3	25
18	6 6 21	15 57	1913.16	190.0	0.74	9.3 10.8	3	26
19	6 7 4	19 0	1913.06	132.6	2.41	8.9 10.8	3	27
37	6 8 17	7 56	1913.08	317.4	2.70	9.1 10.7	3	28
38	6 13 3	6 20	1913.11	190.4	0.87	9.3 9.5	3	29
53	6 17 31	2 44	1913.09	124.9	1.85	7.2 10.8	4	30
54	6 33 35	-1 39	1913.10	203.4	2.53	9.0 9.8	3	31
39	6 35 17	9 19	1913.12	274.7	1.01	9.2 10.3	3	32
40	6 35 40	-0 9	1913.06	102.1	2.16	9.7 10.9	3	33
55	6 43 22	2 8	1913.94	178.9	2.55	9.8 10.2	3	34
20	6 45 2	0 32	1913.97	131.6	0.73	10.1 10.9	3	35
56	6 46 43	2 53	1913.95	328.8	1.16	8.4 9.3	4	36
41	6 50 1	6 38	1913.22	225.2	4.19	9.4 10.1	3	37
21	6 55 42	10 41	1913.12	274.6	3.02	9.5 9.8	3	38
22	6 56 20	12 44	1913.08	332.6	1.10	9.1 9.3	3	39
57	7 3 48	3 6	1913.95	126.1	2.24	9.5 10.7	3	40 AB.
			1913.92	62.9	20.36	10.9	2	AC.
58	7 5 44	-0 18	1914.947					41
			1914.953					
23	7 6 47	2 25	1913.96	295.3	4.90	9.5 9.9	3	42
59	7 6 52	3 3	1913.97	22.9	1.13	9.4 9.5	3	43
60	7 7 12	-2 26	1913.95	30.7	1.30	8.4 9.0	4	44
42	7 8 50	8 6	1913.03	80.6	2.00	9.2 9.6	3	45
61	7 9 39	-0 25						46
43	7 17 21	8 30	1913.03	67.1	3.15	9.7 10.1	3	47
62	7 22 40	7 1	1913.17	305.6	0.46	9.6 10.3	4	48
44	7 24 16	7 4	1913.18	197.5	3.03	10.1 10.9	3	49
63	7 30 30	2 14	1913.17	159.4	1.26	8.5 8.9	4	50
64	7 31 38	3 30	1913.17	176.0	4.09	9.6 10.2	3	51
45	7 35 4	9 18	1913.93	185.0	1.87	9.2 10.8	3	52
65	7 38 6	-0 58	1913.12	211.9	1.35	9.2 10.3	3	53
66	7 41 35	-0 39	1913.06	198.0	2.81	9.6 10.8	3	54
67	7 48 22	0 34	1913.03	51.5	3.60	9.5 9.7	3	55
46	7 48 58	12 59	1913.11	284.0	1.12	9.1 12.0	4	56
68	7 49 53	1 11	1913.08	24.2	1.08	9.3 9.8	3	57
69	7 50 5	3 43	1914.03	258.9	1.79	9.4 11.5	3	58
70	7 52 41	6 40	1913.13	308.3	2.23	8.1 10.4	4	59
71	8 1 25	6 2	1913.13	321.5	3.31	9.2 9.9	3	60
72	8 8 45	0 21	1914.03	116.3	2.64	9.5 9.9	3	61 AB.
			1914.02	121.6	30.96	10.1	2	AC.
73	8 19 31	8 12	1913.97	213.7	3.87	8.2 11.5	3	62
74	8 44 7	1 51	1913.07	98.2	3.54	9.4 9.7	3	63
75	8 46 56	2 4	1913.16	34.5	3.70	8.6 11.5	3	64
76	8 53 51	9 45	1914.03	261.9	4.29	9.3 12.2	3	65
77	8 58 35	10 58	1913.99	113.3	1.16	8.8 9.1	4	66
78	9 32 39	3 14	1912.22	209.1	3.56	9.5 11.0	3	67
83	9 41 16	4 10	1913.05	233.2	1.66	9.7 10.0	3	68
84	10 30 0	1 18	1912.25	60.1	0.67	8.6 8.7	4	69
88	10 32 9	-5 19	1913.15	238.4	8.63	9.4 12.2	3	70
79	10 33 11	8 3	1912.23	146.1	1.56	8.1 8.7	3	71
89	10 37 14	-3 15	1912.24	86.4	4.03	9.5 10.8	3	72
80	10 45 23	7 13	1912.22	217.9	4.03	8.8 9.6	3	73

J.	Right Asc.	Decl.	Date	θ	ρ	Mags.	n	Notes
90	10 ^h 47 ^m 34 ^s	-7° 4'	1913.16	237° .1	4'' .01	9.3 9.5	3	74 AB.
			1913.16	190 .3	10 .69	11.8	3	BC.
81	11 1 43	10 51	1912.23	140 .8	1 .78	9.3 9.8	4	75
82	11 3 44	11 33	1912.23	110 .9	1 .79	9.4 9.9	3	76
85	11 25 13	3 44	1913.18	161 .3	4 .43	7.3 13.6	3	77
86	11 28 12	4 47	1912.24	91 .3	2 .12	8.8 9.8	3	78
87	11 34 47	5 37	1912.26	127 .4	1 .90	7.6 12.2	3	79
92	11 52 55	1 12	1913.15	173 .0	14 .23	6.6 13.5	3	80
94	18 1 15	13 57	1912.40	309 .8	3 .33	9.5 10.0	3	81
95	18 22 17	7 5	1912.43	125 .5	1 .60	9.5 10.3	3	82 AB.
			1912.40	31 .0	33 .89	10.7	2	AC.
96	18 27 9	6 32	1912.43	149 .2	2 .80	9.1 10.2	3	83
98	18 27 53	17 24	1912.53	148 .2	3 .28	9.4 10.7	3	84
99	18 30 49	6 35	1912.46	323 .2	26 .88	5.2 13.5	2	85
101	18 33 28	6 54	1913.59	233 .7	3 .26	9.9 12.7	3	86
102	18 33 47	6 59	1913.64	47 .1	2 .68	9.5 13.1	3	87
105	18 38 45	-7 6	1912.45	344 .5	7 .13	8.6 10.7	3	88
106	18 40 29	-9 46	1912.43	336 .0	3 .87	9.8 10.1	3	89
107	18 44 42	-6 25	1912.45	189 .0	5 .69	8.7 11.8	3	90
108	18 45 4	13 43	1912.43	76 .7	1 .40	9.6 9.9	3	91
109	18 45 24	29 41						92
110	18 46 47	35 15	1913.60	168 .8	1 .62	9.5 11.0	3	93
111	18 52 20	-6 58	1913.13	89 .3	4 .60	9.6 11.4	4	94
113	19 0 32	-8 2	1913.80	4 .3	6 .04	8.9 12.0	3	95
114	19 11 10	-0 38	1913.80	204 .1	4 .97	9.4 11.3	3	96
115	19 15 30	-1 34	1912.87	2 .1	3 .66	9.9 11.3	3	97 AB.
			1912.87	47 .1	7 .03	12.1	3	AC.
116	19 15 33	-1 36	1912.99	95 .7	3 .59	9.4 10.3	5	98
24	19 30 42	20 27	1913.61	256 .1	2 .55	9.3 10.9	3	99
119	19 32 24	-1 32	1913.80	253 .3	5 .67	9.0 12.3	3	100
120	19 32 8	-1 33	1913.81	91 .2	1 .91	8.7 10.1	3	101
123	19 40 33	10 17						102
125	19 46 54	41 22	1913.62	213 .4	1 .80	8.8 9.2	3	103
126	19 46 32	10 3						104
25	19 48 33	29 4	1913.64	2 .1	0 .94	9.5 10.4	4	105
127	20 7 18	34 48	1913.61	2 .3	2 .59	10.1 10.3	3	106
128	20 10 26	34 38	1913.60	196 .8	6 .62	8.8 11.2	3	107
130	20 25 46	41 25	1913.59	265 .3	2 .39	9.1 9.5	3	108
3	20 25 52	11 26	1913.81	134 .6	1 .26	10.2 11.0	4	109
1	20 27 11	11 20	1913.81	34 .3	1 .02	8.8 9.6	4	110
4	20 29 34	47 27	1913.59	358 .3	1 .96	9.7 9.9	3	111
2	21 49 25	18 35	1913.82	30 .2	3 .11	10.1 10.1	4	112

NOTES

1. Several measures by J and V, who reverse the quadrant. Fixed. 2. Three prior measures. Fixed. 3. Called A.G. Cam. 1492 instead of 1942 in J.A. No certain change. 4. Change is still uncertain. 5. The magnitudes differ slightly in J.A. and (4406). 6. Probably fixed. 7. Angle differs in J.A. and (4406). 8. No change is indicated. 9. Apparently fixed. 10. The distance by J is smaller. The magnitudes differ in J.A. and (4406).

11. No change is indicated. 12. A rather difficult pair. Fixed. 13. There is no certain change. 14. Fixed. 15. The large change indicated by the earliest measures is not confirmed. 16. Fixed. 17. Fixed. 18. Several measures. Fixed. 19. The N. star of a N. and S. row. A not very difficult, but an interesting pair. Some increase of angle is indicated. 20. Fixed. 21. Fixed. A 10.5 mag. star is in 213°:20". 22. Apparently fixed. 23. Fixed. 24. Slight changes are

indicated, which are probably not real. 25. A bright and easy pair: it is strange that this was not noticed earlier. Fixed. 26. Probably fixed. 27. In a loose cluster. Some change is indicated. 28. Probably fixed. 29. Fixed. 30. There is a more distant companion: 1913.04 103°.1 33".99 9.5 2*n*. The close pair is probably fixed.

31. J obtains a smaller distance. 32. DM + 9°.1357 should be 9°.1257 in *J.A.* and (4406). Probably fixed. 33. The magnitudes differ in *J.A.* and (4406). Probably fixed. 34. J has a much smaller distance. 4*n* should be 2*n* in (4406). 35. No certain change. 36. Several prior measures. Probably fixed. 37. Fixed. While searching for this the two following pairs were found: —

Anon	6 ^h 53 ^m 17 ^s	6°20'	1913.90	322°.4	6".24	9.8	10.1	2 <i>n</i>	
Anon	6 52 1	6 50	1913.13	149 .1	2 .34	9.4	10.0	4 <i>n</i>	+6°.1463

38. This quadrant certainly correct. Fixed. 39. Probably fixed. 40. A more distant 13.5 mag. star is in 174°. The fainter stars were not mentioned by J. J's distance in AB is much less.

41. On these and two other nights the companion was clearly seen, but it is rather too faint for measures of accuracy. Fully measured by BURNHAM. 42. Distance by J is much smaller. 43. Fixed. 44. Many prior measures. Fixed. 45. Fixed. 46. The position is exactly that of HOWE 17, but the descriptions differ. 47. No change is indicated. 48. An interesting but difficult pair. Change is still not entirely certain. J makes the

magnitudes practically equal. 49. J reverses the quadrant and makes the distance less. 50. Increase in angle and decrease in distance are clearly indicated.

51. The distance by J is much smaller. 52. No certain change is indicated. 53. Probably fixed. 54. J's distance is much less. 55. Probably fixed. 56. Considerably more difficult than would be inferred from the description. A decided increase in angle is indicated. 57. Fixed. 58. A large change. Forms with a 9.5 mag. star an E. and W. pair. Identified from a chart. Certainly the right star. 59. Fixed. A similar, but closer pair was measured by mistake for this: —

Anon	7 ^h 53 ^m 42 ^s	6°54'	1913.13	312°.0	1".23	9.3	10.0	3 <i>n</i>	+6°.1845
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60. J's distance is much less.

61. No certain change. The third star was not noticed by J. 62. Increase in distance is indicated. 63. Change is not yet certain. 64. J's distance is much less. 65. J's distance is only about half as great. 66. Fixed. 67. A 9.5 star is 200" N. in the same field. Probably fixed, though J gets a smaller distance. 68. Several prior measures by J who always reverses the quadrant. Fixed. 69. Fixed. 70. An 8.0 mag. star is 200" sp. J makes the distance one third as great.

71. Fixed. 72. 100" up a bright 30" pair. Fixed. 73. Fixed. 74. J reverses the quadrant of AB and calls the second measure AC. The distances of AC extraordinarily discordant: otherwise fixed. 75. Fixed. 76. The p star of a 200" pair. Fixed. 77. Probably fixed, though J obtains a smaller distance. 78. Fixed. 79. Fixed. 80. There is no certain change.

81. 100", 137°, from an 8.5 mag. star. Fixed. 82. The third star has not before been measured. AB is probably fixed. 83. Fixed. 84. Fixed. 85. The measures are yet hardly sufficient to show whether the faint star shares in the known proper motion of A. 86. J obtains an angle 6° less and a smaller distance. 87. The angle by J is less. 88. The W. star of a faint pair. J's estimate does not agree well with the measures. 89.

J's distance is less than half as great as here. A 9.0 mag. star is 40" sp. 90. J's distance is about half as great as here.

91. Fixed. 92. This pair is H 1352. 93. Fixed. 94. J's distance is less than half of this. 95. J's distance is less than half of this. 96. J makes the distance much less. 97. The third star was not noticed by J. He makes the distance of AB less than half of that here found. 98. J's distance is about one-third of this. 99. Probably fixed. 100. An examination at the telescope shows that J's identification is wrong. This should be -1°.3797, and not -1°.3788. The latter star has no companion. J's distance is about half of that here found.

101. Fixed. 102. This is BURNHAM 55. 103. A change in angle of about 10° is indicated. 104. This is STRUVE 2590. 105. J's first distance was 0".59, but this is probably an error. Fixed. 106. J's distance is less than half as great. 107. This is ESPIN 205, but the star was incorrectly identified by ESPIN. Apparently fixed. 108. Fixed. 109. An 11.5 mag. star is 40" distant, nearly in the direction of B. Probably fixed. 110. Angles and magnitudes differ in *J.A.* and (4406).

111. Prior measures of distance very inconsistent. Probably fixed. 112. Fixed.

The Flower Observatory, May 20, 1915.

ON THE SYSTEM OF THE BRIGHTER A STARS,

AS STUDIED FROM SPECTROGRAPHIC DETERMINATIONS OF THE SOLAR VELOCITY,

By C. A. MANEY.

The data for the present investigation comprise the values of the radial velocities of 284 stars of type A; besides the 212 stars used by CAMPBELL in his article of 1912, 72 more have been added. Also, additional observations for 58 out of the 212 stars used in 1912 have been made use of. Most of the new data is from measures, made by others and by myself, in the regular program of observations of the BRUCE spectrograph attached to the 40-inch refractor of the Yerkes Observatory, which I am permitted to use here in advance of their formal publication.

THE SOLAR MOTION

Table I summarizes the investigations of the solar motion as determined from stars of this type.

TABLE I

Solar Motion from Stars of Class A.

Stars Used No. of	V_0	K	Place	Year
33	-17		Yerkes ²	1910
177	-16.8	+0.95	Lick ³	1911
212	-15.3	-0.6	Lick ¹	1912
284	-18.5	+0.8	Yerkes	1915

V_0 and K are quantities in the well-known equation

$$V_0 \cos d + K - V = 0$$

and are expressed in kilometers.

In the present solution, " d " represents the angular distance of given star from the assumed apex at $\alpha = 270^\circ$, $\delta = +30^\circ$; V_0 is the unknown value of the solar motion; K is an assumed systematic error; and V is the adopted value of the given star's radial velocity. The values of " d " for 212 of the stars are taken from L. O. B. VII, 211, 1912. The values for the remaining 72 stars were computed by the writer and are included in the table at the end of this article. The formulæ used are developed and explained in L. O. B. VII, 214, 1912.

Thus, 284 equations were formed and solved by the method of least squares.

The residual velocities of all the stars were computed, using the values (as given in Table I) of

$$V_0 = -18.5 \text{ and } K = +0.8$$

These form the basis for the following table showing the relation between velocity and subdivisions of this class of stars.

TABLE II

138 stars	Class A	Avg. numerical velocity 10.69 km.
89 stars	Class A1-A4	Avg. numerical velocity 11.58 km.
57 stars	Class A5-A9	Avg. numerical velocity 12.34 km.
284 stars	Class A0-A9	Avg. numerical velocity 11.30 km.

REFERENCES:

1. W. W. CAMPBELL: "Preliminary Radial Velocities of 212 Brighter Class A Stars." Lick Obs., Bulletin VII, 211, 1912.
2. E. B. FROST: "Preliminary Note on the Sun's Velocity with Respect to the Stars of Spectral Type A." Read at the meeting of the National Academy of Sciences, Nov. 8, 1910. *Mem. Spett. Ital.* (2) 1, 26, 1912.

For convenience of comparison, I have copied a similar table* comprising CAMPBELL's results for 212 stars.

98 stars	Class A	Avg. numerical velocity 10.27 km.
67 stars	Class A1-A4	Avg. numerical velocity 11.38 km.
47 stars	Class A5-A9	Avg. numerical velocity 12.58 km.
212 stars	Class A0-A9	Avg. numerical velocity 11.1 km.

Both tables show an increase of velocity with advancing subdivisions, but the larger number of stars indicates a less marked relation.

THE K TERM

A significant fact in Table I is that three solutions, involving a considerable number of stars in each case, give very different values for the assumed error, K . This fact alone is sufficient ground for attributing the source of this error to other causes than to the use of incorrect values of wave lengths affecting all stars alike with a positive or negative velocity. The writer does not propose to show that K is due entirely to any preferential motion, but that preferential motion is the cause of the discordance in the various values of K .

In order to study the effect of various solutions upon the value of K , the sky was divided up into areas. When properly selected areas, each containing fairly large numbers of stars, give widely different values of K , the differences thus obtained being irrespective of type subdivision, the derived values must be assigned to preferential motion. I would say that the areas are properly selected if the effect of individual motion is practically nullified and if the coefficient ($\cos d$) has a wide range in value. An accurate solution of the stars cannot be made for V_0 and K when the stars in a group of small area are selected; the values of ($\cos d$) will be too similar and the effect of errors will be greatly enhanced. Where the distribution is uniform, by selecting diametrically opposite areas of stars for use in solution, the effect of any stream motion of the stellar system is lost in the case of the constant K and shows itself in the value of V_0 . Where the distribution is irregular and there is streaming, there will be a preponderance of either positive or negative stream-velocities which will affect the value of K .

The considerable difference in distribution of the stars used by CAMPBELL in 1912 and those used by the writer seems ample to account for the difference in the derived values of K . Of the 72 stars added to the number that CAMPBELL used, 60 were of north declination and only 12 of south declination; whereas of CAMPBELL's 212 stars, 115 were of north declination and 97 of south declination. From these considerations, a uniform distribution of stars seems desirable.

3. W. W. CAMPBELL: "Some Peculiarities in the Motions of the Stars." L. O. B. VI, 196, 1911.

*L.O.B. No. 211, 1912.

THE MILKY WAY

It has for some time been recognized that because of their distribution, the *A* stars are peculiarly related to the galaxy. It seemed desirable to study this peculiar relation also from the standpoint of the distribution of the radial velocities. The angular distances of the stars from the equator of the galaxy were measured on a globe and estimated to the nearest degree. The stars were first divided into two groups; one containing the stars within twenty-four degrees of the galactic equator, and the other group, the remainder of the stars nearer the poles. There were 142 stars in each group. The results of least-squares solutions for solar motion are

given in Table III.

TABLE III

Group	Number of Stars	V_0	K
0° to 24°	142	-21.28	+1.05
24° to 90°	142	-15.42	+0.42
0° to 90°	284	-18.46	+0.82

The effect of star streaming is quite marked.

Subdivisions of groups were made to study the problem more in detail. The results of these solutions with related quantities are included in Table IV.

TABLE IV

Group	Latitudes inclusive	$\sin \lambda_1 - \sin \lambda_2$	No. of stars	Density VI = 1.0	V_0	K	Galaxy Distribution	
I	0° to 7°	0.139	45	1.6	-18.88	+0.23	19	26
II	8 to 13	0.103	50	2.4	-24.09	+1.13	28	22
III	14 to 24	0.181	47	1.3	-20.67	+2.07	18	29
IV	25 to 34	0.151	46	1.5	-16.87	-0.11	18	28
V	35 to 49	0.192	47	1.2	-11.70	-0.44	18	30
VI	50 to 90	0.233	48	1.0	-19.53	+3.10	27	21
Totals and Means		0.999	283*	1.5	-18.62	+1.00	128	156
								284

*One star was omitted by mistake.

The area of the zone of a sphere between the angular distances λ_1 and λ_2 from a great circle is given by the formula

$$A = \int_{\lambda_2}^{\lambda_1} 2\pi r \cos \lambda \cdot r \delta \lambda = 2\pi r^2 (\sin \lambda_1 - \sin \lambda_2)$$

The values of $(\sin \lambda_1 - \sin \lambda_2)$ are given in the third column.

The densities in column 5 are reduced to the scale of VI = 1.0.

In the last column, stars in that hemisphere of galactic coördinates containing the north celestial pole are considered of positive latitude.

A general consideration of the above table reveals some striking facts and relations.

Because of irregularities in distribution and the small number of stars in each group, too much significance should not be attached to the values of K . With regular distribution, positive values would indicate expansion and negative values, contraction. By expansion is meant a common systematic radial motion of the stars in an outward direction away from the sun, by contraction, a like motion toward the sun.

In groups I and VI, the values of V_0 being near to its mean value, there is an indication of an absence of streaming among stars of these very low and very high galactic latitudes. From the small value of K in group I, the stars near the galactic equator seem to be free from radial stream motion; on the other hand, the large value of K in group VI seems to indicate expansion in high galactic latitudes.

The large value of V_0 in group II indicates a large preferential motion relative to the sun. In this connec-

tion, the great density of stars in this region should not be overlooked.

The small value of V_0 in group V indicates a systematic motion nearer the galactic poles of opposite direction to the systematic motion in group II. The low velocities of the stars here and the higher relative velocities in the latitudes on either side, indicate strongly a relation of the sun to the stars in this latitude.

An extensive investigation¹ has been carried out by H. C. PLUMMER, based upon the assumption that the stars of class *A* move in the plane of the galaxy and hence have velocities which are merely cosine functions of galactic latitudes. The values of V_0 in groups I and VI, indicate that this assumption needs to be modified. Moreover, upon this hypothesis, since the stellar velocities in two latitudes λ_1 and λ_2 are proportional to the cosines of those latitudes, the cosine of the latitude dividing the group latitude into two equal areas was computed for each of the groups from II to V. Starting with the value of -24.09 km. for the stars of group I, the theoretical values of V_0 were computed and with the derived values are placed in Table V for comparison.

TABLE V

Group	Mean Latitude	$\cos \lambda$	Derived V_0	Theoretical V_0	$D-T$
II	10°.5	0.98	-24.09	-24.09	0.00
III	18.9	.95	20.67	23.18	+2.51
IV	29.3	.87	16.87	21.37	4.50
V	41.5	.75	11.70	18.36	6.66

The regular progression in the last column is signifi-

cant and suggests that the velocities are more complex functions of galactic latitudes and that this function still contains the cosine. Moreover the formula

$$V_i = V_{II} \cos \lambda_i + C (\lambda_i - \lambda_{II})$$

where i has the values III, IV, and V, and $C = +0.235$, and where λ is in degrees, adapts itself very well to the velocities in groups II to V.

In Table VI the derived values of V_0 are given, together with values of V_0 computed by the above formula.

TABLE VI

Group	$\lambda_i - \lambda_{II}$	Derived	Theoretical	$D - T$
II	0.0	-24.1	-24.1	0.0
III	8.4	-20.7	-21.2	+0.5
IV	18.8	-16.9	-17.0	+0.1
V	31.0	-11.7	-11.1	-0.6

The agreement is excellent.

Assuming a uniform distribution of stars in space, the system of the A stars has an approximately lenticular form. The mean distance of stars of larger galactic latitude is thus less than that of stars of smaller galactic latitudes. Since (as Table VI indicates) the velocity decreases with increase of galactic latitude, this quantity varies approximately directly as the distance.

The near-to-average values of K in the regions of the poles and equator can be explained by a scarcity of nearer stars in the directions of the galactic poles and a preponderance of nearer stars close to the galactic equator.

Facts support these inferences. WILSON, in L. O. B. VII, 214, 1912, discusses the real motions of 100 stars of large proper motion whose radial velocities have been determined. Among these 100 stars are 7 stars of type A . It is safe to conclude that these 7 stars are among the nearest of the A stars. Table VII shows the distribution of these stars with respect to the Milky Way.

TABLE VII

Star	Mag.	Parallax	Galactic Latitude	Galactic Latitude of Apex
θ Cassiopeia	4.5	0".23	- 7°	+13°
δ Cassiopeia	2.8	0 .03	- 2	+ 9
α Canis Majoris	-1.6	0 .376	- 8	-10
i Ursæ Majoris	3.1	0 .06	- 8	-32
β Leonis	2.2	0 .13	- 5	+15
β Leonis	...	0 .03	...	...
α Aquila	0.9	0 .24	-10	+15
ϵ Cephei	4.2	...	0	-10±
Mean			0	

That all of these seven nearer A stars should be within 10 degrees of the galactic equator, (in fact, within 10 degrees of -5° galactic latitude), and that the mean

distance of their apices from the galactic equator should turn out to be zero, and that 4 out of the 7 are within the latitudes of group I, is surely more than a mere coincidence. Yet conclusions cannot be final until the parallax of a larger number of the brighter A stars has been determined.

It seems advisable to compare my stellar densities with those given by LEWIS BOSS in his classical paper in the *Astronomical Journal* 26, 187, 1911. This article has "to do with the discussion of stars down to H.R.P. 6^m.5 and only a part of the stars down to that magnitude when they are fainter than H.R.P. 5^m.7. His "Table IV shows an enumeration of these stars arranged by types and in galactic zones 20° broad. After the zones +10° to -10°, the others contain one-half the sum of the two corresponding zones, +10° to +30° and -10° to -30°, and so on." In his Table V he gives the relative areal densities in percentages.

The 284 stars used in this paper are distributed as indicated in the third column of Table VIII. The numbers of stars in column four correspond to Boss's plan of enumeration of his A stars in column five, and the percentages in column six correspond to those of Boss in column seven.

TABLE VIII

Zone	Limits	No. of Stars	No. M	No. B	Percentage M	Percentage B
I	+10° to -10°	72	72	422	100	100
II	±10 to ±30	93	46	310	73	78
III	30 to 50	73	36	170	66	53
IV	50 to 70	43	22	100	60	47
V	70 to 90	3	(2)	32	(13)	44
		284	178	1034		

A comparison of the densities in Tables IV and VIII is suggestive of the necessity of a more extensive investigation of the distribution of stars in the galaxy. From the standpoint of my work, the densities in zones I and II of Table VIII agree as nearly as can be expected to those of Boss.

At most this investigation is preliminary. The amount of profitable work that can be done along the lines pursued and suggested in this article seems almost without limit.

Table IX contains the data for the above discussion. The magnitudes and spectral classifications in columns four and five respectively, are those of HARVARD. In column six, λ represents the star's angular distance from the assumed solar apex $\alpha = 270^\circ$, $\delta = +30^\circ$. In column seven, v' is the star's residual velocity as determined from the first general solution involving 284 equations. In the last column is given the star's approximate galactic latitude.

In conclusion I wish to thank PROFESSOR FROST for valuable suggestions in the preparation of this manuscript and for placing the unpublished data at my disposal.

TABLE IX

Star	1900		Mag.	Spec.	λ	v	v'	Gal. Lat.
	α	δ						
α <i>Andromedæ</i>	0 ^h 3 ^m .2	+28°32'	2.2	A0	77°	-13.0	- 9.6	-32
θ <i>Andromedæ</i>	11.9	+38 8	4.4	A0	74	+ 6.2	+10.5	-24
B.D. 43° 92'	22.9	+43 50	5.2	A2	73	+ 2.0	+ 6.5	-17
β_2 <i>Tucanæ</i>	27.0	-63 31	4.5	A2	120	+12.0	+ 2.1	-55
A.G.C. 467	28.1	-63 35	5.2	A2	120	+ 9.0	- 1.0	-55
θ <i>Tucanæ</i>	29.2	-71 49	6.1	A5	120	+ 4.0	- 6.2	-47
η <i>Phæniciæ</i>	38.8	-58 1	4.5	A0	120	-11.0	-21.1	-58
μ <i>Andromedæ</i>	51.2	+37 57	3.9	A2	81	+ 8.0	+10.1	-23
v <i>Phæniciæ</i>	1 3.2	-42 1	5.2	A3	121	- 6.0	-16.2	-77
θ <i>Cassiopeiæ</i>	5.0	+54 37	4.5	A5	74	+12.0	+16.2	- 7
82g <i>Piscium</i>	1 5.6	+30 54	5.0	A5	87	+ 6.0	+ 6.1	-30
v <i>Piscium</i>	14.0	+26 44	4.7	A2	91	+10.0	+ 8.8	-34
δ <i>Cassiopeiæ</i>	19.3	+59 43	2.8	A5	74	+ 7.0	+11.5	- 2
43 <i>Cassiopeiæ</i>	34.9	+67 32	5.5	A0p	71	+ 8.0	+13.3	+ 6
q ₂ <i>Eridani</i>	42.3	-54 1	5.1	A0	128	+ 8.0	- 4.1	-62
γ <i>Arietis</i>	48.0	+18 49	4.8	A0p	102	- 1.2	- 5.9	-40
β <i>Arietis</i>	49.1	+20 19	2.7	A5	101	- 1.0	- 5.5	-39
λ <i>Arietis</i>	52.4	+23 7	4.8	A5	100	+ 1.1	- 3.0	-36
48 <i>Cassiopeiæ</i>	53.8	+70 25	4.6	A2	70	- 4.0	+ 1.4	+ 9
50 <i>Cassiopeiæ</i>	54.9	+71 56	4.1	A0	70	- 9.0	- 3.4	+11
α <i>Piscium</i>	1 56.9	+ 2 17	4.3	A2p	114	+10.0	+ 1.8	-56
v <i>Fornacis</i>	2 0.0	-29 47	4.7	A0p	129	+20.0	+ 7.7	-79
12k <i>Arietis</i>	1.0	+22 10	5.1	A0	102	+12.0	+ 7.2	-36
58 <i>Andromedæ</i>	2.4	+37 23	4.8	A4	93	+ 9.0	+ 7.3	-21
β <i>Trianguli</i>	3.6	+34 31	3.1	A5	95	- 2.0	- 4.3	-25
9 γ <i>Trianguli</i>	11.4	+33 23	4.1	A0	97	+34.0	+31.0	-18
62c <i>Andromedæ</i>	12.8	+46 55	5.1	A0	88	-32.0	-32.0	-12
9i <i>Persei</i>	15.4	+55 23	5.2	A2	82	- 9.0	- 7.3	- 5
i <i>Cassiopeiæ</i>	20.8	+66 57	4.6	A5	75	+ 2.0	+ 6.1	+ 6
ξ_2 <i>Ceti</i>	22.8	+ 8 1	4.3	A0	116	+ 8.0	- 0.8	-46
φ <i>Fornacis</i>	2 23.8	-34 16	5.2	A2	135	+21.0	+ 7.2	-67
86 γ <i>Ceti</i>	38.1	+ 2 49	3.6	A0	122	+ 3.0	- 7.5	-48
μ <i>Ceti</i>	39.5	- 9 42	4.4	A5	118	+26.0	+16.6	-42
RZ <i>Cassiopeiæ</i>	39.9	+69 13	var 6.5	A0	74	-38.3	-34.1	+ 9
A.G.C. 2919	40.1	-32 57	6.1	A0	138	+11.0	- 3.5	-64
ζ <i>Hydri</i>	44.0	-68 3	4.9	A2	133	+ 4.0	- 9.3	-46
21 <i>Persei</i>	51.2	+31 32	5.2	A0p	104	+14.0	+ 8.8	-23
48e <i>Arietis</i>	53.5	+20 56	4.6	A2	111	- 7.0	-14.3	-30
θ_2 <i>Eridani</i>	54.5	-40 42	4.4	A2	141	+20.0	+ 4.8	-60
β <i>Horologii</i>	56.9	-64 28	5.1	A5	135	+25.0	+11.0	-49
37 <i>Her. Cass.</i>	3 1.1	+74 1	4.9	A2	72	+ 5.0	+10.0	+14
ζ <i>Arietis</i>	9.2	+20 40	5.0	A0	115	+19.0	+10.4	-28
+30° 512 <i>Persei</i>	9.2	+30 11	5.5	A2	107	+ 2.0	- 4.3	-13
v <i>Eridani</i>	25.6	- 5 25	4.8	A0	136	+ 9.0	- 5.1	-41
+70° 257 <i>Camelopardalis</i>	38.8	+70 34	5.4	A0	76	+25.0	+28.6	+12
5 <i>Her. Camelopardalis</i>	39.8	+71 1	4.7	A0	76	-12.0	- 8.3	+13
τ_1 <i>Eridani</i>	43.4	-24 11	5.0	A2	149	+21.0	+ 4.3	-50
v <i>Tauri</i>	57.8	+ 5 43	3.9	A0	134	- 4.0	-17.6	-31
41 <i>Tauri</i>	4 0.5	+27 20	5.3	A0p	116	0.0	- 8.9	-16
b <i>Persei</i>	10.7	+50 3	4.6	A2	96	+23.1	+20.2	+ 1
ω <i>Tauri</i>	4 11.4	+20 20	4.8	A5	123	+18.0	+ 7.0	-20
56 <i>Tauri</i>	13.7	+21 32	5.3	A0p	122	+14.0	+ 3.3	-18

TABLE IX — *Continued*

Star	(1900)		Mag.	Spec.	λ	v	v'	Gal. Lat.
	α	δ						
A.G.C. 4858	4 ^h 16 ^m .3	-20° 52'	5.3	A0	155°	- 9.0	-26.6	-40
63 <i>Tauri</i>	17.7	+16 33	5.7	A2	127	+36.5	+24.5	-20
κ <i>Tauri</i>	19.4	+22 4	4.4	A3	123	+39.0	+28.2	-17
68 <i>Tauri</i>	19.7	+17 42	4.2	A0	127	+36.4	+24.6	-19
θ_2 <i>Tauri</i>	23.0	+15 39	3.6	A5	129	+42.9	+30.5	-20
88 <i>Tauri</i>	30.2	+ 9 57	4.4	A2	135	+29.4	+15.6	-22
α <i>Doradus</i>	31.8	-55 15	3.5	A0p	150	+26.0	+ 9.1	-41
κ <i>Doradus</i>	42.9	-59 55	5.4	A5	147	-18.0	-34.4	-38
7 <i>Camelopardalis</i>	4 49.3	+53 35	4.4	A2	95	- 8.9	-11.3	+ 2
i <i>Tauri</i>	57.1	+21 27	4.7	A5	126	+43.0	-31.2	-10
11 <i>Orionis</i>	58.9	+15 16	4.6	A0	132	+14.0	+ 0.7	-13
β <i>Eridani</i>	5 2.9	- 5 13	2.9	A2	152	- 8.0	-25.1	-26
μ <i>Leoporis</i>	8.4	-16 19	3.3	A0p	162	+29.0	+10.6	-27
+33° 1008 <i>Aurigæ</i>	12.4	+33 39	5.4	A0p	115	+32.0	+23.3	00
19 <i>Aurigæ</i>	13.4	+33 51	5.2	A5	115	0.0	- 8.7	00
A.G.C. 6141	16.2	-21 20	4.7	A0	167	+31.5	+12.7	-28
θ_1 <i>Orionis</i>	30.4	- 5 27	6.8	A0	154	+19.0	+ 1.5	-18
130 <i>Tauri</i>	41.6	+17 41	5.5	A0	132	+13.0	- 0.2	- 3
ξ <i>Aurigæ</i>	5 46.5	+55 41	4.9	A2	94	-13.6	-15.8	+16
β <i>Aurigæ</i>	52.2	+44 56	2.1	A0p	105	-18.1	-23.7	+11
A.G.C. 7034	52.6	-52 39	5.3	A5	157	+27.0	+ 9.1	-36
θ <i>Aurigæ</i>	52.9	+37 12	2.7	A0p	113	+31.0	+23.0	+ 8
μ <i>Orionis</i>	56.9	+ 9 39	4.2	A2	140	+41.0	+26.0	- 5
2 <i>Lyncis</i>	6 10.8	+59 3	4.4	A0	91	- 4.0	- 5.1	+21
8 <i>Monocerotis</i>	18.5	+ 4 39	4.5	A5	145	+21.0	+ 5.0	- 1
13 <i>Monocerotis</i>	27.5	+ 7 24	4.5	A0	142	+13.0	- 2.4	+ 1
ξ_2 <i>Canis Major.</i>	30.9	-22 53	4.5	A0	170	+36.0	+17.0	-13
γ <i>Geminorum</i>	31.9	+16 29	1.9	A0	133	-11.0	-24.4	+ 6
N <i>Carinæ</i>	6 32.8	-52 53	4.4	A0	156	+25.0	+ 7.3	-23
26 <i>Geminorum</i>	36.6	+17 45	5.1	A0	131	+34.0	+21.0	+ 8
α <i>Canis Major.</i>	40.7	-16 35	1.6	A0	164	- 7.4	-26.0	- 8
36d <i>Geminorum</i>	45.6	+21 53	5.2	A0	127	+43.0	+31.1	+11
θ <i>Geminorum</i>	46.2	+34 5	3.6	A2	115	+18.0	+ 9.4	+17
A.G.C. 8984	7 2.4	-56 36	5.3	A0	151	+33.0	+16.0	-20
L_1 <i>Puppis</i>	10.2	-45 0	5.0	A0p	160	+ 3.5	-14.6	-24
λ <i>Geminorum</i>	12.3	+16 43	3.6	A2	130	- 9.0	-21.7	-44
21 <i>Lyncis</i>	19.2	+49 25	4.4	A0	99	+25.8	+22.2	-12
5 η <i>Can. Minor.</i>	22.7	+ 7 9	5.3	A5	138	+16.0	+ 1.5	-53
ρ <i>Geminorum</i>	7 22.7	+31 59	4.2	A5	115	- 4.0	-12.6	-29
64 <i>Geminorum</i>	23.1	+28 19	5.0	A2	118	+32.0	+22.4	-33
A.G.C. 9632	25.6	-22 49	4.8	A3p	160	+37.1	+19.0	-13
α <i>Geminorum</i>	28.2	+32 6	2.8	A0	114	+ 2.6	- 5.9	-29
τ <i>Puppis</i>	39.8	-28 43	4.2	A2p	158	+28.0	-10.0	-13
φ <i>Geminorum</i>	47.4	+27 1	5.0	A2	117	-22.0	-31.3	-32
85 <i>Geminorum</i>	49.8	+20 9	5.4	A0	123	+26.0	+15.0	-38
A.G.C. 10482	53.7	-30 4	4.8	A2	155	+28.5	+10.9	00
A.G.C. 10913	8 8.0	-42 41	4.9	A3	152	+19.3	+ 2.2	- 4
q <i>Puppis</i>	14.8	-36 21	4.4	A5	151	+ 8.0	- 9.0	00
30 <i>Monocerotis</i>	8 20.7	- 3 35	4.0	A0	138	+14.0	- 0.5	+19
2A <i>Ursæ Majoris</i>	25.7	+65 29	5.4	A0	80	-12.0	- 9.7	+ 5
e <i>Velorum</i>	34.2	-42 38	4.1	A5	147	+21.2	+ 4.9	00
γ <i>Canceri</i>	37.5	+21 50	4.7	A0	116	+29.0	+20.2	-31
a <i>Velorum</i>	42.6	-45 40	4.1	A0	145	+24.2	+ 8.3	- 1
ρ <i>Hydræ</i>	43.1	+ 6 13	4.4	A0	127	+14.0	+ 2.1	-44
δ <i>Pyxidis</i>	51.3	-27 18	4.9	A0	143	+ 2.0	-13.5	+12

TABLE IX — *Continued*

Star	(1900)		Mag.	Spec.	λ	ν	ν'	Gal. Lat.
	α	δ						
<i>o Caneri</i>	8 ^h 51 ^m .7	+15° 42'	5.2	A3	118°	0.0	- 9.6	-34
<i>i Ursæ Majoris</i>	52.4	+48 26	3.1	A5	93	+14.0	+12.4	- 8
<i>a Cancrī</i>	53.0	+12 15	4.3	A5	121	-14.0	-24.2	-37
<i>v Caneri</i>	8 56.9	+24 51	5.4	A0	111	-12.0	-19.3	-27
<i>a Volantis</i>	9 0.9	-66 0	4.2	A5	135	+ 6.0	- 7.8	-13
<i>f Ursæ Major.</i>	1.9	+52 0	4.5	A3p	89	- 1.1	- 1.5	- 4
<i>e Ursæ Major.</i>	9.0	+54 26	4.9	A5	86	-19.0	-18.6	- 1
<i>β Carinæ</i>	12.1	-69 18	1.8	A0	132	-16.0	-29.2	-14
38 <i>Lyncis</i>	12.6	+37 14	3.8	A0	99	+ 4.0	+ 0.3	+46
τ_2 <i>Hydræ</i>	26.9	- 0 44	4.5	A3	123	+ 8.0	- 2.8	+34
φ <i>Ursæ Major.</i>	45.4	+54 32	4.5	A2	83	-12.0	-10.4	+49
ν_2 <i>Hydræ</i>	10 0.3	-12 35	4.7	A0	122	+25.0	+14.4	+33
η <i>Leonis</i>	1.9	+17 15	3.6	A0p	105	+ 1.8	- 3.8	+52
15 <i>Sextantis</i>	10 2.9	+ 0 7	4.5	A0	115	+ 7.0	+ 1.6	+42
<i>q Velorum</i>	10.5	-41 38	4.1	A2	129	+12.0	- 0.5	+12
λ <i>Ursæ Major.</i>	11.1	+43 25	3.5	A0	87	+18.0	+18.2	+57
22e <i>Sextantis</i>	12.7	- 7 34	5.4	A0	117	+14.0	+ 4.8	+39
A.G. Chris. 1608	16.9	+66 4	4.9	A0	72	- 2.7	+ 2.1	+46
30 <i>Leo Minor.</i>	20.2	+34 18	4.8	A5	91	+14.0	+12.8	+61
<i>K Carinæ</i>	27.8	-71 29	4.9	A2	126	+ 8.8	- 2.8	-12
ω <i>Ursæ Maj.</i>	48.2	+43 43	4.8	A0	81	-18.4	-16.4	+ 1
<i>i Velorum</i>	55.5	-41 41	4.6	A2	121	- 6.0	-16.3	+16
<i>e Leonis</i>	55.6	+ 6 38	5.1	A5	100	-16.0	-20.1	-18
β <i>Urs. Maj.</i>	10 55.8	+56 55	2.4	A0	73	-12.0	- 7.5	+10
<i>b Leonis</i>	57.0	+20 43	4.4	A0	92	-10.0	-11.6	-10
A.G.C. 15269	11 4.4	-61 24	5.4	A0	123	-25.0	-35.8	00
β <i>Crateris</i>	6.7	-22 17	4.5	A2	112	+ 6.0	- 1.7	+35
δ <i>Leonis</i>	8.8	+21 4	2.6	A2	90	-18.0	-18.8	- 8
θ <i>Leonis</i>	9.0	+15 59	3.4	A0	93	+ 7.0	+ 5.3	-10
77b <i>Leonis</i>	16.0	+ 6 35	4.1	A0	96	- 8.0	-10.8	-14
γ <i>Crateris</i>	19.9	-17 8	4.1	A5	107	+ 8.0	+ 1.8	+41
α_2 <i>Centauri</i>	27.1	-58 58	5.3	A2p	120	-16.8	-26.7	+ 1
A.G.C. 15901	33.5	-61 16	5.3	A0	119	+ 5.0	- 4.8	00
λ <i>Musca</i>	11 40.9	-66 10	3.8	A5	119	+17.0	+ 7.2	- 5
β <i>Leonis</i>	44.0	+15 8	2.2	A2	86	+ 2.0	+ 2.5	- 5
γ <i>Urs. Maj.</i>	48.6	+54 15	2.5	A0	68	-10.0	- 3.8	+15
67 <i>Urs. Maj.</i>	57.0	+43 36	5.1	A3	70	+11.0	+16.4	+12
δ <i>Ursæ Maj.</i>	12 10.5	+57 35	3.4	A2	64	-11.0	- 3.6	+20
6 <i>Comæ Berenices</i>	10.0	+15 28	5.1	A2	80	+ 5.0	+ 7.4	+ 1
η <i>Virginis</i>	14.8	- 0 7	4.0	A0	87	+ 2.2	+ 2.4	- 5
14 <i>Comæ Ber.</i>	21.4	+27 49	5.2	A5	72	- 4.0	+ 0.8	+ 9
16 <i>Comæ Ber.</i>	22.0	+27 23	5.0	A2	72	+ 5.0	+ 9.8	+10
17 <i>Comæ Ber.</i>	23.9	+26 28	5.4	A0p	72	0.0	+ 4.8	+10
74 <i>Ursa Major.</i>	12 25.3	+58 57	5.4	A5	61	+ 6.0	+14.0	+22
23 <i>Comæ Ber.</i>	29.9	+23 11	4.8	A0	73	-20.0	-15.2	+ 9
γ <i>Centauri</i>	36.0	-48 25	2.4	A0	106	- 9.0	-15.1	+14
η <i>Centauri</i>	47.9	-39 38	4.3	A5	100	- 1.0	- 5.1	+22
ϵ <i>Ursæ Major.</i>	49.6	+56 30	1.7	A0p	59	-10.0	+ 1.2	+61
α_2 <i>Can. Ven.</i>	51.4	+38 52	2.9	A0p	62	- 1.0	+ 6.8	+79
θ <i>Virginis</i>	13 4.8	- 5 0	4.4	A0	79	+ 0.6	+ 3.4	+55
20 <i>Can. Ven.</i>	13.1	+41 6	4.7	A5	58	+ 9.5	+18.6	+75
21 <i>Can. Ven.</i>	14.0	+50 12	5.1	A0	56	- 5.0	+ 4.6	+67
<i>i Centauri</i>	15.0	-36 11	2.9	A2	91	+ 2.0	- 0.1	+25
ζ <i>Ursæ Major.</i>	13 19.9	+55 27	2.4	A0p	55	- 7.0	+ 2.9	+63
η <i>Ursæ Major.</i>	21.2	+55 30	4.0	A5	54	-12.0	- 2.0	+61

TABLE IX — *Continued.*

Star	(1900)		Mag.	Spec.	λ	v	v'	Gal. Lat.
	α	δ						
<i>K Centauri</i>	13 ^h 23 ^m .3	-50° 39'	5.3	A0	101°	-26.0	-30.4	+10
<i>o Virginis</i>	29.1	+ 4 10	4.9	A0 _p	69	- 6.0	- 0.1	+64
79 <i>Virginis</i>	29.6	- 0 5	3.4	A2	71	- 7.0	- 1.7	+60
24 <i>Can. Ven.</i>	30.4	+49 32	4.6	A3	53	-19.0	+ 8.8	+66
25 <i>Can. Ven.</i>	33.0	+36 48	4.9	A0	55	- 6.0	+ 3.8	+72
Lal. 25224	34.6	+11 15	5.5	A4	64	-23.0	-15.7	+69
τ <i>Virginis</i>	56.6	+ 2 2	4.3	A2	64	-10.0	- 2.7	+58
α <i>Draconis</i>	14 1.7	+64 51	3.6	A0	50	-16.7	- 5.7	+50
η <i>Apodis</i>	14 5.7	-80 32	5.1	A1	115	- 8.0	-16.6	-19
κ <i>Bootis</i>	9.9	+52 16	4.6	A5	47	-19.0	- 7.2	+61
i <i>Bootis</i>	12.6	+51 50	4.8	A5	47	-18.0	- 6.1	+61
19 λ <i>Bootis</i>	12.6	+46 33	4.3	A0	46	-13.0	- 1.0	+63
λ <i>Virginis</i>	13.7	-12 55	4.6	A2	69	- 6.0	- 0.3	+44
22 f <i>Bootis</i>	21.8	+19 41	5.4	A5	50	-29.0	-17.9	+66
A.G.C. 19597	23.7	-49 4	5.5	A2	93	+ 9.0	+ 7.4	+ 9
π <i>Bootis</i>	36.0	+16 51	4.9	A0	48	+ 1.0	+12.5	+61
ζ <i>Bootis</i>	36.4	+14 9	4.4	A2	49	- 8.0	+ 3.3	+60
7 μ <i>Librae</i>	43.8	-13 44	5.4	A0	64	0.0	+ 7.2	+38
16 <i>Libra</i>	14 52.0	- 3 56	4.6	A2	56	+20.0	+29.5	+45
δ <i>Librae</i>	55.6	- 8 7	4.8	A0	58	-45.0	-36.1	+41
c <i>Bootis</i>	15 2.9	+25 16	5.0	A5	39	- 4.0	+ 9.5	+58
i <i>Librae</i>	6.5	-19 25	4.7	A0 _p	65	-17.0	- 9.9	+31
β <i>Circini</i>	9.6	-58 26	4.2	A3	95	+10.2	+ 7.7	- 2
χ <i>Bootis</i>	10.3	+29 32	5.3	A0	37	-18.0	- 3.9	+57
v <i>Lupi</i>	18.2	+39 22	5.4	A0	79	- 9.0	- 6.2	+13
μ <i>Bootis</i>	20.7	+37 44	4.5	A3	34	- 8.0	+ 6.6	+55
γ <i>Urse Min.</i>	20.9	+72 11	3.1	A2	47	-12.0	- 0.2	+40
α <i>Cor. Bor.</i>	30.5	+27 3	2.3	A0	33	+ 0.4	+15.1	+52
28 β <i>Serpentis</i>	15 41.6	+15 44	3.7	A2	35	+ 3.0	+17.4	+47
32 μ <i>Serpentis</i>	44.4	- 3 7	3.6	A0	46	-11.0	+ 1.0	+31
ϵ <i>Serpentis</i>	45.8	+ 4 47	3.8	A0	40	-10.0	+ 3.3	+40
53 <i>Herculis</i>	49.2	+31 52	5.4	A0	15	-18.0	- 0.9	+49
δ <i>Normæ</i>	59.5	-44 54	4.8	A3 _p	80	-16.0	-13.5	+ 5
φ <i>Herculis</i>	16 5.6	+45 12	4.3	A0	27	-14.0	+ 1.7	+45
ψ <i>Scorpii</i>	6.6	- 9 48	4.9	A2	48	- 8.9	- 2.6	+28
d <i>Scorpii</i>	12.1	-28 22	4.9	A0	64	-11.4	- 4.0	+13
o <i>Scorpii</i>	14.6	-23 56	4.8	A3	60	- 8.0	+ 0.6	+16
ω <i>Herculis</i>	20.8	+14 16	4.5	A0 _p	28	- 7.0	+ 8.6	+37
v <i>Ophiuchi</i>	16 22.4	- 8 9	4.7	A2	45	-28.0	-15.7	+26
45 <i>Herculis</i>	42.8	+ 5 25	5.3	A0 _p	31	-14.0	+ 1.1	+29
52 <i>Herculis</i>	46.3	+46 10	4.9	A2 _p	22	- 2.0	+14.4	+39
ϵ <i>Herculis</i>	56.5	+31 4	3.9	A0	14	-24.0	- 6.8	+34
60 <i>Herculis</i>	17 0.7	+12 53	4.9	A3	22	- 4.0	+12.4	+28
c <i>Herculis</i>	4.5	+36 4	5.4	A0	13	-28.0	-10.8	+33
η <i>Ophiuchi</i>	4.6	-15 36	2.6	A0	48	0.0	+11.7	+13
e <i>Herculis</i>	14.2	+37 24	4.8	A0	12	- 4.0	+13.3	+32
v <i>Serpentis</i>	15.2	-12 45	4.4	A0	44	+ 8.0	+20.5	+11
+25° 3246 <i>Herc.</i>	16.1	+25 38	5.3	A2	11	-12.0	+ 5.4	+28
ρ <i>Herculis</i>	17 20.2	+37 14	4.5	A0	11	-18.0	- 0.6	+31
π <i>Aræ</i>	29.0	-54 26	5.3	A0	85	-13.0	-12.1	-13
v_1 <i>Draconis</i>	30.2	+55 15	5.0	A5	26	-14.0	+ 1.8	+32
25 v^2 <i>Draconis</i>	30.3	+55 14	5.0	A5	26	-12.0	+ 3.8	+32
ζ <i>Serpentis</i>	31.9	-15 20	3.6	A5	46	-42.8	-30.7	+ 8
o <i>Serpentis</i>	35.8	-12 49	4.4	A2	43	-33.0	-20.3	+ 8
i_2 <i>Scorpii</i>	43.1	-40 4	4.9	A2 _p	70	-18.0	-12.5	- 2

TABLE IX — *Continued.*

Star	(1900)		Mag.	Spec.	λ	ρ	ρ'	Gal. Lat.
	α	δ						
95 <i>Herculis</i>	18 ^h 57 ^m .2	+21° 36'	5.1	A0	8°	-27.0	- 9.5	+19
π <i>Paronis</i>	58.9	-63 40	4.4	A5	94	-19.6	-21.6	-20
72 <i>Ophiuchi</i>	18 2.6	+ 9 33	3.7	A2	20	-24.0	- 7.5	+12
103o <i>Herculis</i>	18 3.6	+28 45	3.8	A0	1	-27.0	- 9.3	+20
236 <i>Ursæ Min.</i>	4.5	+86 37	4.4	A0	57	0.0	+ 9.4	+27
ϵ <i>Sagittarii</i>	17.5	-34 26	2.0	A0	65	-11.0	- 3.9	-11
α <i>Lyrae</i>	33.6	+38 41	0.1	A0	11	-13.0	+ 4.3	+17
A.G.C. 25500	35.6	-64 58	4.9	A2	95	- 1.0	- 3.5	-24
46e <i>Draconis</i>	40.7	+55 26	5.1	A0	26	-35.0	-19.2	+23
θ <i>Serpentis</i>	51.2	+ 4 4	4.5	A5	29	-48.0	-32.6	00
ξ_1 <i>Sagittarii</i>	51.4	-20 47	5.1	A0	52	+ 3.1	+13.6	-12
γ <i>Lyrae</i>	55.2	+32 33	3.3	A0	12	-20.0	- 2.7	+12
ζ <i>Sagittarii</i>	56.3	-30 1	2.7	A2	62	+22.0	+30.0	-17
17 ξ <i>Aquila</i>	19 0.8	+13 43	3.0	A0	21	-27.0	-10.6	+ 2
28A <i>Aquila</i>	15.0	+12 11	5.4	A0	25	+ 6.0	+22.0	+11
π <i>Draconis</i>	20.2	+65 31	4.6	A2	38	-27.5	-13.6	+28
ν <i>Aquila</i>	21.4	+ 0 9	4.9	A8	36	- 1.0	+13.2	-10
A.G.C. 27100	42.2	-59 26	5.5	A2	92	+ 6.0	+ 4.6	-32
α <i>Aquila</i>	45.9	+ 8 36	0.9	A5	93	-33.0	-18.2	-10
A.G.C. 27237	48.7	-59 10	5.4	A0	92	- 0.1	- 1.5	-32
ϵ <i>Paronis</i>	49.0	-73 10	4.1	A0	105	0.0	- 5.5	-32
13 <i>Vulpecule</i>	49.2	+23 50	4.5	A0	25	-28.0	-12.0	- 2
61c <i>Aquila</i>	51.5	+11 9	5.3	A2	32	-42.0	-27.1	-10
θ_2 <i>Sagittarii</i>	19 53.3	-34 58	5.3	A3	70	-16.0	-10.5	-30
15 <i>Vulpecule</i>	57.5	+27 29	4.7	A5	26	-21.0	- 5.1	- 3
θ <i>Aquila</i>	20 6.1	- 1 7	3.4	A0	43	-28.0	-15.3	-20
b_3 <i>Cygni</i>	10.8	+36 30	5.0	A0	28	-18.0	-25.0	00
ν <i>Capricorni</i>	15.1	-13 4	4.8	A0	54	- 1.3	+ 8.8	-27
ζ <i>Delphini</i>	30.6	+14 20	4.7	A0	38	-22.0	- 8.2	-17
29 <i>Vulpecule</i>	34.1	+20 51	4.8	A0	36	-14.0	+ 0.2	-13
β <i>Paronis</i>	36.0	-88 34	3.6	A5	101	+ 9.4	+ 5.1	-37
α <i>Cygni</i>	38.0	+44 55	1.3	A2	34	- 4.0	+10.5	+ 3
δ <i>Delphini</i>	38.8	+14 43	4.5	A2	40	+10.0	+23.5	-19
i <i>Microscopii</i>	20 41.7	-44 22	5.1	A5	83	-19.0	-17.5	-40
ϵ <i>Aquarii</i>	42.3	- 9 52	3.8	A0	56	-16.0	- 6.4	-32
η <i>Capricorni</i>	58.7	-20 15	4.9	A3	66	+26.0	+32.7	-39
θ <i>Capricorni</i>	21 0.3	-17 38	4.2	A0	64	-31.0	-23.8	-39
ϵ <i>Microscopii</i>	11.9	-32 35	4.8	A0	77	+ 5.0	+ 8.3	-46
θ <i>Indi</i>	12.7	-53 52	4.6	A5	94	-14.0	-16.0	-45
67 σ <i>Cygni</i>	13.5	+38 59	4.3	A0 _p	40	- 3.0	+10.3	- 7
θ_1 <i>Microscopii</i>	14.4	-41 14	4.9	A2 _p	84	+ 5.0	+ 6.1	-47
5a <i>Cephei</i>	16.2	+62 10	2.6	A5	45	- 8.0	+ 4.3	+ 9
103 <i>Equulei</i>	17.9	+ 6 23	5.1	A0	52	- 9.0	+ 1.6	-31
Brad. 2792	21 21.7	+46 17	5.5	A2	42	+ 1.0	+13.9	- 3
ξ <i>Aquarii</i>	32.4	- 8 18	4.8	A5	64	-20.0	-12.6	-42
i <i>Pisces Austr.</i>	39.0	-33 29	4.4	A0	82	+ 6.0	+ 7.8	-52
ν <i>Cephei</i>	42.6	+60 40	4.5	A2 _p	48	-21.0	- 9.3	+ 6
14 <i>Pegasi</i>	45.4	+29 43	5.0	A0	48	-22.0	-10.5	-18
γ <i>Gruis</i>	47.9	-37 50	3.2	A0	86	- 3.0	- 2.6	-53
η <i>Pisces Austr.</i>	55.1	-28 56	5.4	A0	81	-10.0	- 8.0	-54
32 <i>Aquarii</i>	59.6	- 1 23	5.2	A2	65	+23.0	+30.0	-45
ξ <i>Cephei</i>	22 0.9	+64 8	4.4	A8	50	- 4.5	+ 6.5	+ 7
A.G.C. 30290	4.1	-34 31	5.5	A5	86	-30.0	-29.7	-48
λ <i>Pisces Austr.</i>	22 8.6	-28 16	5.4	A0	83	- 1.3	+ 0.1	-57
ϵ <i>Cephei</i>	11.3	+56 33	4.2	A5	51	- 0.2	+19.7	00

TABLE IX — *Continued.*

Star	(1900)		Mag.	Spec.	λ	v	v'	Gal. Lat.
	α	δ						
β <i>Pisces Austr.</i>	22 ^h 25 ^m .9	-32° 52'	4.4	A0	89°	+ 8.0	+ 7.6	-61
29 σ <i>Cephei</i>	29.0	+78 19	5.5	A0	56	+ 7.0	+16.5	+17
9 <i>Lacerta</i>	33.2	+51 1	4.8	A5	54	+12.0	+22.1	- 6
α <i>Pegasi</i>	37.0	+28 48	4.8	A0	59	+ 8.7	+17.3	-25
γ <i>Pisces Austr.</i>	47.0	-33 24	4.5	A0	93	+18.0	+16.3	-66
δ <i>Aquarii</i>	49.3	-16 21	3.5	A2	84	+22.0	+23.3	-63
ρ <i>Pegasi</i>	50.2	+18 17	5.0	A0	71	-10.0	- 4.7	-45
α <i>Pisces Austr.</i>	52.1	-30 9	1.3	A3	92	+ 6.7	+ 5.3	-67
τ <i>Pegasi</i>	23 15.7	+23 12	4.6	A5	70	-22.0	-16.3	-34
κ <i>Piscium</i>	21.8	+ 0 42	4.9	A2	81	0.0	+ 2.0	-55
i <i>Phœnicis</i>	29.7	-43 10	4.8	A2 _p	105	+26.0	+20.4	-69
λ <i>Piscium</i>	36.9	+ 1 14	4.6	A5	84	8.1	+ 9.1	-57

Yerkes Observatory, August, 1915.

OBSERVATIONS OF ASTEROIDS.

MADE WITH THE PHOTOGRAPHIC TELESCOPE AT THE U. S. NAVAL OBSERVATORY,

By GEORGE H. PETERS.

[Communicated by Captain J. A. HOOGEWERFF, U. S. Navy, Superintendent.]

Name	Mag.	Date	G. M. T.	α	δ
		1914		1914.0	1914.0
(696) <i>Leonora</i>	11.8	Oct. 19	^h ^m 14 46.5	^h ^m ^s 0 42 11.5	[°] ['] ["] +27 31 26
(367) <i>Amicitia</i>	12.6	Oct. 19	16 42.2	2 12 26.0	8 54 19
(731) [1912 OQ]	12.3	Oct. 19	16 42.2	2 10 8.5	7 55 58
(367) <i>Amicitia</i>	12.6	Oct. 21	16 40.0	2 10 23.0	8 44 51
(731) [1912 OQ]	12.3	Oct. 21	16 40.0	2 8 18.2	7 53 58
(361) <i>Bononia</i>	12.5	Oct. 22	16 35.0	2 35 49.9	21 48 31
(595) <i>Polyxena</i>	12.2	Oct. 22	16 35.0	2 43 51.1	23 28 1
(709) [1911 LK]	12.0	Oct. 27	16 23.0	3 1 54.3	42 38 34
(709) [1911 LK]	12.0	Nov. 6	14 8.8	2 52 17.3	42 29 42
(619) <i>Triberga</i>	11.8	Nov. 9	17 3.0	3 15 44.2	2 35 26
(551) <i>Ortrud</i>	12.2	Nov. 17	16 47.0	3 8 6.0	18 17 54
(126) <i>Velleda</i>	11.2	Nov. 17	16 47.0	3 5 11.0	19 58 8
(712) <i>Boliviana</i>	10.5	Nov. 17	16 47.0	3 17 20.2	17 13 28
(161) <i>Athor</i>	11.2	Nov. 19	16 47.1	4 7 21.5	31 32 50
(382) <i>Dodona</i>	13.0	Nov. 19	16 47.1	4 1 48.6	30 37 59
Asteroid	14.0	Nov. 21	16 5.0	3 15 7.9	23 16 37
(762) [1913 SQ]	11.4	Dec. 21	13 16.5	3 58 52.7	36 17 28
(769) [1913 TA]	13.6	Dec. 22	14 42.0	5 7 13.2	+29 31 16
		1915		1915.0	1915.0
(769) [1913 TA]	13.6	Jan. 5	14 22.0	4 56 23.3	+29 19 57
(536) <i>Merapi</i>	12.1	Jan. 8	15 38.7	9 18 46.2	40 5 52
(769) [1913 TA]	13.6	Jan. 9	13 29.4	4 53 51.6	29 15 37
(536) <i>Merapi</i>	12.1	Jan. 10	16 3.9	9 17 24.5	40 20 14
(764) [1913 SU]	12.8	Jan. 15	15 57.0	7 6 0.4	15 16 12
(536) <i>Merapi</i>	12.1	Feb. 8	14 3.1	8 52 36.1	42 42 35
(536) <i>Merapi</i>	12.1	Feb. 10	13 14.0	8 50 52.4	42 46 45
(766) [1913 SW]	12.6	Feb. 10	14 39.0	8 22 20.0	33 6 27
(536) <i>Merapi</i>	12.1	Feb. 17	13 39.0	8 44 57.0	42 55 26
(766) [1913 SW]	12.6	Feb. 17	15 10.8	8 16 38.4	32 54 39
Asteroid*	13.0	Feb. 18	17 10.0	9 57 48.6	9 6 34
(773) [1913 TV]	12.7	Feb. 18	17 10.0	10 2 11.0	8 51 10
(191) <i>Kolga</i>	12.3	Feb. 18	17 10.0	10 2 36.4	8 44 49

OBSERVATIONS OF ASTEROIDS — *Continued*

Name	Mag.	Date	G.M.T.	α	δ
		1914		1914.0	
(270) <i>Anahita</i>	11.8	Feb. 18	^h 17 ^m 10.0	^h 10 ^m 6 ^s 53.4	[°] 7 ['] 36 ["] 1
(260) <i>Huberta</i>	12.6	Feb. 18	17 10.0	10 4 5.4	9 4 11
Asteroid*	13.0	Feb. 19	16 39.1	9 56 55.6	9 9 12
(773) [1913 TV]	12.7	Feb. 19	16 39.1	10 1 13.1	8 51 39
(191) <i>Kolga</i>	12.3	Feb. 19	16 39.1	10 1 51.0	8 52 20
(270) <i>Anahita</i>	11.8	Feb. 19	16 39.1	10 5 51.8	7 41 27
(260) <i>Huberta</i>	12.6	Feb. 19	16 39.1	10 3 25.9	9 8 53
(733) [1912 PF]	12.8	Feb. 21	16 51.0	9 59 25.3	20 57 25
(733) [1912 PF]	12.8	Mar. 9	14 6.0	9 45 20.2	20 30 13
Asteroid*	13.0	Mar. 9	15 43.5	9 41 59.3	9 54 11
(773) [1913 TV]	12.7	Mar. 9	15 43.5	9 44 36.3	8 59 17
(270) <i>Anahita</i>	11.8	Mar. 9	15 43.5	9 48 13.6	9 19 41
(494) <i>Virtus</i>	12.2	Mar. 13	16 11.0	10 55 22.5	16 23 40
(234) <i>Barbara</i>	12.9	Mar. 13	16 11.0	10 53 24.3	15 21 29
(128) <i>Nemesis</i>	11.1	Mar. 13	16 11.0	10 50 51.2	17 17 12
(530) <i>Turandot</i>	13.2	Mar. 14	17 48.0	11 33 11.9	11 3 38
(530) <i>Turandot</i>	13.2	Mar. 17	15 51.0	11 31 11.1	11 19 38
(494) <i>Virtus</i>	12.2	Mar. 17	14 27.0	10 52 12.8	16 33 45
(234) <i>Barbara</i>	12.9	Mar. 17	14 27.0	10 50 5.8	15 53 49
(128) <i>Nemesis</i>	11.1	Mar. 17	14 27.0	10 47 43.3	+17 31 35
(779) [1914 UB]	12.2	Mar. 21	17 23.0	12 34 45	-26 53.9
(779) [1914 UB]	12.2	Mar. 23	17 16.0	12 32 58	-26 49.5

* Observations of an unidentified asteroid.

The accompanying observations of asteroid positions were made with the photographic telescope of the U. S. Naval Observatory. The photographic equipment consists of a pair of triple combinations of 10 inches aperture and about 113 inches focal length, practically on the plan of the Cook stellar lens. They were constructed of Jena glass at this observatory, by the writer, from computations by Mr. E. D. TILLYER. While the optical surfaces are quite perfect, and fine stellar images are obtained, the collimation of the different components to secure good definition over the entire field gave considerable trouble, but was finally satisfactorily accomplished. The photographic plates employed are 8 in. by 10 in. (203 by 254 mm.) giving a field of 20 minutes in right ascension and 4 degrees in declination.

The method of exposure is to follow on the computed motion of the moving object with the micrometer of the 9.6-inch visual telescope, guiding on a convenient star.

The plates were measured on the Stackpole measuring engine, and in case of asteroids in high declinations comparison stars of nearly the same right ascension were selected to avoid plate curvature errors. In most of the observations two or more pairs of comparison stars have been employed, and the means of the resulting positions published. On several of the plates asteroids were photographed which have not been identified. On the plate of Oct. 22, 1914, (361) *Bononia*, 12.5 magnitude, is shown, as well as another which was thought to be (595) *Polyxena*,

12.2 magnitude. From the published results of Dr. WOLF (595) should be near (361) but is not found. On Dec. 21, 1914, while photographing for (659) *Nestor*, a 14th magnitude asteroid was observed in the position given. A second plate was not obtained as no other clear night was available. *Nestor* could not be found although located near the edge of this plate, according to WOLF's observation. On the plates of Feb. 18, 19, and March 9, 1915, an asteroid was observed which has not been identified.

In a letter dated August 18, last, Dr. CONN, Director of the Royal Astronomical Computing Bureau of Berlin sends the following notes regarding the above observations:

(1) 709 (1911 L K). The right ascensions of the two observations of October 27 and November 6 do not quite correspond. Since the latter observation is confirmed by Heidelberg, it must be that the right ascension of October 27 is in error.

(2) In the place of (551) *Ortrud*, of November 17, (384) *Burdigala* is to be read.

(3) Whether (595) *Polyxena*, October 22, is correctly identified, is still not quite certain. However, it is possible that WOLF's observation is to be identified as something else. We are keeping the matter under observation, and as soon as the calculations are finished will communicate regarding it.

(4) The object of November 21 is new. According to the observation of THULE at Bergedorf (see circular 55), and according to the ephemeris in the *Astronomische*

Nachrichten No. 4765 must *Nestor* have been at about $3^{\text{h}} 22^{\text{m}} .4$ right ascension and $+23^{\circ} 35'$ declination.

(5) The object of February 18, 19, March 9 is new.

The observation referred to in (1), of October 27, has been rereduced. The corrected position of 709 (1911 LK) for that date is

Greenwich M. T., October 27, $16^{\text{h}} 23^{\text{m}} .0$, 1914.0,
 $\alpha = 3^{\text{h}} 03^{\text{m}} 4^{\text{s}} .5$ $\delta = +42^{\circ} 38' 48''$

With regard to (3) a second communication from Dr. Conn states that the Washington observation is probably correct.

OBSERVATIONS OF COMET 1913f (DELAN) (DELAN)

MADE WITH THE FILAR MICROMETER OF THE 433MM EQUATORIAL OF LA PLATA OBSERVATORY.

By P. T. DELAVAN.

1915	La Plata M. T.	*	Comp.	$\Delta\alpha$ — $\Delta\delta$		α 's Apparent δ		Log $p'\Delta$	
				$\Delta\alpha$	$\Delta\delta$	α	δ	for α	for δ
Feb. 18	15 42 36	2	10,10	+ 3.93	-4 6.5	17 41 53.97	-26 28 33.9	9.6748 n	0.4694 n
20	14 42 29	3	10,10	- 2.95	-6 18.6	17 43 23.72	-27 5 5.9	9.7186 n	0.5588 n
22	15 49 49	5	10,10	+13.00	+3 51.0	17 44 52.24	-27 43 21.1	9.6580 n	0.4133 n
27	16 35 13	6	10,10	- 9.86	+3 54.4	17 48 5.08	-29 18 20.7	9.5613 n	0.2325 n
27	16 58 4	7	10,10	-19.59	+3 32.0	17 48 5.49	-29 18 40.2	9.5062 n	0.1722 n
Mar. 1	14 49 42	8	10,10	+11.41	-4 57.8	17 49 9.27	-29 55 16.1	9.7048 n	0.4564 n
4	13 49 23	10	10,10	- 1.72	-0 57.8	17 50 37.30	-30 52 2.9	9.8131 n	0.5534 n
8	16 4 5	12	10,10	+ 4.84	-3 13.2	17 52 15.68	-32 11 55.0	9.5701 n	0.1206 n
10	14 8 21	13	10,10	+ 3.10	+4 22.1	17 52 52.24	-32 49 42.3	9.7263 n	0.4402 n
10	14 26 13	13	10,10	+ 3.34	+4 7.0	17 52 52.48	-32 49 57.4	9.7101 n	0.3907 n
14	14 21 25	15	10,10	-15.64	-3 15.4	17 53 46.32	-34 9 50.5	9.7098 n	0.3469 n
15	14 10 44	17	10,10	- 1.12	+5 2.9	17 53 59.51	-34 28 52.9	9.7184 n	0.3622 n
17	15 3 19	19	10,10	-14.85	+5 49.7	17 54 6.49	-35 10 30.9	9.6415 n	0.1166 n
18	14 31 1	20	10,10	-21.36	+7 32.2	17 54 9.08	-35 31 5.6	9.6855 n	0.2221 n
19	13 40 44	23	10,10	- 4.92	-2 30.3	17 54 9.54	-35 50 55.5	9.7372 n	0.3780 n
19	14 3 45	22	10,10	-27.60	+4 44.0	17 54 9.51	-35 51 14.4	9.7158 n	0.3028 n
22	15 55 40	24	10,10	+28.63	+1 6.2	17 53 57.03	-36 54 49.3	9.4789 n	9.3886 n
23	14 33 34	25	10,10	-21.10	+1 29.2	17 53 49.31	-37 14 23.9	9.6659 n	0.0705 n
25	13 37 28	26	10,10	- 6.66	-2 53.9	17 53 25.99	-37 55 22.9	9.7307 n	0.2668 n
25	14 45 45	27	10,10	+36.87	+1 33.1	17 53 25.29	-37 56 23.6	9.6315 n	9.8933 n
27	13 49 31	28	10,10	-28.37	-3 36.5	17 52 52.44	-38 37 33.2	9.7098 n	0.1484 n

All observations of $\Delta\alpha$ have been made by direct micrometer measures.

Mean Places of Comparison Stars

*	α 1915.0	Red. to App. Pl.	δ 1915.0	Red. to App. Pl.	Authority
1	17 42 16.24	+0.60	-26 18 54.3	-7.3	Washington Catalogue of 23521 Stars, 15402.
2	17 41 49.42	+0.62	-26 24 20.2	-7.2	Connected with Star 1.
3	17 43 26.00	+0.67	-26 58 40.3	-7.0	Argelander Southern Zones 17227.
4	17 42 12.56	+0.73	-27 47 58.4	-6.7	Boss' Preliminary General Catalogue 4493.
5	17 44 38.52	+0.72	-27 47 5.4	-6.7	Connected with Star 4.
6	17 48 14.05	+0.89	-29 22 8.8	-6.3	Cordoba Zone Catalogue 3087.
7	17 48 24.19	+0.89	-29 22 5.9	-6.3	Cordoba Zone Catalogue 3101.
8	17 48 56.91	+0.95	-29 50 12.1	-6.2	Washington Catalogue of 23521, Stars 15578.
9	17 48 42.72	+1.06	-30 50 34.5	-5.8	Cordoba Zone Catalogue 3123.
10	17 50 37.97	+1.05	-30 50 59.4	-5.7	Connected with Star 9.
11	17 51 56.96	+1.21	-32 2 19.9	-5.1	Argentine General Catalogue 24359.
12	17 52 9.64	+1.20	-32 8 36.7	-5.1	Connected with Star 11.
13	17 52 47.86	+1.28	-32 53 59.4	-5.0	Cordoba Zone Catalogue 3388.

Mean Places of Comparison Stars — Continued.

*	α 1915.0	Red. to App. Pl	δ 1915.0	Red. to App. Pl	Authority
	^h ^m ^s	^s	[°] ' "	"	
14	17 55 20.25	+1.43	-34 00 9.5	-4.6	Argentine General Catalogue 24453.
15	17 54 0.52	+1.44	-34 6 30.5	-4.6	Connected with Star 14.
16	17 54 28.80	+1.48	-34 40 55.1	-4.4	Argentine General Catalogue 24429.
17	17 53 59.15	+1.48	-34 33 51.4	-4.4	Connected with Star 16.
18	17 55 30.15	+1.55	-35 18 51.9	-4.1	Cordoba Zone Catalogue 3562.
19	17 54 19.78	+1.56	-35 16 16.5	-4.1	Connected with Star 18.
20	17 54 28.84	+1.60	-35 38 33.8	-4.0	Cordoba Zone Catalogue 3497.
21	17 54 25.86	+1.64	-35 49 44.3	-3.8	Cordoba Zone Catalogue 3493.
22	17 54 35.47	+1.64	-35 55 54.6	-3.8	Cordoba Zone Catalogue 3502.
23	17 54 12.82	+1.64	-35 48 21.4	-3.8	Connected with Star 21.
24	17 53 26.62	+1.78	-36 55 52.1	-3.4	Argentine General Catalogue 24396.
25	17 54 8.59	+1.82	-37 15 49.8	-3.3	Cordoba Zone Catalogue 3468.
26	17 53 30.74	+1.91	-37 52 25.8	-3.2	Cordoba Zone Catalogue 3425.
27	17 52 46.50	+1.92	-37 57 53.5	-3.2	Argentine General Catalogue 24378.
28	17 53 18.81	+2.00	-38 33 53.7	-3.0	Argentine General Catalogue 24391.

La Plata, Argentina, March 30, 1915.

PROVISIONAL ORBIT ELEMENTS, AND EPHEMERIS,

COMET 1915a (MELLISH),

By C. J. MERFIELD.

The appended orbit elements of this comet have been determined from twelve condition equations, the absolute terms of which were formed from observations between the dates 1915, Feb. 25th, and 1915, June 2d.

The results are as follows:

$$T = 1915 \text{ July } 17.155190 - [6.40583]de. \text{ G.M.T.}$$

$$\begin{aligned} \omega &= 247^\circ 46' 25''.9 + 1.033 de \\ \Omega &= 72 \quad 16 \quad 21 \quad .5 + 0.139 de \\ \iota &= 54 \quad 47 \quad 10 \quad .4 + 0.375 de \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} 1915.$$

$$q = 1.0052775 + [4.33565] de.$$

$$\text{Log } q = 0.0022860.$$

$$e = 1.0 + [5.0000] de.$$

The value of "de" to be expressed in units of the fifth decimal place.

The value of this quantity determined from the pres-

ent calculation is 3.5743, therefore the eccentricity will be 1.0000357.

CO-ORDINATES

$$\begin{aligned} x &= [9.7979610] r \sin (38^\circ 46' 16''.0 + v). \\ y &= [9.9489556] r \sin (348 \quad 24 \quad 24 \quad .7 + v). \\ z &= [9.9556047] r \sin (372 \quad 35 \quad 57 \quad .8 + v). \end{aligned}$$

RESIDUALS

	1915	Cos δ $d\alpha$	$d\delta$
Feb. 26	-0.02	-0''.2	
Apr. 12	+0.25	+4 .2	
Apr. 20	-0.36	+1 .6	
Apr. 30	+0.21	-2 .6	
May 17	-0.01	0 .0	
June 1	-0.01	+0 .1	

AUGUST METEORS.

By EDWIN F. SAWYER,

Short watches, aggregating 3h. 30m. taken on the 9-10-11-13 of August (8-12 cloudy) between the hours 10-11 P.M., revealed only 19 meteors; 15 of these being recorded on the 11th. Of the 19 observed, 15 were *Perseids*, 14 being noted on the 11th, and only 1 on the

10th. A few mapped with sufficient accuracy on the 11th indicated the Radiant at 45° , $+57^\circ$, center of observation *Cassiopeia*. Sky generally clear; $\frac{1}{8}$ of visible heavens covered. Very meagre displays for some years.

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THE MOON'S MEAN LONGITUDE AND THE ECLIPSE OF FEB. 3, 1916,

By FRANK E. ROSS.

[Communicated by CAPT. J. A. HOOGEWERFF, U. S. Navy, Superintendent U. S. Naval Observatory.]

The present paper is a continuation of that in the *Astronomical Journal* No. 667. In addition to utilizing the best of the Washington occultations from Dec., 1913, to March, 1915, a number of those of the brighter stars observed at Greenwich, published in *Monthly Notices*, Vol. 75, No. 3, was used. The Washington occultations up to 1914, June 6, have been published in *A. J.*, Nos. 673-4. Those later than this date have not yet appeared in print.

As in the previous reductions, only "central" occultations were utilized. Care was taken in selection, especially where bright limb emersions were in question. In each such case, the time obtained with the 26-inch equatorial was exclusively employed, the time of the phenomenon secured with the 12-inch telescope being given weight zero. For all dark limb phenomena, the agreement between the two instruments was very satisfactory.

The equation for determining the correction to the

mean longitude, δM , is

$$E \cdot \delta M = s' - D,$$

where s' is the semi-diameter of the Moon, and D the computed distance between the occulted star and the moon's center. Each occultation therefore gives a determination of δM of weight E^2 .

The tabular *Ephemeris* position of the Moon has been corrected by a large number of terms, whose sum in the three coördinates is given in the fifth, sixth and seventh columns below, computed from manuscript tables in the possession of the *American Ephemeris* (*A. J.* 667, p. 155). In the present reduction the short period empirical terms are omitted. The values of δM in the following tabulation accordingly give the correction to: pure theory plus NEWCOMB's long period empirical term. They are therefore identical with the "minor residuals" of NEWCOMB. The last column, δv , contains the residuals obtained by substituting in the equations of condition the smooth-curve values of δM .

TABLE I
Moon's LONGITUDE FROM OCCULTATIONS, 1913-15

Place	Date	Star	Phen.	Tabular Corrections to HANSEN-NEWCOMB			E	$s'-D$	δM	δv
				$\delta \lambda$	$\delta \beta$	$\delta \pi$				
W	1913 Dec. 5	96 <i>Aquarii</i>	DD	+6.52	+ .33	+.40	-.90	-7.3	+8.1	-1.0
W	12	38 <i>B Aquarii</i>	DD	5.83	.57	.44	-.98	-6.3	6.5	+0.6
W	13	406 <i>B Tauri</i>	DB	5.43	.46	.45	-.95	-6.6	7.0	0.0
W	21	ι <i>Virginis</i>	DB	4.15	1.44	.50	-1.07	-7.9	7.4	-0.4
W	21	ι <i>Virginis</i>	RD	4.15	1.44	.50	+1.01	+5.1	5.0	-2.0
W	1914 Jan. 6	μ <i>Arietis</i>	DD	7.62	.65	.41	-.80	-5.7	7.2	-0.1
W	6	μ <i>Arietis</i>	RB	7.62	.65	.41	+.91	+6.9	7.6	+0.5
W	7	16 <i>Tauri</i>	DD	7.30	.60	.42	-.70	-6.6	9.4	-1.7
W	7	η <i>Tauri</i>	DD	7.30	.60	.42	-.95	-7.8	8.2	-1.1
W	7	20 <i>Tauri</i>	DD	7.30	.60	.42	-.83	-7.1	8.5	-1.3
W	10	49 <i>Aurigæ</i>	DD	5.75	.36	.45	-.62	-5.3	8.5	-1.0
W	10	49 <i>Aurigæ</i>	RB	5.75	.36	.45	+.62	+2.9	4.7	-1.4
W	15	83 <i>Leonis</i>	RD	3.41	1.24	.50	+.64	+4.4	6.9	-0.1
W	17	49 <i>Virginis</i>	RD	3.44	1.70	.50	+.98	+4.9	5.0	-1.9
G	Feb. 3	ϵ <i>Arietis</i>	DD	8.06	.37	.41	-.91	-7.5	8.3	-1.1
W	7	47 <i>Geminorum</i>	DD	6.32	.14	.46	-.99	-9.0	9.1	-2.0
W	11	ϵ <i>Leonis</i>	RD	+4.19	+1.03	+.50	+.98	+6.7	+6.9	-0.3

TABLE I—Moon's LONGITUDE FROM OCCULTATIONS, 1913-1915 (Continued.)

Place	Date	Star	Phen.	Tabular Corrections to HANSEN-NEWMOMB			<i>E</i>	<i>s' - D</i>	δM	<i>i</i>
				$\delta\lambda$	$\delta\beta$	$\delta\pi$				
W	Mar. 3	18 <i>Tauri</i>	DD	+8.30	+ .14	+ .42	— .91	—8.1	+8.9	—1.6
W	9	8 <i>Leonis</i>	DD	6.29	+ .15	.48	—1.07	—9.4	8.8	—1.8
W	14	231 <i>G Virginis</i> . .	RD	3.97	+1.82	.49	+1.02	+2.9	2.8*	—4.4
W	14	236 <i>G Virginis</i> . .	RD	3.97	+1.82	.49	+ .75	+3.1	4.1	—2.2
W	Apr. 9	ψ <i>Virginis</i>	DD	6.12	+1.06	.50	—1.11	—7.7	6.9	+0.3
W	12	<i>b Scorpionii</i>	RD	4.58	+1.61	.49	+ .85	+5.1	6.0	—1.1
W	12	4 <i>Scorpionii</i>	DB	4.58	+1.61	.49	—1.10	—6.7	6.1	+1.3
W	12	<i>b Scorpionii</i>	RD	4.58	+1.61	.49	+1.00	+5.5	5.5	—1.7
W	18	δ <i>Capricorni</i>	RD	4.01	+ .88	.43	+ .61	+4.7	7.8	+0.3
G	29	136 <i>Tauri</i>	DD	7.17	+ .10	.44	— .64	—5.0	7.8	—0.4
W	May 1	5 <i>B Cancri</i>	DD	7.42	— .20	.46	— .82	—7.6	9.3	—1.7
G	4	49 <i>Leonis</i>	DD	7.65	+ .03	.49	—1.07	—7.6	7.1	+0.1
W	16	42 <i>Aquarii</i>	RD	4.12	— .07	.42	+ .82	+6.9	8.5	+1.0
W	17	81 <i>Aquarii</i>	RD	4.48	— .08	.41	+ .91	+6.3	7.0	—0.2
W	28	κ <i>Geminorum</i>	DD	6.82	+ .16	.45	— .95	—5.0	5.3	+1.8
W	June 6	4 <i>Scorpionii</i>	DD	6.74	+1.12	.49	—1.11	—7.5	6.7	+0.6
W	July 8	17 <i>Capricorni</i>	DB	4.97	— .17	.45	—1.00	—8.5	8.5	—1.2
W	8	17 <i>Capricorni</i>	RD	4.97	— .17	.45	+ .98	+7.8	7.9	+0.6
W	15	20 <i>H¹ Arietis</i>	DB	5.48	+ .10	.40	— .79	—7.1	9.0	—1.3
W	17	16 <i>Tauri</i>	RD	5.86	+ .39	.41	+ .84	+6.1	7.3	0.0
W	17	<i>q Tauri</i>	DB	5.86	+ .39	.41	— .86	—6.7	7.8	—0.5
W	17	<i>q Tauri</i>	RD	5.86	+ .39	.41	+ .92	+8.5	9.3	+1.8
W	17	20 <i>Tauri</i>	DB	5.86	+ .39	.41	— .90	—6.1	6.8	+0.5
W	17	20 <i>Tauri</i>	RD	5.86	+ .39	.41	+ .80	+6.7	8.4	+0.8
W	17	21 <i>Tauri</i>	DB	5.86	+ .39	.41	— .82	—6.5	7.9	—0.5
W	17	22 <i>Tauri</i>	DB	5.86	+ .39	.41	— .88	—8.1	9.3	—1.7
W	Aug. 7	λ <i>Aquarii</i>	RD	5.04	— .68	.42	+ .89	+7.1	8.0	+0.6
W	7	78 <i>Aquarii</i>	DB	5.04	— .68	.42	— .95	—8.2	8.7	—1.2
W	7	78 <i>Aquarii</i>	RD	5.04	— .68	.42	+ .85	+8.2	9.7	+1.9
W	30	τ <i>Sagittarii</i>	DD	6.89	+ .98	.47	— .99	—6.0	6.0	+1.3
W	30	τ <i>Sagittarii</i>	RB	6.89	+ .98	.47	+ .89	+5.3	6.0	—1.2
W	Sept. 12	406 <i>B Tauri</i>	DB	4.84	+ .04	.42	— .94	—6.9	7.4	0.0
W	12	406 <i>B Tauri</i>	RD	4.84	+ .04	.42	+ .86	+5.9	6.9	—0.4
G	Dec. 1	17 <i>Tauri</i>	DD	5.94	+ .58	.40	— .79	—5.4	6.8	+0.6
G	1	19 <i>Tauri</i>	DD	5.94	+ .58	.40	— .63	—3.5	5.6	+1.3
G	1	20 <i>Tauri</i>	DD	5.94	+ .58	.40	— .89	—6.6	7.4	+0.2
G	1	22 <i>Tauri</i>	RB	5.94	+ .58	.40	+ .82	0.0	0.0*	—6.2
G	1	22 <i>Tauri</i>	DD	5.94	+ .58	.40	— .67	—4.2	6.3	+0.9
G	1	21 <i>Tauri</i>	DD	5.94	+ .58	.40	— .54	—4.3	7.9	—0.2
G	1	16 <i>Tauri</i>	RB	5.94	+ .58	.40	+ .90	+3.1	3.4	—3.8
G	1	19 <i>Tauri</i>	RB	5.94	+ .58	.40	+ .78	+4.2	5.4	—1.8
G	1	20 <i>Tauri</i>	RB	5.94	+ .58	.40	+ .90	+5.0	5.6	—1.8
G	1	21 <i>Tauri</i>	RB	5.94	+ .58	.40	+ .73	+3.1	4.3	—2.4
G	21	<i>c Aquarii</i>	DD	5.18	— .62	.44	— .68	—6.3	9.3	—1.1
W	23	21 <i>Piscium</i>	DD	5.82	— .64	.42	— .86	—6.9	8.0	—0.4
W	23	21 <i>Piscium</i>	RB	5.82	— .64	.42	+ .93	+5.7	6.2	—1.3
W	1915 Jan. 1	<i>A Geminorum</i>	DB	5.05	+ .70	.43	— .94	—5.7	6.0	+1.5
W	1	<i>A Geminorum</i>	RD	5.05	+ .70	.43	+ .88	+6.2	7.0	—0.5
W	8	75 <i>Virginis</i>	DB	3.64	+1.14	.50	— .74	—3.2	4.3	+2.4
W	29	9 <i>Cancri</i>	DD	5.99	+ .52	.44	— .94	—8.2	8.7	—1.0
W	Feb. 9	55 <i>G Sagittarii</i>	RD	2.72	+ .63	.49	+1.06	+8.6	8.1	+0.4
W	26	η <i>Cancri</i>	DD	6.86	+ .23	.44	— .57	—5.5	9.7	—1.1
W	26	η <i>Cancri</i>	RB	6.84	+ .23	.44	+ .78	+5.9	7.6	—0.1
W	26	102 <i>B Cancri</i>	DD	6.78	+ .23	.44	— .99	—7.6	7.7	0.0
W	26	ϵ <i>Cancri</i>	DD	6.78	+ .23	.44	— .94	—8.0	8.5	—0.8
W	Mar. 31	75 <i>Virginis</i>	DB	5.14	+ .93	.50	—1.09	—8.3	7.6	+0.1
W	31	75 <i>Virginis</i>	RD	+5.11	+ .93	+ .50	+1.03	+4.7	+4.6	—3.2

* Not subsequently used.

The values of the short period empirical terms which have been applied to the occultations in *A. J.*, 667, are as follows:

TABLE II
EMPIRICAL SHORT PERIOD TERMS

Year		Year	
1908.0	+2".51	1912.0	+3".01
1909.0	2 .76	1913.0	2 .91
1910.0	2 .91	1914.0	2 .78
1911.0	2 .99	1915.0	2 .60

These enable the results obtained there to be reduced to the datum of the present paper.

Minor residuals, mean values.—The values of δM in the above table have been combined by weights into three mean values, at approximately equidistant epochs, the breaks in the table showing the limiting dates. The results are exhibited in the following table, which includes as well the results of the previous investigation (*loc. cit.*). Bright limb emersions have not been used. The weight of dark limb immersions has been taken to be

TABLE III
MEAN VALUES OF MINOR RESIDUALS

Mean Epoch	δM	Mean Epoch	δM
1908.58	+3".33	1913.22	+6".96
1909.46	3 .64	1913.66	6 .81
1910.79	3 .72	1914.10	7 .29
1911.29	4 .49	1914.51	7 .68
1912.64	5 .81	1915.08	7 .33

TABLE IV.
MINOR AND MAJOR RESIDUALS IN THE *Moon's* LONGITUDE

Year	δM	δT	Year	δM	δT	Year	δM	δT	Year	δM	δT
1681.0	−0.2	−10.8	1852.5	+1.8	+4.2	1875.5	−2.2	−6.5	1899.5	−2.3	−12.4
1710.0	+0.1	−3.7	1853.5	+1.6	+3.7	1876.5	−2.3	−6.9	1900.5	−0.8	−11.0
1727.0	+0.1	+1.2	1854.5	+2.1	+3.9	1878.0	−2.4	−7.5	1901.5	−0.8	−11.2
1737.0	0.0	+4.0	1855.5	+2.6	+4.1	1879.5	−1.3	−6.7	1902.5	−0.2	−10.8
1747.0	−0.4	+6.3	1856.5	+2.2	+3.4	1880.5	−2.3	−8.0	1903.5	+0.5	−10.3
1755.0	−0.3	+8.3	1857.5	+2.0	+2.9	1881.5	−2.5	−8.5	1904.5	+1.3	−9.6
1771.0	+0.7	+12.2	1858.5	+3.2	+3.8	1882.5	−2.1	−8.3	1905.5	+1.9	−9.2
1784.7	+1.0	+13.8	1859.5	+3.3	+3.6	1883.5	−2.7	−9.2	1906.5	+1.9	−9.3
1792.0	+0.2	+13.2	1860.5	+3.1	+3.1	1884.5	−2.4	−9.1	1907.5	+2.8	−8.6
1801.5	−0.6	+12.0	1861.5	+2.4	+2.1	1885.5	−2.4	−9.4	1908.3	+3.0	−8.5
1809.0	−0.4	+11.6	1862.5	+3.0	+2.4	1886.5	−2.5	−9.7	1908.6	+3.3	−8.2
1813.0	−0.6	+10.9	1863.5	+2.3	+1.4	1887.5	−2.0	−9.5	1909.5	+3.6	−8.0
1821.0	−0.2	+10.0	1864.5	+2.4	+1.2	1888.5	−2.7	−10.4	1910.8	+3.7	−8.1
1822.5	0.0	+9.9	1865.5	+2.3	+0.8	1889.5	−2.6	−10.6	1911.3	+4.5	−7.4
1829.5	−1.6	+6.9	1866.5	+2.7	+1.0	1890.5	−2.5	−10.7	1912.6	+5.8	−6.2
1833.5	−0.7	+6.8	1867.5	+1.4	−0.6	1891.5	−2.4	−10.8	1913.2	+7.0	−5.1
1838.5	−0.5	+5.8	1868.5	+1.6	−0.7	1892.5	−2.1	−10.7	1913.7	+6.8	−5.3
1843.0	+0.2	+5.3	1869.5	+2.3	−0.3	1893.5	−2.4	−11.2	1914.1	+7.3	−4.9
1846.5	+0.6	+4.7	1870.5	+1.6	−1.3	1894.5	−2.9	−11.9	1914.5	+7.7	−4.5
1848.5	+2.0	+5.5	1871.5	−0.4	−3.6	1895.5	−2.5	−11.8	1915.1	+7.3	−5.0
1849.5	+0.5	+3.8	1872.5	−0.9	−4.4	1896.5	−1.8	−11.3			
1850.5	+1.6	+4.6	1873.5	−1.1	−4.8	1897.5	−2.8	−12.5			
1851.5	+1.6	+4.3	1874.5	−1.7	−5.7	1898.5	−2.4	−12.3			

$4E^2$, of dark limb emersions and bright limb immersions to be $3E^2$, as in the previous paper.

The rapid increase in δM during 1911 and 1912 is not in evidence in 1913 and 1914. Apparently a maximum has been reached in the latter year, or possibly during 1915. This is a conclusion, however, which the complete curve of minor residuals does not bear out. (*See Diagram.*)

The table of minor residuals, δM , of the *Moon's* longitude from 1681 to 1908, given by NEWCOMB in *Monthly Notices*, Vol. 69, p. 166, requires a small correction

$$\Delta_1(\delta M) = +0''.40 \sin \Omega - 0''.07 \cos \Omega$$

due to an error in his transformation of HANSEN's node term. A further correction

$$\Delta_2(\delta M) = +0''.48 \sin \Omega$$

is necessary to reduce the node term from HILL's to HAYFORD's ellipticity of the *Earth*. The minor residuals, δM , thus corrected are given in the following table, in which for completeness the above found values for 1908–15 are included. In addition the outstanding differences between observation and pure theory, δT , or major residuals, are given. These are obtained from δM by adding to it NEWCOMB's long period empirical term.

The diagrams appended were drawn from the data in the above table. The tabular values of δM and δT are represented by dots in the first and second diagrams respectively. NEWCOMB's long period empirical term is

shown as a broken curve. That it will fail to represent future observations even in a general way is increasingly apparent since 1910.

Eclipse of 1916, Feb. 3. — From the curve of minor residuals, δM , a tolerably accurate value of the correction to the predicted mean longitude of the *Moon* can be extrapolated for the date of the eclipse of Feb. 3, 1916, which will furnish a useful correction to the predicted time of totality. For this date is found from the curve

$$\delta M = +8''.5$$

A correction of $\delta M = +2''.4$ has been applied in the computation of the time of totality of the *American Ephemeris*. The additional correction to be applied to the mean longitude adopted in the *American Ephemeris* eclipse data is then $+6''.1$, or $+6''.9$ in true longitude, which transforms to $+0^s.47$ in right ascension. A correction of $+0^s.09$ is to be applied to the *Sun's* right ascension (see *A. J.* 667). From these numbers results:

Correction to *American Eph.* time . . . -10^s .

Discussion of the occultations. — The occultations tabulated and reduced in the present paper, and in the previous one in *A. J.* 667, have been compared with a tolerably correct theory. On account of this circumstance, and the large number of occultations reduced, it is possible to make a comparative study of the *Moon's* longitude separately for the four classes of phenomena: dark limb disappearances, DD; bright limb disappearances, BD; dark limb reappearances, DR; and bright limb reappearances, BR.

All of the occultations (1908–15) should be reduced to one epoch. 1912.0 was chosen. To obtain data for this reduction, the mean values of δM found above were plotted, and a smooth curve drawn. The values for the beginning of each year from 1908 to 1915 were read off. For the occultations in *A. J.* 667, it is necessary to combine with these the values given in Table II above, to withdraw the short period empirical terms which had been included. The results are as follows:

Correction to δM (1908, Apr. 13 — 1913, Nov. 17)		Correction to δM (1913, Dec. 5* 1915, Mar. 31)	
1908.0	$+4''.1$	1913.0	$-1''.4$
1909.0	$+4''.3$	1914.0	$-2''.3$
1910.0	$+4''.2$	1915.0	$-2''.6$
1911.0	$+3''.8$		
1912.0	$+3''.0$		
1913.0	$+1''.5$		
1914.0	$+0''.5$		

The value of δM obtained from each occultation was corrected from the numbers in the above table. The corrected values were collected according to the phe-

nomenon, and means taken, with results shown in the following table. In forming the means the weight of the individual values of δM was taken as E^2 .

TABLE V
CORRECTIONS TO THE MOON'S LONGITUDE GIVEN
BY THE FOUR PHENOMENA

Phen.	δM	p. e. (δM)	p. e. (one occ. of unit wt.)
DD	$+5''.07$	$\pm 0''.11$	$\pm 0''.72$
DB	$+5''.12^1$	0 .14	0 .73
RD	$+4''.73^2$	0 .14	0 .82
RB	$+2''.92^3$	0 .26	0 .96

¹ Occ. of 1913, Apr. 29, discarded.

² Occ. of 1914, Mar. 14 (231 *G Virg.*) discarded.

³ Occ. of 1914, Dec. 1 (22 *Tauri*) discarded.

Reappearances at the bright limb. — Comparison of the value of δM for the phenomenon RB with the others discloses the fact that reappearances at the bright limb are subject to a very large systematic error, they being observed on the average four seconds late. That this should be true for observations of occultations of bright stars with an instrument as powerful as the 26-inch Washington equatorial, is surprising. Comparison of the times of emersions at the bright limb of stars observed with 12-inch and 26-inch in the published lists reveals the fact that the phenomenon is observed with the 12-inch almost invariably later by several seconds than with the 26-inch. The size of the error depends, therefore, upon the power of the telescope, as well as the magnitude of the star. It does not appear that observations of this phenomenon, even with a large telescope, serve any useful end. The results have been rejected in the reductions of this paper.

Semidiameter of the Moon. — Comparison of the values of δM in Table V leads to a value of the "occultation semidiameter" of the *Moon* of considerable weight. The value adopted in the computations is NEWCOMB'S (*Researches*, II¹, p. 40), namely,

$$s = [4.74977] \sin \pi$$

$$(\pi = \text{HANSEN'S parallax} + 0''.40)$$

Comparing δM for DB and RD above, it is seen that the adopted semidiameter is too small by $0''.20 \pm 0''.10$. The present series of occultations accordingly gives for s

$$s = [4.74988] \sin \pi$$

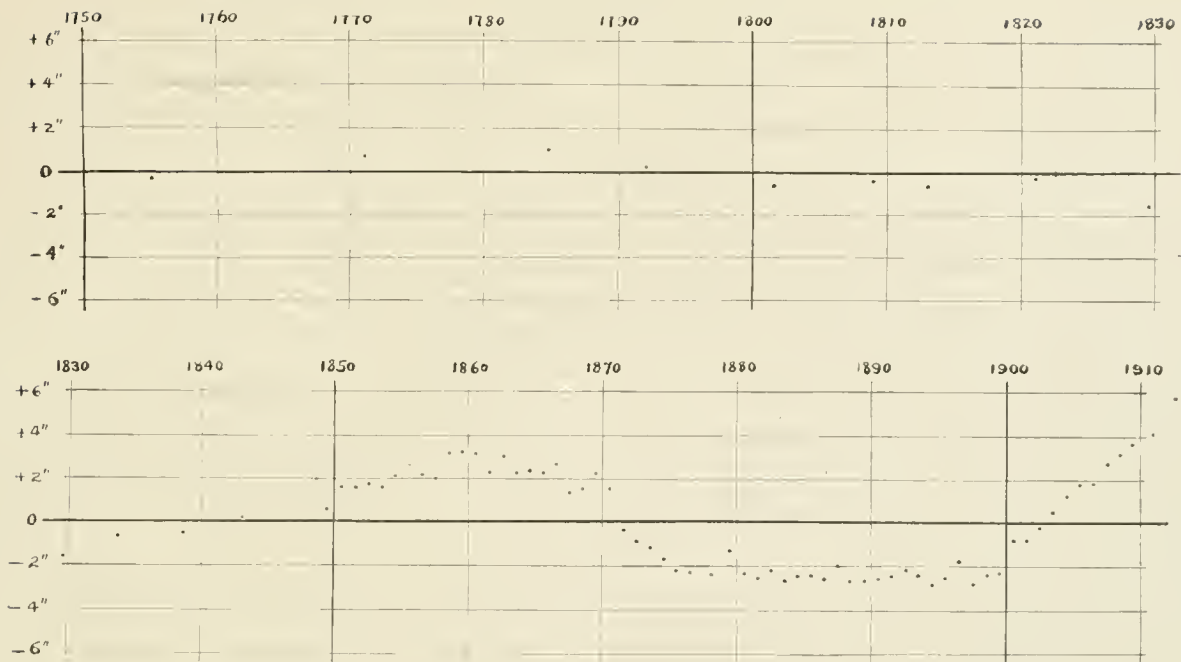
$$(\pi = \text{HANSEN'S parallax} + 0''.45)$$

Apparently the phenomena DD and DB give identical results. The question of their agreement is bound up with that of the true value of the lunar parallactic equation, which will be discussed in another paper.

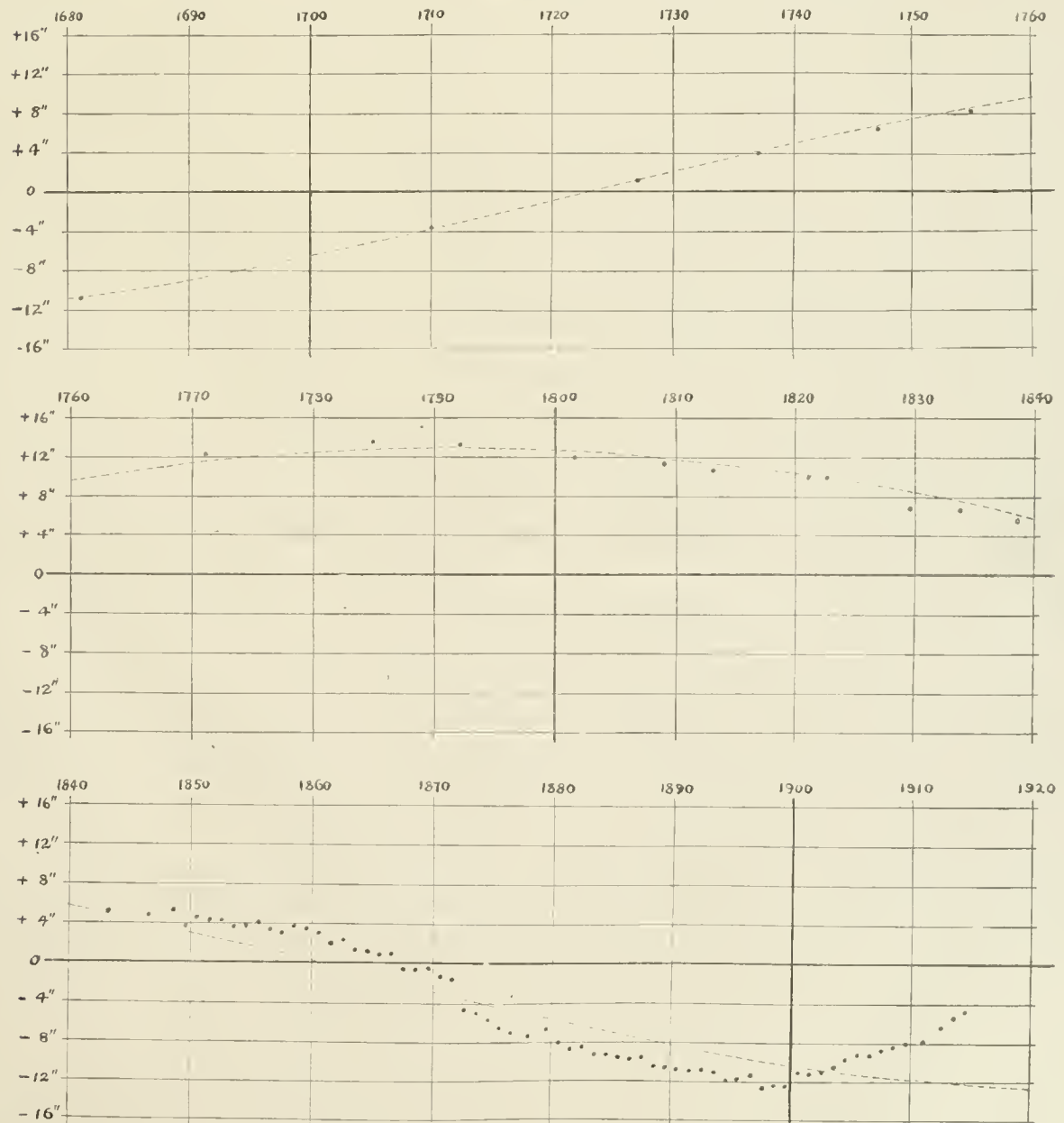
Nautical Almanac Office, U. S. Naval Obs., Sept. 15.

¹ *Astronomical Papers of the American Ephemeris*, Vol. IX, Part I

MINOR RESIDUALS, $0 - C$, *Moon's* MEAN LONGITUDE, 1750-1915.



MAJOR RESIDUALS, $0 - C$, *Moon's* MEAN LONGITUDE, 1681-1915.



TABLES FOR PARALLAX FACTORS IN LONGITUDE.

By J. H. PITMAN.

In the determination of stellar parallaxes, a great amount of time is used in computing parallax factors when each is determined separately. The following tables were constructed in order that parallax factors sufficiently accurate for present methods of investigation might be obtained with greater ease. These tables give the parallax factors in the case where the axis of measurement has been chosen parallel to the ecliptic. The annual parallax multiplied by these factors gives the shift in this direction.

The horizontal argument, $\odot$, is the longitude of the *Sun* at the date of observation. The vertical argument is, $\odot - \lambda$, or the longitude of the *Sun* minus the longitude of the star. It was not thought necessary to complete the tables so as to include the factors for $160^\circ - 200^\circ$ and $340^\circ - 20^\circ$.

The computation was made by means of the equation

$$P = -R \sin (\lambda - \odot) = R \sin (\odot - \lambda)$$

where R is the radius vector of the earth at the date of observation. In the latter form the signs of the factors are the same as $\sin (\odot - \lambda)$ or in other words P is positive when $(\odot - \lambda)$ is less than 180° and negative when $(\odot - \lambda)$ is greater than 180° . In case the value of $(\odot - \lambda)$ is not given in the table, look up the value of the factor corresponding to $(\lambda - \odot)$ and take the sign the same as for $(\odot - \lambda)$.

$\lambda = 153^\circ$, $\odot = 45^\circ$, $\odot - \lambda = 252^\circ$. P is negative.

This value of $(\odot - \lambda)$ is not given in the table, therefore take $\lambda - \odot = 108^\circ$. We then find $P = -0.960$.

The computations were carried out to four decimal places for the even angles from 80° to 160° inclusive and were checked by the method of differences. The values of the factors for the odd angles within the above limits were obtained by interpolation to halves. Three decimal places were then tabulated and again checked by differences. The values given are correct to .001.

Longitude of the Sun.

$\odot - \lambda$	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°	$\odot - \lambda$
260° 80°	.981	.986	.990	.994	.997	.999	1.001	1.001	1.000	.998	.996	.992	.988	80° 260°
262 82	.987	.991	.996	1.000	1.003	1.005	1.007	1.007	1.006	1.004	1.001	.998	.994	82 262
264 84	.991	.995	1.000	1.004	1.007	1.009	1.011	1.011	1.010	1.008	1.005	1.002	.998	84 264
266 86	.994	.998	1.003	1.007	1.010	1.012	1.014	1.014	1.013	1.011	1.008	1.005	1.001	86 266
268 88	.996	1.000	1.004	1.008	1.012	1.014	1.016	1.016	1.015	1.013	1.010	1.007	1.003	88 268
270 90	.996	1.001	1.005	1.009	1.012	1.015	1.016	1.017	1.016	1.014	1.011	1.007	1.003	90 270
271 91	.996	1.001	1.005	1.009	1.012	1.015	1.016	1.016	1.016	1.014	1.011	1.007	1.003	91 271
272 92	.996	1.000	1.004	1.008	1.012	1.014	1.016	1.016	1.015	1.013	1.010	1.007	1.003	92 272
273 93	.995	.999	1.003	1.007	1.011	1.013	1.015	1.015	1.014	1.012	1.009	1.006	1.002	93 273
274 94	.994	.998	1.002	1.006	1.010	1.012	1.014	1.014	1.013	1.011	1.008	1.005	1.001	94 274
275 95	.993	.997	1.001	1.005	1.009	1.011	1.013	1.013	1.012	1.010	1.007	1.004	1.000	95 275
276 96	.991	.995	1.000	1.004	1.007	1.009	1.011	1.011	1.010	1.008	1.005	1.002	.998	96 276
277 97	.989	.993	.998	1.002	1.005	1.007	1.009	1.009	1.008	1.006	1.003	1.000	.996	97 277
278 98	.987	.991	.996	1.000	1.003	1.005	1.007	1.007	1.006	1.004	1.001	.998	.994	98 278
279 99	.984	.989	.993	.997	1.000	1.002	1.004	1.004	1.003	1.001	.999	.995	.991	99 279
280 100	.981	.986	.990	.994	.997	.999	1.001	1.001	1.000	.998	.996	.992	.988	100 280
281 101	.978	.983	.987	.991	.994	.996	.998	.998	.997	.995	.993	.989	.985	101 281
282 102	.975	.979	.983	.987	.990	.993	.994	.995	.994	.992	.989	.985	.981	102 282
283 103	.971	.975	.979	.983	.986	.989	.990	.991	.990	.988	.985	.981	.977	103 283
284 104	.967	.971	.975	.979	.982	.985	.986	.987	.986	.984	.981	.977	.973	104 284
285 105	.963	.967	.971	.975	.978	.981	.982	.982	.982	.980	.977	.973	.969	105 285
286 106	.958	.962	.966	.970	.973	.976	.977	.977	.977	.975	.972	.968	.964	106 286
287 107	.953	.957	.961	.965	.968	.971	.972	.972	.972	.970	.967	.963	.959	107 287
288 108	.948	.952	.956	.960	.963	.966	.967	.967	.967	.965	.962	.958	.954	108 288
289 109	.942	.946	.950	.954	.957	.960	.961	.961	.961	.959	.956	.953	.949	109 289

Longitude of the Sun (Continued.)

$\odot - \lambda$	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°	$\odot - \lambda$
290° 110°	.936	.940	.944	.948	.951	.954	.955	.955	.955	.953	.950	.947	.943	110° 290°
291 111	.930	.934	.938	.942	.945	.948	.949	.949	.949	.947	.944	.941	.937	111 291
292 112	.924	.928	.932	.936	.939	.941	.943	.943	.942	.940	.937	.934	.930	112 292
293 113	.917	.921	.925	.929	.932	.934	.936	.936	.935	.933	.931	.927	.923	113 293
294 114	.910	.914	.918	.922	.925	.927	.929	.929	.928	.926	.924	.920	.916	114 294
295 115	.903	.907	.911	.915	.918	.920	.922	.922	.921	.919	.917	.913	.909	115 295
296 116	.896	.900	.903	.907	.910	.912	.914	.914	.913	.911	.909	.905	.902	116 296
297 117	.888	.892	.895	.899	.902	.904	.906	.906	.905	.903	.901	.897	.894	117 297
298 118	.880	.884	.887	.891	.894	.896	.898	.898	.897	.895	.893	.889	.886	118 298
299 119	.872	.876	.879	.883	.886	.888	.889	.889	.889	.887	.884	.881	.878	119 299
300 120	.863	.867	.870	.874	.877	.879	.880	.880	.880	.878	.875	.872	.869	120 300
301 121	.854	.858	.861	.865	.868	.870	.871	.871	.871	.869	.866	.863	.860	121 301
302 122	.845	.849	.852	.856	.859	.861	.862	.862	.862	.860	.857	.854	.851	122 302
303 123	.836	.840	.843	.846	.849	.851	.853	.853	.852	.851	.848	.845	.842	123 303
304 124	.826	.830	.833	.836	.839	.841	.843	.843	.842	.841	.838	.835	.832	124 304
305 125	.816	.820	.823	.826	.829	.831	.833	.833	.832	.831	.828	.825	.822	125 305
306 126	.806	.810	.813	.816	.819	.821	.823	.823	.822	.821	.818	.815	.812	126 306
307 127	.796	.800	.803	.806	.809	.811	.812	.812	.811	.810	.808	.805	.801	127 307
308 128	.785	.789	.792	.795	.798	.800	.801	.801	.800	.799	.797	.794	.790	128 308
309 129	.774	.778	.781	.784	.787	.789	.790	.790	.789	.788	.786	.783	.779	129 309
310 130	.763	.767	.770	.773	.776	.778	.779	.779	.778	.777	.775	.772	.768	130 310
311 131	.752	.756	.759	.762	.764	.766	.767	.768	.767	.765	.763	.761	.757	131 311
312 132	.741	.744	.747	.750	.752	.754	.755	.756	.755	.753	.751	.749	.746	132 312
313 133	.729	.732	.735	.738	.740	.742	.743	.744	.743	.741	.739	.737	.734	133 313
314 134	.717	.720	.723	.726	.728	.730	.731	.732	.731	.729	.727	.725	.722	134 314
315 135	.705	.708	.711	.714	.716	.718	.719	.719	.719	.717	.715	.713	.710	135 315
316 136	.693	.696	.698	.701	.703	.705	.706	.706	.706	.704	.703	.700	.697	136 316
317 137	.680	.683	.685	.688	.690	.692	.693	.693	.693	.691	.690	.687	.684	137 317
318 138	.667	.670	.672	.675	.677	.679	.680	.680	.680	.678	.677	.674	.671	138 318
319 139	.654	.657	.659	.662	.664	.666	.667	.667	.667	.665	.664	.661	.658	139 319
320 140	.641	.644	.646	.649	.651	.653	.654	.654	.653	.652	.650	.648	.645	140 320
321 141	.628	.630	.633	.635	.637	.639	.640	.640	.639	.638	.636	.634	.632	141 321
322 142	.614	.616	.619	.621	.623	.625	.626	.626	.625	.624	.622	.620	.618	142 322
323 143	.600	.602	.605	.607	.609	.611	.612	.612	.611	.610	.608	.606	.604	143 323
324 144	.586	.588	.591	.593	.595	.597	.598	.598	.597	.596	.594	.592	.590	144 324
325 145	.572	.574	.577	.579	.581	.583	.584	.584	.583	.582	.580	.578	.576	145 325
326 146	.558	.560	.563	.565	.566	.568	.569	.569	.568	.567	.566	.564	.561	146 326
327 147	.543	.545	.548	.550	.551	.553	.554	.554	.553	.552	.551	.549	.546	147 327
328 148	.528	.530	.533	.535	.536	.538	.539	.539	.538	.537	.536	.534	.531	148 328
329 149	.513	.515	.518	.520	.521	.523	.524	.524	.523	.522	.521	.519	.516	149 329
330 150	.498	.500	.503	.505	.506	.508	.509	.509	.508	.507	.506	.504	.501	150 330
331 151	.483	.485	.488	.490	.491	.493	.494	.494	.493	.492	.491	.489	.486	151 331
332 152	.468	.470	.473	.474	.476	.477	.478	.478	.477	.476	.475	.474	.471	152 332
333 153	.453	.455	.467	.458	.460	.461	.462	.462	.461	.460	.459	.458	.456	153 333
334 154	.437	.439	.441	.442	.444	.445	.446	.446	.445	.444	.443	.442	.440	154 334
335 155	.421	.423	.425	.426	.428	.429	.430	.430	.429	.428	.427	.426	.424	155 335
336 156	.405	.407	.409	.410	.412	.413	.414	.414	.413	.412	.411	.410	.408	156 336
337 157	.389	.391	.393	.394	.396	.397	.398	.398	.397	.396	.395	.394	.392	157 337
338 158	.373	.375	.377	.378	.380	.381	.381	.381	.381	.380	.379	.378	.376	158 338
339 159	.357	.359	.360	.362	.363	.364	.364	.364	.364	.363	.362	.361	.360	159 339

Longitude of the Sun (Continued.)

☉ — λ		180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°	☉ — λ	
260°	80°	.988	.983	.979	.975	.972	.970	.969	.968	.969	.971	.974	.977	.981	80°	260°
262	82	.994	.989	.985	.981	.978	.976	.974	.974	.975	.977	.979	.983	.987	82	262
264	84	.998	.993	.989	.985	.982	.980	.978	.978	.979	.981	.983	.987	.991	84	264
266	86	1.001	.996	.992	.988	.985	.983	.981	.981	.982	.984	.986	.990	.994	86	266
268	88	1.003	.998	.994	.990	.987	.985	.983	.983	.984	.985	.988	.992	.996	88	268
270	90	1.003	.999	.994	.991	.987	.985	.984	.983	.984	.986	.989	.992	.996	90	270
271	91	1.003	.999	.994	.991	.987	.985	.984	.983	.984	.986	.989	.992	.996	91	271
272	92	1.003	.998	.994	.990	.987	.985	.983	.983	.984	.986	.988	.992	.996	92	272
273	93	1.002	.997	.993	.989	.986	.984	.982	.982	.983	.985	.987	.991	.995	93	273
274	94	1.001	.996	.992	.988	.985	.983	.981	.981	.982	.984	.986	.990	.994	94	274
275	95	1.000	.995	.991	.987	.984	.982	.980	.980	.981	.983	.985	.989	.993	95	275
276	96	.998	.993	.989	.985	.982	.980	.978	.978	.979	.981	.983	.987	.991	96	276
277	97	.996	.991	.987	.983	.980	.978	.976	.976	.977	.979	.981	.985	.989	97	277
278	98	.994	.989	.985	.981	.978	.976	.974	.974	.975	.977	.979	.983	.987	98	278
279	99	.991	.986	.982	.978	.975	.973	.972	.971	.972	.974	.977	.980	.984	99	279
280	100	.988	.983	.979	.975	.972	.970	.969	.968	.969	.971	.974	.977	.981	100	280
281	101	.985	.980	.976	.972	.969	.967	.966	.965	.966	.968	.971	.974	.978	101	281
282	102	.981	.977	.973	.969	.966	.963	.962	.962	.963	.965	.967	.971	.975	102	282
283	103	.977	.973	.969	.965	.962	.960	.958	.958	.959	.961	.963	.967	.971	103	283
284	104	.973	.969	.965	.961	.958	.956	.954	.954	.955	.957	.959	.963	.967	104	284
285	105	.969	.965	.961	.957	.954	.952	.950	.950	.951	.953	.955	.959	.963	105	285
286	106	.964	.960	.956	.952	.949	.947	.945	.945	.946	.948	.950	.954	.958	106	286
287	107	.959	.955	.951	.947	.944	.942	.940	.940	.941	.943	.945	.949	.953	107	287
288	108	.954	.950	.946	.942	.939	.937	.935	.935	.936	.938	.940	.944	.948	108	288
289	109	.949	.945	.940	.937	.934	.931	.930	.930	.931	.932	.935	.938	.942	109	289
290	110	.943	.939	.934	.931	.928	.926	.924	.924	.925	.926	.929	.932	.936	110	290
291	111	.937	.933	.928	.925	.922	.920	.918	.918	.919	.920	.923	.926	.930	111	291
292	112	.930	.926	.922	.919	.916	.914	.912	.912	.913	.914	.917	.920	.924	112	292
293	113	.923	.919	.916	.912	.909	.907	.906	.905	.906	.908	.910	.914	.917	113	293
294	114	.916	.912	.909	.905	.902	.900	.899	.898	.899	.901	.903	.907	.910	114	294
295	115	.909	.905	.902	.898	.895	.893	.892	.891	.892	.894	.896	.900	.903	115	295
296	116	.902	.898	.894	.890	.888	.885	.884	.884	.885	.887	.889	.892	.896	116	296
297	117	.894	.890	.886	.883	.880	.878	.876	.876	.877	.879	.881	.884	.888	117	297
298	118	.886	.882	.878	.875	.872	.870	.868	.868	.869	.871	.873	.876	.880	118	298
299	119	.878	.874	.870	.867	.864	.862	.860	.860	.861	.863	.865	.868	.872	119	299
300	120	.869	.865	.861	.858	.855	.853	.852	.852	.853	.854	.857	.860	.863	120	300
301	121	.860	.856	.852	.849	.846	.844	.843	.843	.844	.845	.848	.851	.854	121	301
302	122	.851	.847	.843	.840	.837	.835	.834	.834	.835	.836	.839	.842	.845	122	302
303	123	.842	.838	.834	.831	.828	.826	.825	.825	.826	.827	.830	.833	.836	123	303
304	124	.832	.828	.825	.821	.819	.817	.816	.815	.816	.818	.820	.823	.826	124	304
305	125	.822	.818	.815	.811	.809	.807	.806	.805	.806	.808	.810	.813	.816	125	305
306	126	.812	.808	.805	.801	.799	.797	.796	.795	.796	.798	.800	.803	.806	126	306
307	127	.801	.798	.795	.791	.789	.787	.786	.785	.786	.788	.790	.793	.796	127	307
308	128	.790	.787	.784	.781	.778	.777	.775	.775	.776	.777	.779	.782	.785	128	308
309	129	.779	.776	.773	.770	.767	.766	.764	.764	.765	.766	.768	.771	.774	129	309
310	130	.768	.765	.762	.759	.756	.755	.753	.753	.754	.755	.757	.760	.763	130	310
311	131	.757	.754	.751	.748	.745	.744	.742	.742	.743	.744	.746	.749	.752	131	311
312	132	.746	.742	.739	.736	.734	.732	.731	.731	.732	.732	.735	.738	.741	132	312
313	133	.734	.730	.727	.724	.722	.720	.720	.719	.720	.721	.723	.726	.729	133	313
314	134	.722	.718	.715	.712	.710	.708	.708	.707	.708	.709	.711	.714	.717	134	314

Longitude of the Sun (Continued.)

$\odot - \lambda$		180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°	$\odot - \lambda$	
315° 135°		.710	.706	.703	.700	.698	.696	.696	.695	.696	.697	.699	.702	.705	135° 315°	
316 136		.697	.694	.691	.688	.686	.684	.684	.683	.684	.685	.687	.690	.693	136 316	
317 137		.684	.681	.678	.676	.674	.672	.671	.671	.672	.673	.675	.677	.680	137 317	
318 138		.671	.668	.665	.663	.661	.659	.658	.658	.659	.660	.662	.664	.667	138 318	
319 139		.658	.655	.652	.650	.648	.646	.645	.645	.646	.647	.649	.641	.654	139 319	
320 140		.645	.642	.639	.637	.635	.633	.632	.632	.633	.634	.636	.638	.641	140 320	
321 141		.632	.629	.626	.623	.622	.620	.619	.619	.620	.621	.623	.625	.628	141 321	
322 142		.618	.615	.612	.610	.608	.607	.606	.606	.606	.607	.609	.611	.614	142 322	
323 143		.604	.601	.598	.596	.594	.593	.592	.592	.592	.593	.595	.597	.600	143 323	
324 144		.590	.587	.584	.582	.580	.579	.578	.578	.578	.579	.581	.583	.586	144 324	
325 145		.576	.573	.570	.568	.566	.565	.564	.564	.564	.565	.567	.569	.572	145 325	
326 146		.561	.559	.556	.554	.552	.551	.550	.550	.550	.551	.553	.555	.558	146 326	
327 147		.546	.544	.542	.540	.538	.537	.536	.536	.536	.537	.539	.541	.543	147 327	
328 148		.531	.529	.527	.525	.524	.522	.522	.522	.522	.523	.524	.526	.528	148 328	
329 149		.516	.514	.512	.510	.509	.507	.507	.507	.507	.408	.509	.511	.513	149 329	
330 150		.501	.499	.497	.495	.494	.492	.492	.492	.492	.493	.494	.496	.498	150 330	
331 151		.486	.484	.482	.480	.479	.477	.477	.477	.477	.478	.479	.481	.483	151 331	
332 152		.471	.469	.467	.465	.464	.462	.462	.462	.462	.463	.464	.466	.468	152 332	
333 153		.456	.454	.452	.450	.449	.447	.447	.447	.447	.448	.449	.451	.453	153 333	
334 154		.440	.438	.436	.435	.434	.432	.432	.432	.432	.433	.434	.436	.437	154 334	
335 155		.424	.422	.420	.419	.418	.417	.416	.416	.417	.417	.418	.420	.421	155 335	
336 156		.408	.406	.404	.403	.402	.401	.400	.400	.401	.401	.402	.404	.405	156 336	
337 157		.392	.390	.388	.387	.386	.385	.384	.384	.385	.385	.386	.388	.389	157 337	
338 158		.376	.374	.372	.371	.370	.369	.368	.368	.369	.369	.370	.372	.373	158 338	
339 159		.360	.358	.356	.355	.354	.353	.352	.352	.353	.353	.354	.356	.357	159 339	

*Sprout Observatory, Sept. 9, 1915.*OBSERVATIONS OF THE CONDENSATIONS NEAR THE NUCLEUS
OF COMET 1915 α (MELLISH).POSITIONS OF THE CONDENSATIONS MEASURED ON PHOTOGRAPHS
OF THE COMET OBTAINED WITH THE 40-INCH REFLECTOR
OF THE LOWELL OBSERVATORY,

By C. O. LAMPLAND.

1915 Gr. M.T.	P.A. nuc. - (C ₁ + C ₂)	DISTANCE nuc. - C ₁ nuc. - C ₂	
May 20 20 ^h 46 ^m		29"	48"
May 21 21 23	290°	32	52

For the outermost condensation the visual and photographic observations (see Mr. Slipher's measures below) may not refer to precisely the same point. This condensation appears on the photographs as an elongated mass, with its greatest length in the direction of the line to the nucleus, and involved in its nebosity there are apparently several small condensations. In the measurements of the photographs settings were made near the middle point of the mass. During the short interval covered by

Lowell Observatory, Flagstaff, Arizona, May 25, 1915.

these observations both the visual and photographic measures agree in showing a progressive motion of the condensations away from the nucleus.

MEASURES OF THE CONDENSATIONS NEAR THE COMET'S NUCLEUS,
WITH THE 24-INCH REFRACTOR AND FILAR MICROMETER,

By E. C. SLIPHER.

1915 Gr. M.T.	P.A.		DISTANCE		
May	C ₁	C ₂	C ₃	C ₁	C ₂
21 21 ^h 40 ^m			290°.1		53".8
22 22 30	294°		290 .1	33".1	57".6
23 21 55	294 .6			36 .8	
23 22 10			290 .0		63 .4
23 22 18		290°.9			53".
23 22 45				37 .3	
23 22 55					53 .31
23 23 5					63 .7

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OBSERVATIONS OF THE CONDENSATIONS NEAR THE NUCLEUS OF COMET 1915 α (MELLISH), BY C. O. LAMPLAND AND E. C. SLIPHER.

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NO. 10

OBSERVATIONS OF THE SIXTH SATELLITE OF JUPITER,

By E. E. BARNARD.

The following observations of PERRINE's sixth satellite of *Jupiter* have been made with the large telescope. Under fair conditions the satellite is an extremely easy object and can be measured with much accuracy. The following table contains the estimates of its brightness, which were made during the observations for position.

ESTIMATES OF BRIGHTNESS

1915 Oct. 5	13	magnitude
Oct. 26	14	"
Nov. 2	13.7	"
Nov. 4	14	"
Nov. 11	13½	"
Dec. 1	14	"

Mean = 13.7 magnitude

Positions of the Sixth Satellite.

1915	C. S. T.	$\Delta \alpha \cos \delta$	$\Delta \alpha$	$\Delta \delta$	Comps.	α Appt.	δ Appt.	Red. to Appt.	*
Oct. 5	13 ^h 1 ^m 15 ^s	-110.49	-0 ^m 7.39 ^s	-1 ['] 29.6 ["]	6-3	23 30 46.3	-4 57.7	+4.28 +26.2	1
26	8 6 45	-110.66	-0 7.41	-3 14.1	10-5	23 21 21.53	-5 36 49.2	+4.17 +24.9	2†
28	8 12 41	+31.62	+0 2.13	+1 17.6	9-5	23 20 42.94	-5 38 59.9	+4.16 +24.8	3‡
Nov. 2	7 16 50	+171.61	+0 11.50		4	23 19 21.2		+4.12	4
2	7 22 41			+3 14.2	4		-5 43.1	 +24.4	4
2	7 28 34	+170.06	+0 11.40		4	23 19 21.1		+4.12	4
2	7 32 6			+3 14.2	4		-5 43.1	 +24.4	4
4	7 37 29	-229.84	-0 15.40	+2 7.9	10-5	23 18 54.3	-5 44.2	+4.10 +24.2	4
11	7 14 49	-128.22	-0 8.59		4	23 18 49.4		+4.03	5
11	7 21 26			+1 24.0	5		-5 45.5	 +24.0	5
11	7 28 7	-128.67	-0 8.62		5	23 18 49.4		+4.03	5
11	7 34 57			+1 24.8	5		-5 45.5	 +24.0	5
Dec. 1	8 57 40	+206.84	+0 13.85	-1 28.4	9-5	23 18 6.3	-5 23.8	+3.81 +22.6	6

† Seeing very bad. Measures difficult.

‡ Satellite very difficult from haze and bad seeing.

Mean Places of Comparison Stars.

*	α 1915.0	δ 1915.0	Authority
1	23 ^h 30 ^m 49 ^s .4	-4° 56' 6"	B.D. -5°6017.
2	23 21 24.77	-5 34 0.0	Strassburg A. G. C. 8053.
3	23 20 36.65	-5 40 42.3	Wien-Ottakring A. G. C. 8311.
4	23 19 5.6	-5 46.7	B.D. -6°6199.
5	23 18 54.0	-5 47.3	13 mag. Compared with Star 4.
6	23 17 48.6	-5 22.7	12 mag. Compared with B.D. -5°5975.

Measured Positions of Stars 5 and 6.

			$\Delta\alpha$	Comps.	$\Delta\delta$	Comps.
1915	Nov. 11	Star 5 — Star 4	$-0^m 11^s.58$	14 tr.	$-0' 38''.5$	4
	Dec. 1	Star 6 — B.D. $-5^\circ 59' 75$	$[209''.19] +0 14.01$	3	$+2 37.2$	3

DR. EICHELBERGER, Superintendent of the *Nautical Almanac*, informs me that the observations of October 5 and 26 give the following correction to the ephemeris:

$$\text{In } \alpha - 4^s, \quad \text{In } \delta + 0'.1$$

During the observations of December 1, a small double star was noticed in the field with the satellite.

$$P. A. 351^\circ.5 \quad \text{Dist. } 3''.29 \quad (1 \text{ night}) \quad 12^m - 12\frac{1}{2}^m$$

The approximate position is:

$$1915.0 \quad \alpha 23^h 18^m.0 \quad \delta - 5^\circ 22'.1$$

Yerkes Observatory, Williams Bay, Wisconsin, 1915, December 24.

THE INFLUENCE OF QUADRATIC TERMS IN REDUCTIONS OF STELLAR PARALLAXES.

By OLIVER J. LEE.

In determining stellar parallaxes from plates taken with telescopes of short focal length, such as those of the astrographic type, corrections have usually been applied for terms of the second order in refraction and aberration. The large field, about two degrees square, and sometimes large hour angles of observing, have rendered this additional labor desirable or necessary. Thus KAPTEYN,* in reducing the Helsingfors plates, deduced the corrections of second order directly from the measures, while KOSTINSKY† derived them *a priori* for refraction alone and applied them after making the solution with linear terms. When the plates are taken with a telescope of relatively very long focus and uniformly very close to the meridian, it has reasonably been supposed that the influence of the quadratic and higher powers of the coördinates is negligible for all plates exposed under normal conditions.

In his classical discussion of the whole problem, KAPTEYN* remarks: "There are probably no measures of high precision in which sources of error have not been found sooner or later which had not at first been suspected, and it is well known that in most of them there is evidence of errors of which we know nothing even now." Obviously a reduction with linear terms alone takes care of all unknown as well as known corrections that can be represented by a linear formula. Similarly, a reduction which includes also the quadratic terms, will provide for unknown as well as known sources of error, the corrections of which can be represented by such a reduction formula.

In order to ascertain whether or not the second-order corrections are of any consequence in our work with plates taken with the 40-inch telescope, reductions have been made of two sets of measures, using for each, first the linear relation of the form

$$a + bx + cy = n, \quad (1)$$

and then the longer one of the form

$$a + bx + cy + dx^2 + exy + fy^2 = n. \quad (2)$$

The first set of measures pertains to the field K 35 ($15^h 49^m, +45^\circ$) of the Selected Areas. This field has only eighteen well measurable stars on our plates, which cover rectangular fields in the sky 45 minutes of arc east and west by 35 minutes of arc north and south. From two suitable plates the proper motion of each star was derived, using first (1), and then (2). Then the parallax of each star was derived from eleven plates, using again the two forms of reduction. Each plate has one exposure of 30–40 minutes, made immediately east of the meridian. The constants $a, b \dots f$ were derived from least squares solutions. The unit in the quantities given below is the second of arc. The quantity l is the upper limit of the probable error, obtained on the assumption that the real relative parallaxes and proper motions are zero, and that the observed quantities are due to accidental errors alone.

For the proper motions, the solution of

$$\text{equations (1) gave } [v] = 0.006661 \quad l = \pm 0.0112$$

$$\text{equations (2) gave } [v] = 0.004865 \quad l = \pm 0.0136$$

* *Groningen Publications*, 1, 1900.

† *Publications de l'Observatoire Central Nicolas*, Serie II, 17, Pt. 2, 1905.

For the parallaxes,

equations (1) gave $[vv] = 0.011623 \quad l = \pm 0.0188$

equations (2) gave $[vv] = 0.010352 \quad l = \pm 0.0198$

The reduction in $[vv]$ for proper-motions is 27%.

The reduction in $[vv]$ for parallaxes is 11%.

The loss of weight, however, due to the large number of unknowns, almost equals the appreciable reduction in the first case and actually overbalances the small reduction in the second case. This field has about the minimum number of stars with which it is practicable to determine general parallaxes and it was chosen to show the effects of the second-order corrections at this limit.

The second set of measures relates to the field $K39$ ($19^h 47^m, +45^\circ$) of the Selected Areas. Ninety stars were used in this case. As before, two plates were used to derive the proper-motions while the parallaxes are based on ten plates.

For the proper motions

equations (1) gave $[vv] = 0.025358 \quad l = \pm 0.0115$

equations (2) gave $[vv] = 0.023856 \quad l = \pm 0.0114$

For the parallaxes,

equations (1) gave $[vv] = 0.083419 \quad l = \pm 0.0209$

equations (2) gave $[vv] = 0.081631 \quad l = \pm 0.0210$

The reduction in $[vv]$ for proper-motions is 6%.

The reduction in $[vv]$ for parallaxes is 2%.

The loss of weight in the solutions of equations (2) is less for this field because of the large number of equations of condition.

The conclusions are:

1. The reduction of the quantity $[vv]$ obtained by including the quadratic terms in the solution is in most cases negligible, and will in no case justify the additional labor involved in deriving it.

2. The loss of weight in the parallaxes and proper-motions occasioned by determining six unknowns instead of three will on the average about equal the advantage gained by using them.

It is a pleasure to acknowledge the excellent assistance of Miss EUDORA MAGILL of the observatory staff in the above computations.

Yerkes Observatory, Williams Bay, Wisconsin, December, 1915.

A NEW VARIABLE STAR,

1915.0 α 22^h 19^m 44^s.36 δ + 55° 4' 52".6

By E. E. BARNARD.

I have found on my photographs a variable star, which PROFESSOR E. C. PICKERING kindly informs me is new. The position with respect to $BD + 54^\circ 2755$, which is Cambridge (U. S.) A. G. C. 7651, is:

1915.806 $P = 181^\circ.21, \Delta = 152''.68$ (3 nights)

from which the position at the head of this paper is derived.

It has a range of probably six magnitudes or more, and perhaps passes beyond the reach of the 10-inch Bruce photographic telescope. The star is yellowish and the period is probably long. Visual observations show that from October 18 of this year to December 18, it rose from 12.6 to 10.4 magnitude. An 11th magnitude star precedes it in the position:

1915.806 $P = 297^\circ.42, \Delta = 103''.13$ (3 nights)

The variable at maximum is brighter than this star.

On one of my Lick Observatory plates, 1893, October 11, the star is perhaps at full brightness. It is not shown on a plate taken with the 10-inch Bruce lens on 1909, August 22 (exposure 5^h 55^m). It is visible on a plate made 1911, January 21, but below maximum.

Yerkes Observatory, Williams Bay, Wisconsin, 1915, December 24.

NOTE: Further observations indicate that the variable reached its maximum (about $\frac{1}{2}$ magnitude brighter than the 11th magnitude star preceding, or about $10\frac{1}{2}$ magnitude) somewhere near December 25, and that it is now on the decline.

1916, January 10.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A 4½-INCH REFRACTOR.

By A. W. QUMBY.

1915	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.	1915	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.	1915	Time	New Grs.	Total Grs.	Spots	Fac. Grs.	Def.			
July	1	6	—	3	28	2	fair	Sept.	1	6	1	5	11	2	fair	Oct.	31	8	—	3	10	—	fair
	2	6	—	2	13	1	fair		2	6	—	5	10	2	fair		Nov. 1	7	—	2	6	—	poor
	3	6	—	2	11	2	fair		3	6	—	4	17	2	fair		2	7	—	2	12	1	fair
	4	6	—	2	7	1	fair		4	6	2	5	15	2	fair		3	7	—	2	5	1	poor
	5	4	1	2	28	2	v. g.		5	6	—	4	24	4	fair		4	8	—	1	3	1	poor
	6	6	—	2	20	2	fair		6	7	—	3	22	4	fair		5	7	—	1	3	1	poor
	7	6	—	2	38	2	fair		7	4	1	4	20	—	fair		6	9	—	1	4	1	fair
	8	5	—	2	40	2	fair		8	7	1	5	17	2	fair		7	4	2	2	8	1	fair
	9	4	2	3	42	3	fair		9	7	—	4	10	2	fair		8	9	1	3	8	1	fair
	10	6	1	4	54	3	fair		10	7	—	4	8	4	fair		9	8	—	3	8	—	fair
	11	6	1	5	52	2	fair		11	7	—	3	8	2	fair		10	8	—	3	7	0	poor
	12	5	1	6	40	3	fair		12	7	—	2	6	3	fair		11	10	—	2	7	1	poor
	13	6	—	6	27	3	fair		13	7	—	2	4	2	fair		12	8	—	2	6	1	poor
	14	6	—	5	17	2	fair		14	7	—	2	2	2	fair		13	8	—	1	4	1	fair
	15	6	—	4	16	3	fair		15	7	—	1	3	2	fair		14	8	—	1	4	1	poor
	16	6	1	5	10	2	fair		16	7	1	1	8	2	fair		15	4	1	2	3	1	poor
	17	6	—	4	10	3	fair		17	7	—	1	8	2	fair		16	8	—	2	8	3	poor
	18	6	—	3	7	3	fair		18	7	1	2	11	2	fair		17	8	—	2	4	1	poor
	19	6	1	4	8	3	fair		19	9	—	2	11	2	fair		18	8	—	1	2	1	poor
	20	6	—	4	20	3	fair		20	7	—	2	11	2	fair		19	12	3	3	8	—	poor
	21	6	1	5	23	3	fair		21	4	1	3	16	2	fair		20	8	1	4	19	2	fair
	22	6	—	5	79	3	v. g.		22	7	—	2	16	2	fair		21	1	—	2	6	—	poor
	23	6	1	6	67	2	good		23	7	—	2	13	—	fair		*22	9	—	2	8	—	poor
	24	6	1	6	102	2	v. g.		24	7	—	2	10	—	fair		*23	4	—	2	8	1	poor
	25	6	—	5	84	3	good		25	7	2	3	27	2	fair		*24	8	—	2	8	1	poor
	26	6	1	6	105	3	v. g.		26	7	—	1	10	—	poor		25	8	—	2	2	1	poor
	27	6	—	4	89	2	good		27	7	1	4	22	—	fair		26	8	1	2	4	—	fair
	28	6	—	4	50	2	fair		28	7	—	4	12	1	fair		27	8	—	2	8	—	fair
	29	6	1	4	35	2	fair		29	7	—	3	15	1	fair		28	8	—	2	8	1	fair
	30	6	—	4	31	2	fair		30	7	—	3	17	1	fair		29	10	—	2	9	1	fair
	31	6	—	3	50	2	fair	Oct.	2	10	—	2	6	1	poor		30	8	—	1	12	3	fair
Aug.	1	6	—	3	90	2	fair		3	7	1	2	4	2	fair		Dec. 1	8	—	1	12	—	fair
	2	6	—	3	110	2	v. g.		4	7	—	2	6	3	fair		2	11	—	1	8	—	poor
	4	5	—	3	100	2	v. g.		5	3	—	1	1	—	poor		3	8	1	2	8	2	fair
	5	5	—	2	95	—	fair		6	4	2	3	8	3	fair		4	8	—	1	2	2	fair
	7	8	—	2	66	3	poor		7	7	—	1	1	—	poor		5	8	1	2	7	—	fair
	8	5	2	4	27	4	fair		8	7	2	3	16	2	fair		6	8	1	3	7	1	fair
	*9	8	—	2	7	1	fair		9	7	—	2	15	1	fair		7	8	0	2	3	—	poor
	*10	9	—	2	3	1	fair		10	7	—	2	18	1	fair		8	4	1	3	4	2	fair
	*11	8	—	1	1	1	fair		11	7	1	3	23	1	fair		9	8	—	2	5	2	poor
	*12	9	1	2	3	2	fair		12	7	—	3	16	—	fair		10	4	2	4	10	1	fair
	*13	6	—	2	3	1	fair		13	7	—	3	18	—	fair		11	8	—	4	7	1	fair
	*14	6	—	3	3	1	fair		14	7	—	3	15	1	poor		12	4	1	3	5	1	fair
	*15	6	1	4	5	1	fair		15	9	1	2	28	1	fair		14	4	2	5	20	1	fair
	*16	6	1	4	15	2	fair		16	10	—	2	22	1	poor		15	9	—	4	13	1	fair
	*17	6	—	4	17	2	fair		17	7	1	3	36	2	fair		16	8	—	3	6	1	fair
	*18	6	—	4	17	1	fair		18	7	1	4	36	2	fair		18	3	—	2	12	1	fair
	*19	6	—	4	15	1	fair		19	7	—	3	34	1	fair		19	8	—	2	12	1	fair
	*20	6	—	4	20	1	fair		20	7	—	3	42	2	fair		20	8	1	3	14	3	fair
	21	7	—	3	16	3	fair		21	7	—	3	22	—	poor		21	8	—	2	8	1	fair
	22	6	—	3	21	3	fair		22	7	—	3	15	—	fair		22	8	—	1	1	1	fair
	23	6	2	4	24	3	fair		23	4	1	3	16	2	fair		23	8	—	1	1	—	fair
	24	6	—	4	24	3	fair		24	4	—	3	6	2	poor		24	8	—	1	1	—	fair
	25	6	1	5	26	3	fair		25	7	—	2	12	1	poor		25	8	—	1	1	—	fair
	26	6	—	5	22	4	fair		26	7	—	2	8	1	fair		26	9	—	1	3	—	poor
	27	9	—	3	15	—	poor		27	7	1	3	22	2	fair		27	8	1	2	6	1	fair
	28	7	1	4	17	2	fair		28	7	—	3	16	2	fair		28	8	—	2	4	—	poor
	30	4	—	4	9	3	fair		29	7	—	2	8	2	fair		30	8	1	3	5	—	poor
	31	6	—	4	10	2	fair		30	7	—	2	6	1	fair		31	8	—	3	14	1	fair

*With hand telescope.

OBSERVATIONS OF THE FIFTH SATELLITE OF JUPITER.

By E. E. BARNARD.

The unfavorable position of *Jupiter* during the past few years has made observations of the fifth satellite very difficult. As a consequence very few measures have been obtained of it since those of 1911 (*A. J.* 637). The planet is now in fairly good position, but the elongations of the satellite in the past opposition occurred at large distances from the meridian. This increased the difficulty of seeing it, and only two elongations were covered. The satellite was very difficult — especially in the observations of October 21, which were made near the time of full moon. I have been trying for many years to get a few good elongations near together, for a more accurate determination of the motion of the line of apsides of the satellite's orbit, but this has been hopeless so far. By plating the measures the following results were obtained. The times are Central Standard Time = 6^h 0^m slow of G. M. T.

C. S. Time	App't Distance	At Δ 5.20
1915 Oct. 14 ^d 11 ^h 53 ^m .9	61''.34	48''.11
Oct. 21 11 13 .6	60 .35	48 .07

If these times are corrected for parallax and light equation and reduced to the *Sun* of October 21 we have:

Observation of October 14,	21 ^d 17 ^h 30 ^m .7
Observation of October 21,	21 17 27 .8
Difference	2 ^m .9

This difference is rather large compared with previous observations. The above observations will hardly give any improvement on the period already obtained, which is 0^d.49817906, or 0^d 11^h 57^m 22^s.6708 (*A. N.* 4339).

Following are the observations of the satellite obtained since those printed in *A. J.* 637. Where required, corrections for phase have been applied.

1912 JULY 13 — SATELLITE FOLLOWING

C. S. Time	From Limb	From Center	Comp.	
10 ^h 24 ^m 41 ^s	32''.52	54''.33	3	From fol. limb
10 31 16	76 .55	54 .54	5	" pr. "
10 37 36	32 .85	54 .67	4	" fol. "
10 42 11	33 .62	55 .43	3	" fol. "

Excessively difficult.

1913 JUNE 29 — SATELLITE PRECEDING

C. S. Time	From Limb	From Center	Comp.	
11 ^h 4 ^m 46 ^s	35''.99	59''.95	1	From pr. limb
11 8 58	83 .55	59 .59	3	" fol. "
11 12 56	82 .85	58 .89	3	" fol. "
11 16 46	35 .83	59 .79	2	" pr. "

Seen with certainty, though very faint. A bright satellite (I) was nearer to *Jupiter*.

1915 JULY 20 — SATELLITE FOLLOWING

C. S. Time	From Limb	From Center	Comp.	
14 ^h 3 ^m 21 ^s	32''.96	55''.72	4	From fol. limb
14 10 20	78 .28	55 .90	6	" pr. "
14 16 54	31 .33	54 .09	6	" fol. "
14 23 8	76 .23	53 .86	5	" pr. "

POLAR MEASURES

C. S. Time	From Limb		Appt. β	Comp.	
	Obs'd	Corr'd			
13 ^h 51 ^m 38 ^s	18''.39	21''.36	-0''.03	4	From north limb
13 56 21	25 .10	22 .13	+0 .80	5	" south "
14 31 54	23 .79	21 .06	-0 .27	4	" south "
14 38 9	19 .19	21 .89	-0 .56	4	" north "

The position angle of the wires in the polar measures was 68°.59, but when the position angle of the belts was measured a little later it was found that this was not parallel to the belts. The position angle of the wires should have been 65°.55. I have corrected the measured polar distances for this error of 3°. The slight corresponding corrections to the equatorial distances have been applied.

1915 AUGUST 10 — SATELLITE FOLLOWING

C. S. Time	From Limb	From Center	Comp.	
12 ^h 9 ^m 41 ^s	34''.18	58''.35	5	From fol. limb
12 14 37	82 .44	58 .07	6	" pr. "
12 19 23	33 .05	57 .22	6	" fol. "
12 25 24	81 .62	57 .24	7	" pr. "

Only feebly seen at best. Observations stopped by haze.

1915 OCTOBER 14 — SATELLITE PRECEDING

C. S. Time	From Limb	From Center	Comp.	
10 ^h 53 ^m 51 ^s	28".35	53".09	5	From pr. limb
10 57 57	79 .73	55 .11	5	" fol. "
11 2 41	30 .46	55 .21	6	" pr. "
11 7 25	81 .06	56 .44	6	" fol. "
11 12 35	32 .80	57 .54	7	" pr. "
11 18 7	83 .31	58 .69	5	" fol. "
11 23 0	34 .47	59 .21	6	" pr. "
11 28 20	84 .45	59 .83	6	" fol. "
11 33 19	35 .64	60 .39	6	" pr. "
11 38 21	85 .64	61 .02	6	" fol. "
11 42 46	36 .09	60 .83	6	" pr. "
11 47 27	86 .46	61 .83	7	" fol. "
11 53 1	36 .43	61 .17	6	" pr. "
11 58 11	85 .93	61 .31	8	" fol. "
12 3 54	36 .24	60 .99	7	" pr. "
12 8 45	86 .12	61 .50	8	" fol. "
12 15 12	35 .51	60 .25	9	" pr. "
12 22 5	84 .63	60 .01	6	" fol. "
12 28 5	33 .23	57 .97	5	" pr. "

1915 OCTOBER 21 — SATELLITE PRECEDING

C. S. Time	From Limb	From Center	Comp.	
10 ^h 32 ^m 56 ^s	31".85	56".03	6	From pr. limb
10 38 57	81 .65	57 .65	8	" fol. "
10 44 35	34 .29	58 .48	7	" pr. "
10 49 6	83 .15	59 .15	5	" fol. "
10 53 44	34 .45	58 .63	6	" pr. "
10 57 43	83 .53	59 .52	7	" fol. "
11 1 46	35 .38	59 .57	6	" pr. "
11 5 28	84 .21	60 .21	4	" fol. "
11 8 4	84 .87	60 .87	4	" fol. "
11 10 49	35 .66	59 .84	4	" pr. "
11 12 50	35 .40	59 .59	4	" pr. "
11 15 27	85 .26	61 .25	4	" fol. "
11 18 9	84 .99	60 .98	5	" fol. "
11 21 30	35 .10	59 .29	6	" pr. "
11 25 22	84 .26	60 .26	4	" fol. "
11 28 18	84 .34	60 .34	6	" fol. "
11 31 25	34 .74	58 .92	4	" pr. "
11 34 19	34 .55	58 .64	6	" pr. "
11 38 8	82 .78	58 .78	4	" fol. "
11 40 50	82 .65	58 .64	4	" fol. "
11 44 31	33 .58	57 .76	5	" pr. "
11 47 53	33 .23	57 .41	5	" pr. "
11 51 8	81 .96	57 .96	4	" fol. "
11 53 54	80 .51	56 .51	5	" fol. "

1915 OCTOBER 21 — SATELLITE PRECEDING. (Continued)

C. S. Time	From Limb	From Center	Comp.	
11 ^h 57 ^m 33 ^s	31".16	55".34	4	From pr. limb
12 0 26	30 .71	54 .89	5	" pr. "
12 4 54	78 .46	54 .46	4	" fol. "
12 8 30	77 .68	53 .68	4	" fol. "

Very faint in all the measures, in nearly full moonlight. Seeing fair = 3. There was much drift in the telescope and oscillation of the image, making the bisections of the limb difficult, but the bisection of the satellite was always very good.

1915 OCTOBER 28 — SATELLITE PRECEDING

C. S. Time	From Limb	From Center	Comp.	
10 ^h 5 ^m 11 ^s	32".41	56".58	6	From pr. limb
10 11 40	82 .24	58 .31	6	" fol. "
10 19 10	34 .18	58 .34	1	" pr. "

Excessively faint.

1915 DECEMBER 1 — SATELLITE PRECEDING

C. S. Time	From Limb	From Center	Comp.	
7 ^h 25 ^m 52 ^s	30".95	52".46	2	From pr. limb
7 39 27	31 .26	52 .78	5	" pr. "
7 59 52	31 .46	52 .98	4	" pr. "
8 18 53	29 .60	51 .12	5	" pr. "

Satellite only feebly glimpsed and measures uncertain.

In these and in all the previous measures of the satellite, the wires were placed perpendicular to the belts of *Jupiter* by the position circle. When polar distances have been measured the wires have been placed parallel to the belts.

The following equatorial diameters of *Jupiter* were used in the reductions.

Date	Apparent	Phase
1912 July 13	44".03	0".20 on fol. limb
1913 June 29	47 .92	0 .00
1915 July 20	45 .51	0 .38 " pr. "
1915 Aug. 10	48 .34	0 .21 " pr. "
Oct. 14	49 .48	0 .12 " fol. "
Oct. 21	48 .37	0 .18 " fol. "
Oct. 28	48 .33	0 .24 " fol. "
Dec. 1	43 .03	0 .42 " fol. "

The apparent polar diameter used on 1915, July 20, was 42".67.

On October 14 and 28 the diameters were measured. On the other dates they were reduced to the apparent from the value 38".522 (Δ 5.20). See *A. J.* 325.

As in previous years, when the opportunity has occurred, I have measured the position angles of the belts of *Jupiter*. The following measures are a continuation of those printed in No. 637 and previous numbers of the *Astronomical Journal*. These measures now cover more than two complete revolutions of the planet and should give a very accurate determination of the inclination of the axis of *Jupiter*.

POSITION ANGLES OF THE BELTS OF *Jupiter*

Date	C.S.Time	P. A.	Comp.	Remarks
1909 Mar. 20	12 ^h 20 ^m	114°.11*	5	
1912 July 16	8 55	99 .93	5	
	21 10 40	97 .84†	5	
	23 7 45	99 .47	5	Seeing fair
1913 Sept. 9	7 40	85 .73	5	Seeing very bad
1915 June 2	15 40	64 .68	5	Seeing fair
	July 20 14 45	65 .55	5	
	22 12 50	64 .26	5	

* This was omitted by mistake from the measures printed in *A. N.* 4339.

† This was by a north belt. The north edge of the great south belt did not seem parallel with it.

POSITION ANGLES OF THE BELTS OF *Jupiter*. (Continued)

Date	C.S.Time	P. A.	Comp.	Remarks
1915 Aug. 5	13 ^h 0 ^m	65°.52	3	Lost in clouds.
	10 12 0	64 .89	5	In haze.
	12 15 20	63 .23	5	Seeing very bad
	17 12 50	65 .17	5	Seeing poor.
	19 12 30	65 .17	6	Seeing very bad
	26 14 30	64 .33	5	Seeing poor.
	31 10 30	64 .86	5	Seeing fair.
Sept. 16	11 40	64 .91	5	Faint in clouds.
	28 14 0	63 .82	5	Seeing very bad
	30 10 25	64 .12	5	Seen very feebly through clouds
Oct. 14	10 15	63 .99	6	Seeing fair.
	18 10 0	63 .31	5	Seeing bad.
	21 10 30	63 .76	7	
	26 10 0	63 .77	5	Seeing very bad
	28 8 40	65 .26	6	Seeing bad.
Nov. 2	8 15	63 .89	5	
	4 7 15	66 .71	5	
	9 7 15	62 .68	5	Faint in clouds
	11 6 50	65 .33	5	
Dec. 1	6 50	64 .32	5	Seeing bad.
	4 5 20	65 .15	5	Dim in clouds
	29 5 30	64 .54‡	7	

‡ Measure added to the proof.

Yerkes Observatory, Williams Bay, Wisconsin, 1915, December 24.

THE NUMBER OF THE NEBULÆ,

REMARKS ON A PAPER BY DR. E. A. FATH IN "ASTRONOMICAL JOURNAL," No. 658.

By C. D. PERRINE.

In No. 658 of the *Journal*, DR. FATH in his "A Study of the Nebulæ" finds a large discrepancy between the number of the nebulæ as estimated from the Mount Wilson plates which he used and my estimate from the plates of the Keeter program taken with the Crossley Reflector of the Lick Observatory.

DR. FATH considered the most probable explanation of the discordance to be due to the fact that a larger proportion of the Lick plates were in the great north galactic polar nebular region, whereas the Mount Wilson plates were uniformly distributed over the sky, being taken in the series of KAPTEYN'S Selected Areas.

At first sight this explanation appears plausible, but further examination throws strong doubt on its validity as will be seen from the following considerations —

1. The *proportion* of non-nebulous regions appears to be practically the same in both the Mount Wilson and Lick series, amounting to nearly half.

2. The *largest number* of nebulae on any of the Mount Wilson plates, irrespective of the difference in area is almost exactly the same as in the Lick series.

3. The average number of nebulae per *plate* in the two series (after correcting the Crossley series for the excess of regions within 45° of the north galactic pole) is as follows —

Mount Wilson	7.4
Lick	6.9

4. The average per *plate* for the region within 45° of the north galactic pole is as follows —

Mount Wilson	18.7
Lick	16.5

This leaves for the average for the region *outside* this area —

Mount Wilson	4.8
Lick	4.1

The determination of these values is somewhat rough, but I do not think they are greatly in error.

5. Taking the four quadrants arbitrarily we find the following averages per plate —

	0 ^h -6 ^h	6 ^h -12 ^h	12 ^h -18 ^h	18 ^h -24 ^h
Mount Wilson	5.5	10.1	12.2	1.7
Lick	6.2	9.4	12.9	2.2

These determinations are rather rough also — but not greatly in error.

6. DR. FATH's diagram shows no regular increase in density towards either pole, but rather an irregular distribution over a large area.

An examination shows the distribution of the Crossley plates to be fairly representative of larger areas of the sky, if not entirely uniform, with the exception of the excess in the north galactic polar region. Reducing the density of this excess to the density of the outlying regions (which should make the *distribution* of the Crossley plates sufficiently comparable with the uniform distribution of the Mount Wilson series) I find that it reduces the average number per plate (and therefore my total estimated number) 20%.

Thus, after having the two series comparable so far as the larger areas at least of sky are concerned we still find a discordance between the estimates of $2\frac{1}{2}$ to 1.

It is not known whether the regions about some of the larger spirals or the especially rich regions may be centers of condensation, around which there are fewer small nebulae, a point which has been alluded to also by DR. FATH. This might if well marked have a tendency to cause such a discordance as that noted by DR. FATH. There are reasons for believing, however, that this effect is not large.

The close agreement shown above of the total number of nebulae on the Mount Wilson and Lick plates upon all assumptions of distribution — rich areas, poor areas, areas arbitrarily chosen — notwithstanding the wide

difference in area, leads to the suspicion that the Mount Wilson plates are not fairly representative of the areas covered.

There is a condition which has certainly tended to reduce the actual number of nebulae noted on the plates taken at Mount Wilson which DR. FATH does not appear to have investigated.

The Mount Wilson plates cover an area of sky nearly $2\frac{1}{2}$ times that of the Crossley plates. The images are considerably distorted at the edges of the Crossley plates and must be much more so at the edges of the Mount Wilson plates due not only to the larger area covered, but also to the larger angular aperture of the Mount Wilson mirror, $1/5.0$ as compared with the Crossley $1/5.8$.

Distortion should cause a diminution of the number of small nebulae in two ways —

1. By spreading the faint nebular images and rendering many of them invisible, and

2. By confusing the appearance and rendering it difficult to distinguish faint nebular images from those of stars in the greatly distorted regions on the plates.

A rough test of the two richest regions of the above series of Mount Wilson plates (as deduced from the positions of the nebulae published) shows a decided falling off at the edges of the plates. This tends to confirm the presence of some such effect.

A good test would be to take a plate rich in nebulae and take four others whose *centers* are at the *corners* of that plate.

A strictly fair comparison with the Crossley plates would be to count the nebulae on the Mount Wilson plates in central areas the size of the Crossley.

Until the effect of the additional distortion on the Mount Wilson plates has been determined no direct comparison with the Lick results is possible.

*Observatorio Nacional Argentino
Cordoba, December 13, 1915.*

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ON THE THEORY OF MOTION OF THE SMALL PLANETS WITH A PERIODIC ORBIT FOR THE *HILDA* TYPE,

By LOUIS ALLEN HOPKINS.

PURPOSE

The object of this investigation is to develop a method for the construction of periodic solutions of the differential equations for the plane motion of an infinitesimal body* subject to the attraction of *Jupiter* and the *Sun*, and to present a periodic orbit under perturbations of the first order for small planets whose mean motion is commensurable with the mean motion of *Jupiter* in ratio three to two.† It will be assumed that *Jupiter's* orbit is a circle of unit radius.

PART I. THE GENERAL SOLUTION

The differential equations of motion in canonical units with the *Sun* as origin are,

$$(1) \quad \frac{d^2x}{dt^2} = \frac{\partial \psi}{\partial x}, \quad \frac{d^2y}{dt^2} = \frac{\partial \psi}{\partial y},$$

in which

$$\psi = \frac{\sigma}{r} + \mu \left\{ \frac{1}{\rho} - (xX + yY) \right\}.$$

The σ and μ denote the masses of the *Sun* and *Jupiter* and r and ρ the distances from the small planet to the two finite bodies respectively. With the chosen units

$$(5) \quad \sigma + \mu = 1.$$

$$r_0 = a_0 \left\{ 1 - [\cos \omega (t - T)] e + \left[\frac{1}{2} - \frac{1}{2} \cos 2\omega (t - T) \right] e^2 + \left[\frac{3}{8} \cos \omega (t - T) - \frac{3}{8} \cos 3\omega (t - T) \right] e^3 + \dots \right\},$$

$$\varphi_0 = \lambda - (t - T) + \omega (t - T) + [2 \sin \omega (t - T)] e + \left[\frac{3}{4} \sin 2\omega (t - T) \right] e^2 + \left[-\frac{1}{4} \sin \omega (t - T) + \frac{13}{12} \sin 3\omega (t - T) \right] e^3 + \dots$$

The four arbitrary constants are T the time of perihelion passage, a_0 the major semi-axis, e the eccentricity and λ the longitude of perihelion. The mean angular motion ω is related to a_0 by

*Moulton, *Celestial Mechanics* (Macmillan 1914) p. 277.

†The *Berliner Jahrbuch* (1915) tabulates five small planets with approximately this mean motion. They are Nos. 153, 190, 334, 361, 499. The subtitle of this paper is chosen from the name of the first of the five.

The X and Y are the rectangular coördinates of *Jupiter*. Equations (1) possess one integral due to JACOBI, viz.,

$$\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 = 2(\psi + c). \quad (2)$$

For the immediate purpose, equations (1) are put into polar coördinates rotating with *Jupiter* by the transformations,

$$\begin{aligned} x &= r \cos (\varphi + t), & X &= \cos t, \\ y &= r \sin (\varphi + t), & Y &= \sin t, \end{aligned} \quad (3)$$

in which φ is the angle between the radii vectores of *Jupiter* and the small planet. The equations of motion become,

$$\frac{d^2r}{dt^2} - r \left[\frac{d\varphi}{dt} + 1 \right]^2 = \frac{\partial W}{\partial r}, \quad (4)$$

$$\frac{d}{dt} \left[r^2 \left(\frac{d\varphi}{dt} + 1 \right) \right] = \frac{\partial W}{\partial \varphi},$$

where

$$W = \frac{\sigma}{r} + \mu \left[\frac{1}{\rho} - r \cos \varphi \right].$$

With $\mu = 0$, equations (4) have the elliptic solutions,

$$\omega^2 a_0^3 = \sigma. \quad (6)$$

The initial conditions at $t = T$ are

$$r_0(T) = a_0(1 - e), \quad \varphi_0(T) = \lambda, \quad (7)$$

$$\frac{dr_0}{dt}(T) = 0, \quad \frac{d\varphi_0}{dt}(T) = -1 + \omega E_0$$

in which

$$E_0 = 1 + 2e + \frac{5}{2}e^2 + 3e^3 + \dots$$

(81)

To obtain the solutions of equations (4) with μ different from zero, but sufficiently small, the initial conditions (7) are varied by giving to a_0 an increment α and the independent variable is changed from $t - T$ to $(1 + \delta)\tau$, where δ is an arbitrary parameter.* The solutions of (4) thus transformed may be written as.

$$(8) \quad \begin{aligned} r &= r_0(\tau) + \xi(\tau), \\ \varphi &= \varphi_0(\tau) + \eta(\tau), \end{aligned}$$

in which ξ and η are power series in α , δ and μ vanishing

$$(10) \quad \begin{aligned} & [\xi'' - r_0' \dot{\eta}^2 - 2r_0' \dot{\varphi}_0 \dot{\eta} - 2r_0' \dot{\eta} - \dot{\varphi}_0^2 \xi - \xi \dot{\eta}^2 - \xi - 2\dot{\varphi}_0 \dot{\eta} \xi - 2\dot{\varphi}_0 \xi - 2\xi \dot{\eta}] + \delta [-2r_0'' + 2r_0' \dot{\varphi}_0^2 + 2r_0' \dot{\varphi}_0 - 2r_0' \dot{\eta} - 2\dot{\varphi}_0 \xi - 2\xi \dot{\eta} - 2\xi] \\ & + \delta^2 [r_0' \dot{\varphi}_0^2 - r_0'' - \xi + 2r_0' \dot{\varphi}_0] + \sigma(1 + \delta)^2 \sum_{n=1}^{\infty} (-1)^n \frac{n+1}{r_0^{n+2}} \xi^n = + (1 + \delta)^2 \mu \left[\frac{\partial U}{\partial r_0} + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\xi \frac{\partial}{\partial r_0} + \eta \frac{\partial}{\partial \varphi_0} \right)^n \frac{\partial U}{\partial r_0} \right], \\ & [r_0'' + \dot{\varphi}_0 \xi + \xi \dot{\eta} + 2r_0' \dot{\eta} + 2\dot{\varphi}_0 \xi + 2\xi \dot{\eta} + 2\xi] + \delta [2r_0' + 2\xi] \\ & = + (1 + \delta)^2 \mu \left[\frac{1}{r_0} \frac{\partial U}{\partial \varphi_0} + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\xi \frac{\partial}{\partial r_0} + \eta \frac{\partial}{\partial \varphi_0} \right)^n \frac{1}{r_0} \frac{\partial U}{\partial \varphi_0} \right]. \end{aligned}$$

In these equations the accents denote derivatives with respect to τ and

$$U = \frac{1}{\rho_0} - r_0 \cos \varphi_0.$$

The solutions of equations (10) are,

$$(11) \quad \xi = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \theta_{ijk}(\tau) \alpha^i \delta^j \mu^k,$$

$$(12) \quad \begin{aligned} & \theta_{100}'' - 2r_0'(\dot{\varphi}_0 + 1)\dot{\psi}_{100} - \left[(\dot{\varphi}_0 + 1)^2 + \frac{2\sigma}{r_0^3} \right] \theta_{100} = 0, \\ & r_0 \psi_{100}'' + \dot{\varphi}_0 \theta_{100} + 2r_0' \dot{\psi}_{100} + 2(\dot{\varphi}_0 + 1)\dot{\theta}_{100} = 0; \end{aligned}$$

$$(13) \quad \begin{aligned} & \theta_{010}'' - 2r_0'(\dot{\varphi}_0 + 1)\dot{\psi}_{010} - \left[(\dot{\varphi}_0 + 1)^2 + \frac{2\sigma}{r_0^3} \right] \theta_{010} = 2r_0'' - 2r_0' \dot{\varphi}_0 (\dot{\varphi}_0 + 1), \\ & r_0 \psi_{010}'' + \dot{\varphi}_0 \theta_{010} + 2r_0' \dot{\psi}_{010} + 2(\dot{\varphi}_0 + 1)\dot{\theta}_{010} = -2r_0'; \end{aligned}$$

$$(14) \quad \begin{aligned} & \theta_{001}'' - 2r_0'(\dot{\varphi}_0 + 1)\dot{\psi}_{001} - \left[(\dot{\varphi}_0 + 1)^2 + \frac{2\sigma}{r_0^3} \right] \theta_{001} = \frac{\partial U}{\partial r_0}, \\ & r_0 \psi_{001}'' + \dot{\varphi}_0 \theta_{001} + 2r_0' \dot{\psi}_{001} + 2(\dot{\varphi}_0 + 1)\dot{\theta}_{001} = \frac{1}{r_0} \frac{\partial U}{\partial \varphi_0}; \end{aligned}$$

for $\alpha = \delta = \mu = 0$. The initial conditions for these power series, due to the increment α in (7), are

$$\xi(0) = (1 - e)\alpha, \quad \eta(0) = 0, \quad (9)$$

$$\frac{d\xi}{d\tau}(0) = 0, \quad \frac{d\eta}{d\tau}(0) = E_0 \omega \sum_{n=1}^{\infty} (-1)^n \frac{3 \cdots (2n+1)}{2^n n! a_0^n} \alpha^n.$$

After making the indicated transformation on the independent variable in (4) and subtracting equations (4) with $\mu = 0$, there results the following equations in the variations ξ and η ,

$$\eta = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \psi_{ijk}(\tau) \alpha^i \delta^j \mu^k, \quad (i+j+k > 0),$$

with initial conditions given by (9). On substituting (11) in (10) and equating coefficients of like powers of α , δ , μ , there is obtained an infinite set of differential equations integrable sequentially. The coefficients of α , δ , μ and α^2 , etc., are

*PROFESSOR MOULTON suggested this method in Section 11 of his article on "Closed Orbits of Ejection and Related Periodic Orbits" (*Proceedings of the London Mathematical Society*, Ser. 2, Vol. 11, Part 5).

$$\ddot{\theta}_{200} - 2r_0(\dot{\varphi}_0 + 1)\dot{\psi}_{200} - \left[(\dot{\varphi}_0 + 1)^2 + \frac{2\sigma}{r_0^3} \right] \theta_{200} = r_0\dot{\psi}_{100}^2 - \frac{3\sigma}{r_0^4} \theta_{100}^2 + 2\theta_{100}\dot{\psi}_{100}(\dot{\varphi}_0 + 1), \quad (15)$$

$$r_0\ddot{\psi}_{200} + \ddot{\varphi}_0\theta_{200} + 2\dot{r}_0\dot{\psi}_{200} + 2(\dot{\varphi}_0 + 1)\dot{\theta}_{200} = -\theta_{100}\ddot{\psi}_{100} - 2\dot{\theta}_{100}\dot{\psi}_{100}.$$

The left members of each set of two equations are the same and the right members involve known functions of τ or the solutions of previous differential equations.

Equations (12) are the equations of variation* of which the solutions are the partial derivatives of the generating solution (5) with respect to the four arbitrary constants. Thus the general solutions of (12) are,

$$\theta_{100} = K_1 \frac{\partial r_0}{\partial T} + K_2 a_0 \frac{\partial r_0}{\partial a_0} + K_3 \frac{\partial r_0}{\partial e} + K_4 \frac{\partial r_0}{\partial \lambda},$$

$$\psi_{100} = K_1 \frac{\partial \varphi_0}{\partial T} + K_2 a_0 \frac{\partial \varphi_0}{\partial a_0} + K_3 \frac{\partial \varphi_0}{\partial e} + K_4 \frac{\partial \varphi_0}{\partial \lambda},$$

where after the differentiation $t - T$ is replaced by τ . The partial derivatives of r_0 and φ_0 with respect to a_0 are multiplied by a_0 for sake of symmetry. In forming $\frac{\partial r_0}{\partial a_0}$ it must be noted that a_0 enters both explicitly and implicitly through ω and therefore

$$\frac{\partial r_0}{\partial a_0} = \left(\frac{\partial r_0}{\partial a_0} \right) + \frac{\partial r_0}{\partial \omega} \cdot \frac{\partial \omega}{\partial a_0},$$

$$\frac{\partial \varphi_0}{\partial a_0} = \frac{\partial \varphi_0}{\partial \omega} \cdot \frac{\partial \omega}{\partial a_0}.$$

Furthermore, as r_0 does not contain λ , and φ_0 is linear in λ , it follows that

$$\frac{\partial r_0}{\partial \lambda} = 0, \quad \frac{\partial \varphi_0}{\partial \lambda} = 1.$$

$$r_0(\dot{\varphi}_0 + 1) = a_0 \omega N, \quad (\dot{\varphi}_0 + 1)^2 + \frac{2\sigma}{r_0^3} = \omega^2 P, \quad \ddot{\varphi}_0/r_0 = -\frac{\omega^2}{a_0} Q, \quad (18)$$

$$\dot{r}_0/r_0 = \omega R, \quad \frac{\dot{\varphi}_0 + 1}{r_0} = \frac{2\omega}{a_0} S, \quad \ddot{r}_0 - r_0 \dot{\varphi}_0(\dot{\varphi}_0 + 1) = -a_0 T + a_0 \omega N,$$

where N, P, Q, R, S and T are infinite power series in e with periodic coefficients. The expressions N, P, S and T contain cosines and Q and R sines of multiples of $\omega\tau$. As τ occurs only in the combination $\omega\tau$, for abbreviation let

$$\omega\tau = \beta. \quad (19)$$

Therefore the general solutions of (12) become,

$$\theta_{100} = K_1 \frac{\partial r_0}{\partial T} + K_2 a_0 \frac{\partial r_0}{\partial a_0} + K_3 \frac{\partial r_0}{\partial e}, \quad (16)$$

$$\psi_{100} = K_1 \frac{\partial \varphi_0}{\partial T} + K_2 a_0 \frac{\partial \varphi_0}{\partial a_0} + K_3 \frac{\partial \varphi_0}{\partial e} + K_4.$$

The constants K_j are found from the initial conditions (9) by means of the equations

$$K_1 \frac{\partial r_0}{\partial T}(0) + K_2 a_0 \frac{\partial r_0}{\partial a_0}(0) + K_3 \frac{\partial r_0}{\partial e}(0) = 1 - e,$$

$$K_1 \frac{\partial^2 r_0}{\partial \tau \partial T}(0) + K_2 a_0 \frac{\partial^2 r_0}{\partial \tau \partial a_0}(0) + K_3 \frac{\partial^2 r_0}{\partial \tau \partial e}(0) = 0,$$

$$K_1 \frac{\partial \varphi_0}{\partial T}(0) + K_2 a_0 \frac{\partial \varphi_0}{\partial a_0}(0) + K_3 \frac{\partial \varphi_0}{\partial e}(0) + K_4 = 0,$$

$$K_1 \frac{\partial^2 \varphi_0}{\partial \tau \partial T}(0) + K_2 a_0 \frac{\partial^2 \varphi_0}{\partial \tau \partial a_0}(0) + K_3 \frac{\partial^2 \varphi_0}{\partial \tau \partial e}(0) = -\frac{3}{2} \frac{\omega}{a_0} E_0,$$

to be

$$K_1 = K_3 = K_4 = 0, \quad K_2 = 1/a_0.$$

The solutions of equations (12) are therefore,

$$\theta_{100} = \frac{\partial r_0}{\partial a_0}, \quad (17)$$

$$\psi_{100} = \frac{\partial \varphi_0}{\partial a_0}.$$

The solutions of (13), (14), (15), etc., can now be obtained sequentially from the general solutions (16) of equations (12) by the method of variation of parameters. But before proceeding further let the following abbreviations be adopted:

$$\ddot{\theta}_{100} - 2a_0 N \dot{\psi}_{100} - P \theta_{100} = 0, \quad (20)$$

$$\dot{\psi}_{100} - \frac{1}{a_0} Q \theta_{100} + 2R \dot{\psi}_{100} + \frac{2}{a_0} S \dot{\theta}_{100} = 0.$$

*POINCARÉ. *Les Méthodes Nouvelles de la Mécanique Céleste*, Vol. I, Chapter V.

The dots indicate derivatives with respect to β . The coefficients are periodic of period 2π . Equations (20) may be written as a system of first order differential equations by letting,

$$(21) \quad \begin{aligned} x_1 &= \theta_{100}, \\ x_2 &= \dot{x}_1 = \dot{\theta}_{100}, \\ x_3 &= \psi_{100}, \\ x_4 &= \dot{x}_3 = \dot{\psi}_{100}. \end{aligned}$$

as follows:

$$(22) \quad \begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= Px_1 + 2a_0Nx_4, \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \frac{1}{a_0}Qx_1 - \frac{2}{a_0}Sx_2 - 2Rx_4. \end{aligned}$$

The fundamental set of solutions of equations (22) as already found in (16), are,

$$(23) \quad \begin{aligned} x_{11} &= -a_0\omega A, & x_{12} &= a_0B - \frac{3}{2}a_0\beta A, & x_{13} &= a_0C, & x_{14} &= 0, \\ x_{21} &= -a_0\omega \dot{A}, & x_{22} &= a_0\dot{B} - \frac{3}{2}a_0\dot{A} - \frac{3}{2}a\beta\dot{A}, & x_{23} &= a_0\dot{C}, & x_{24} &= 0, \\ x_{31} &= 1 - \omega D, & x_{32} &= -\frac{3}{2}\beta D, & x_{33} &= E, & x_{34} &= 1, \\ x_{41} &= -\omega\dot{D}, & x_{42} &= -\frac{3}{2}\dot{D} - \frac{3}{2}\beta\dot{D}, & x_{43} &= \dot{E}, & x_{44} &= 0; \end{aligned}$$

where A and E contain only sines and B , C , and D only cosines of multiples of β . The general solutions (16) may now be written as

$$(24) \quad x_i = \sum_{j=1}^4 K_j x_{ij} \quad (i = 1, 2, 3, 4).$$

Equations (13) under the substitutions (18) and (19) are,

$$(25) \quad \begin{aligned} \dot{x}_1 - x_2 &= 0, \\ \dot{x}_2 - (Px_1 + 2a_0Nx_4) &= -2a_0T + 2\frac{a_0}{\omega}N = G, \\ \dot{x}_3 - x_4 &= 0, \\ \dot{x}_4 - \left(\frac{1}{a_0}Qx_1 - \frac{2}{a_0}Sx_2 - 2Px_4\right) &= -\frac{2}{\omega}R = H. \end{aligned}$$

To solve this set let us substitute (24) in (25) considering the K_j as functions of β . The following relations result from which the $\dot{K}_j$ and consequently the K_j can be found.

$$(27) \quad \Delta = -\frac{1}{2}a_0^2\omega e \{1 + [2\cos\beta]e + [1 + \frac{5}{2}\cos 2\beta]e^2 + [\frac{7}{4}\cos\beta + \frac{1}{4}\cos 3\beta]e^3 + \dots\}.$$

The numerators of the solutions of equations (26) for the $\dot{K}_j$ are respectively:

$$(28) \quad \begin{aligned} N_1 &= G(x_{12}x_{43} - x_{13}x_{42}) + H(x_{13}x_{22} - x_{12}x_{23}), \\ N_2 &= G(x_{13}x_{41} - x_{11}x_{43}) + H(x_{11}x_{23} - x_{13}x_{21}), \\ N_3 &= G(x_{11}x_{42} - x_{12}x_{41}) + H(x_{12}x_{21} - x_{11}x_{22}), \\ N_4 &= -x_{31}N_1 - x_{32}N_2 - x_{33}N_3. \end{aligned}$$

$$(26) \quad \begin{aligned} \sum_{j=1}^4 \dot{K}_j x_{1j} &= 0, \\ \sum_{j=1}^4 \dot{K}_j x_{2j} &= G, \\ \sum_{j=1}^4 \dot{K}_j x_{3j} &= 0, \\ \sum_{j=1}^4 \dot{K}_j x_{4j} &= H. \end{aligned}$$

The determinant of this system of equations

$$\Delta = |x_{ij}| \quad (i, j = 1, 2, 3, 4),$$

is the determinant of the fundamental set of solutions (23), its value can be more readily found from the relation,

$$\Delta = \Delta_0 e^{\int_{\beta} R d\beta},$$

where Δ_0 is the value of Δ for $\beta = 0$.* The expression for Δ is thus found to be

The particular integrals of (25) are found on integrating the equations

$$\dot{K}_j = N_j \Delta^{-1} \quad (j = 1, 2, 3, 4),$$

to be $f_i = \sum_{j=1}^4 K_j x_{ij} \quad (i = 1, 2, 3, 4)$.

*MOULTON and MACMILLAN: "Solutions of Certain Types of Linear Differential Equations with Periodic Coefficients," *American Journal of Mathematics*, Vol. XXXIII, 1911.

It is not necessary to develop the f_2 and f_4 , though they have been obtained separately and checked by the relations $\dot{f}_1 = f_2, \dot{f}_3 = f_4$. On carrying out the process indicated it is found that the particular integrals of (25) are

$$f_1 = 4a_0 \left\{ \left[-\frac{1}{2} \right] + [\beta \sin \beta + \frac{1}{2} \cos \beta] e + \dots \right\} + \frac{4a_0}{\omega} \left\{ \left[\frac{1}{2} \right] + \left[-\frac{3}{4} \beta \sin \beta + \frac{1}{8} \cos \beta \right] e + \dots \right\},$$

$$f_3 = 4 \left\{ [\beta] + [2\beta \cos \beta] e + \dots \right\} + \frac{4}{\omega} \left\{ [-\beta] + \left[-\frac{3}{2} \beta \cos \beta - \frac{1}{4} \sin \beta \right] e + \dots \right\}.$$

The general solutions of (25) are therefore,

$$x_i = \sum_{j=1}^4 K_j^{(0)} x_{ij} + f_i \quad (i = 1, 2, 3, 4),$$

where the $K_j^{(0)}$ are arbitrary constants. The initial conditions as given by (9) are,

$$(30) \quad x_i(0) = 0 \quad (i = 1, 2, 3, 4),$$

from which the constants $K_j^{(0)}$ may be determined. As $x_{11}, x_{22}, x_{23}, x_{32}, x_{33}, x_{41}, f_1$ and f_3 are odd functions of β and $x_{14} \equiv x_{24} \equiv x_{44} \equiv 0$, equations (30) reduce to

$$(31) \quad \begin{aligned} (a) \quad & K_2^{(0)} x_{12}(0) + K_3^{(0)} x_{13}(0) = -f_1(0), \\ (b) \quad & K_1^{(0)} x_{21}(0) = 0, \\ (c) \quad & K_1^{(0)} x_{31}(0) + K_4^{(0)} x_{34}(0) = 0, \\ (d) \quad & K_2^{(0)} x_{42}(0) + K_3^{(0)} x_{43}(0) = -f_4(0). \end{aligned}$$

From (31b) it follows that $K_1^{(0)} = 0$ and consequently $K_4^{(0)} = 0$ from (31c), and

$$K_2^{(0)} = \begin{vmatrix} x_{13}(0) & f_1(0) \\ x_{43}(0) & f_4(0) \\ x_{12}(0) & x_{13}(0) \\ x_{42}(0) & x_{43}(0) \end{vmatrix}, \quad K_3^{(0)} = \begin{vmatrix} f_1(0) & x_{12}(0) \\ f_4(0) & x_{42}(0) \\ x_{12}(0) & x_{13}(0) \\ x_{42}(0) & x_{43}(0) \end{vmatrix}.$$

The final solutions of equations (25) or, reversing the transformation (19), the solutions of (13) are thus completed, and are

$$\begin{aligned} \theta_{010} &= K_2^{(0)} x_{12}(\omega\tau) + K_3^{(0)} x_{13}(\omega\tau) + f_1(\omega\tau), \\ \psi_{010} &= K_2^{(0)} x_{32}(\omega\tau) + K_3^{(0)} x_{33}(\omega\tau) + f_3(\omega\tau). \end{aligned} \quad (32)$$

The solutions of equations (14) are obtained in a similar way. The important difference lies in the right hand members. A beginning is made by expanding U_0 in powers of r_0 . Thus,

$$U_0 = 1 + \frac{1}{4}[1 + 3 \cos 2\varphi_0] r_0^2 + \frac{1}{8}[3 \cos \varphi_0 + 5 \cos 3\varphi_0] r_0^3 + \dots$$

Here it has been assumed that $r_0 < 1$ i.e. less than the radius vector of *Jupiter*. For small planets outside the orbit of *Jupiter* the expansion would be made in powers of $1/r_0$. The general formulas for $\cos m\varphi_0$, $\sin m\varphi_0$ and r_0^n (m and n being integers) are as follows:

$$\begin{aligned} \frac{\cos}{\sin} m\varphi_0 &= \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau) + \left[-m \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau - \omega\tau) + m \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau + \omega\tau) \right] e \\ &+ \left[\left(-\frac{5}{8}m + \frac{1}{2}m^2 \right) \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau - 2\omega\tau) - m^2 \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau) \right. \\ &\quad \left. + \left(\frac{5}{8}m + \frac{1}{2}m^2 \right) \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau + 2\omega\tau) \right] e^2 \\ &+ \left[\left(-\frac{13}{24}m + \frac{5}{8}m^2 - \frac{1}{6}m^3 \right) \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau - 3\omega\tau) + \left(\frac{1}{8}m - \frac{5}{8}m^2 + \frac{1}{2}m^3 \right) \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau - \omega\tau) \right. \\ &\quad \left. + \left(-\frac{1}{8}m - \frac{5}{8}m^2 - \frac{1}{2}m^3 \right) \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau + \omega\tau) \right. \\ &\quad \left. + \left(\frac{13}{24}m + \frac{5}{8}m^2 + \frac{1}{6}m^3 \right) \frac{\cos}{\sin} (m\lambda - m\tau + m\omega\tau + 3\omega\tau) \right] e^3 + \dots \end{aligned}$$

$$r_0^n = a_0^n \{ 1 + [-n \cos \omega \tau] e + [\frac{1}{4}(n + n^2) + \frac{1}{4}(-3n + n^2) \cos 2\omega \tau] e^2 \\ + [\frac{1}{4}(3n + n^2 - n^3) \cos \omega \tau + \frac{1}{24}(-17n + 9n^2 - n^3) \cos 3\omega \tau] e^3 \\ + [\frac{1}{64}(20n - 19n^2 - 2n^3 + n^4) + \frac{1}{48}(22n - n^2 - 6n^3 + n^4) \cos 2\omega \tau \\ + \frac{1}{192}(-142n + 95n^2 - 18n^3 + n^4) \cos 4\omega \tau] e^4 + \dots \}.$$

The right hand members of equations (14) after dividing the second by r_0 are

$$G = \frac{\partial U}{\partial r_0} = \frac{1}{2}[1 + 3 \cos 2\varphi_0] r_0 + \frac{3}{8}[3 \cos \varphi_0 + 5 \cos 3\varphi_0] r_0^2 + \frac{1}{6}[9 + 20 \cos 2\varphi_0 + 35 \cos 4\varphi_0] r_0^3 + \dots,$$

$$H = \frac{1}{r_0^2} \frac{\partial U}{\partial \varphi_0} = -\frac{1}{2} \sin 2\varphi_0 + \frac{3}{8}[\sin \varphi_0 + 5 \sin 3\varphi_0] r_0 + \frac{5}{16}[2 \sin 2\varphi_0 + 7 \sin 4\varphi_0] r_0^2 + \dots.$$

The development of the particular integrals proceeds as before, they are found to be

$$(33) \\ F_1 = \frac{2a_0}{\omega^2} \left\{ \left[\frac{1}{4} + \left\{ -\frac{9}{s(-\frac{2}{\omega} + 1)} + \frac{3}{2(-\frac{2}{\omega} + 2)} - \frac{3}{s(-\frac{2}{\omega} + 3)} \right\} \cos(2\lambda - \frac{2}{\omega}\beta + 2\beta) \right] \right. \\ + \left[-\frac{17}{16} \cos \beta + \left\{ \frac{-15}{s(-\frac{2}{\omega})} - \frac{9}{s(-\frac{2}{\omega} + 1)} - \frac{3}{s(-\frac{2}{\omega} + 2)} - \frac{9}{s(-\frac{2}{\omega} + 2)^2} - \frac{3}{s(-\frac{2}{\omega} + 3)} \right\} \cos(2\lambda - \frac{2}{\omega}\beta + \beta) \right] \\ + \left\{ -\frac{9}{s(-\frac{2}{\omega} + 1)} - \frac{3}{4(-\frac{2}{\omega} + 2)} + \frac{9}{s(-\frac{2}{\omega} + 2)^2} + \frac{21}{s(-\frac{2}{\omega} + 3)} - \frac{3}{4(-\frac{2}{\omega} + 4)} \right\} \cos(2\lambda - \frac{2}{\omega}\beta + 3\beta) \\ \left. - \frac{7}{8}\beta \sin \beta \right] e + \dots \} \\ + \frac{2a_0^2}{\omega^2} \left\{ \left[\left\{ \frac{-15}{32(-\frac{1}{\omega})} + \frac{3}{s(-\frac{1}{\omega} + 1)} + \frac{3}{32(-\frac{1}{\omega} + 2)} \right\} \cos(\lambda - \frac{1}{\omega}\beta + \beta) \right] \right. \\ + \left\{ \frac{-45}{32(-\frac{3}{\omega} + 2)}^* + \frac{15}{s(-\frac{3}{\omega} + 3)} - \frac{15}{32(-\frac{3}{\omega} + 4)} \right\} \cos(3\lambda - \frac{3}{\omega}\beta + 3\beta) + \dots \left. \right\} + \dots \\ F_3 = \frac{2}{\omega^2} \left\{ \left[\left\{ \frac{9}{4(-\frac{2}{\omega} + 1)} - \frac{3}{2(-\frac{2}{\omega} + 2)} - \frac{9}{4(-\frac{2}{\omega} + 2)^2} - \frac{3}{4(-\frac{2}{\omega} + 3)} \right\} \sin(2\lambda - \frac{2}{\omega}\beta + 2\beta) - \frac{1}{2}\beta \right] \right. \\ + \left[\frac{9}{8} \sin \beta - \frac{7}{4}\beta \cos \beta + \left\{ \frac{-15}{4(-\frac{2}{\omega})} + \frac{63}{16(-\frac{2}{\omega} + 1)} + \frac{27}{s(-\frac{2}{\omega} + 1)^2} + \frac{3}{4(-\frac{2}{\omega} + 2)} - \frac{9}{1(-\frac{2}{\omega} + 2)^2} \right. \right. \\ \left. \left. - \frac{15}{16(-\frac{2}{\omega} + 3)} \right\} \sin(2\lambda - \frac{2}{\omega}\beta + \beta) + \left\{ \frac{15}{16(-\frac{2}{\omega} + 1)} - \frac{9}{4(-\frac{2}{\omega} + 2)^2} - \frac{21}{16(-\frac{2}{\omega} + 3)} - \frac{27}{s(-\frac{2}{\omega} + 3)^2} \right. \right. \\ \left. \left. - \frac{3}{2(-\frac{2}{\omega} + 4)} \right\} \sin(2\lambda - \frac{2}{\omega}\beta + 3\beta) \right] e + \dots \left. \right\} \\ + \frac{2a_0}{\omega^2} \left\{ \left[\left\{ \frac{15}{16(-\frac{1}{\omega})} - \frac{9}{s(-\frac{1}{\omega} + 1)} - \frac{9}{16(-\frac{1}{\omega} + 1)^2} + \frac{3}{16(-\frac{1}{\omega} + 2)} \right\} \sin(\lambda - \frac{1}{\omega}\beta + \beta) \right] \right. \\ + \left\{ \frac{45}{16(-\frac{3}{\omega} + 2)}^* - \frac{15}{s(-\frac{3}{\omega} + 3)} - \frac{45}{16(-\frac{3}{\omega} + 3)^2} - \frac{15}{16(-\frac{3}{\omega} + 4)} \right\} \sin(3\lambda - \frac{3}{\omega}\beta + 3\beta) \left. \right] + \dots \left. \right\} + \dots$$

*In case $\omega = 3/2$ the terms marked are replaced by $+\frac{4}{3}\frac{5}{2}\beta \sin(3\lambda + \beta)$ and $+\frac{1}{16}\frac{5}{2}\beta \cos(3\lambda + \beta)$.

In the computations the F_i were all obtained separately and then checked by the relations $\dot{F}_1 = F_2$, $\dot{F}_3 = F_4$. The general solutions of (14) are

$$(34) \quad x_i = \sum_{j=1}^4 K_j^{(0)} x_{ij} + F_i \quad (i = 1, 2, 3, 4).$$

Under the initial conditions (9) the

$$x_i(0) = 0 \quad (i = 1, 2, 3, 4),$$

and on choosing perihelion on the line joining the *Sun* and *Jupiter* at $\beta = 0$, that is,

$$\lambda = 0$$

the K_j are found exactly as in (31) except that the F_i replace the f_i . The solutions of (14) are found to be, therefore,

$$(35) \quad \begin{aligned} \theta_{001} &= K_2^{(0)} x_{12}(\omega\tau) + K_3^{(0)} x_{13}(\omega\tau) + F_1(\omega\tau), \\ \varphi_{001} &= K_2^{(0)} x_{32}(\omega\tau) + K_3^{(0)} x_{33}(\omega\tau) + F_3(\omega\tau). \end{aligned}$$

The solutions of equations (15) may be developed similarly with the initial conditions

$$x_i(0) = 0 \quad (i = 1, 2, 3),$$

$$x_4(0) = \frac{15}{8} \frac{\omega}{a_0^2} E_0,$$

and so on. Thus we may proceed sequentially to the solutions of the infinite set of differential equations. The solutions (17), (32), (35), etc., are now substituted in (11) as the successive coefficients of the power series in a , δ and μ . Equations (11) thus formed are the solutions of equations (10).

Let us now impose the conditions that the motion of the small planet shall be periodic. Suppose the mean motion is

$$\omega = q/p,$$

where q and p are integers, $q > p$ for small planets within the orbit of *Jupiter* and $q < p$ for those without. Sufficient conditions for periodicity* are

$$(36) \quad \xi(p\pi) = \eta(p\pi) = 0.$$

These equations can be solved for a and δ as power series in μ , vanishing with μ ,

$$(a) \quad a = \sum_{j=1}^{\infty} a_j \mu^j, \quad (37)$$

$$(b) \quad \delta = \sum_{j=1}^{\infty} \delta_j \mu^j,$$

provided the Jacobian is different from zero. On carrying out the computations the Jacobian is found to be

$$= \frac{3}{2} q^2 \pi^2 e \cos q\pi P(e),$$

where $P(e)$ is a power series in e . The Jacobian vanishes with e , that is, the generating solution must be an elliptic one*. To make the period in τ equal to the period in t put

$$\delta = 0. \quad (38)$$

This equation can be used to determine e in terms of μ . For on dividing by μ equation (38) becomes

$$0 = \sum_{j=1}^{\infty} \delta_j \mu^{j-1}$$

where the δ_j are power series in a_0 with coefficients which are power series in e . On putting $\delta_1 = 0$, there is obtained an equation of the form,

$$0 = \delta_1 = \sum_{j=0}^{\infty} A_j e^{j+1}, \quad (39)$$

in which the A_j are power series in a_0 . From (39) the value of e for the generating solution is to be obtained, call it

$$e_0. \quad (40)$$

Then on substituting

$$e = e_0 + \epsilon \quad (41)$$

in (39), this equation becomes a power series in ϵ and μ vanishing with ϵ and μ and therefore can be solved for ϵ as a power series in μ , vanishing with μ . Thus the expression (41) for e in the three body problem is of the form

$$e = e_0 + \sum_{j=1}^{\infty} e_j \mu^j. \quad (42)$$

The solution of the problem announced would now be completed by substituting in (11) the value of a from

*In a paper entitled, "The Problem of Three Bodies with Application to the Lunar Theory," (*American Mathematical Society Transactions*, Vol. 7, 1906) PROFESSOR MOULTON developed periodic orbits for all cases of commensurability except for those in which the numerator and denominator of the ratio of the mean motions differ by unity. The circle served as the generating conic in this investigation. The present paper treats the exceptional case.

*MOULTON, "Closed Orbits of Ejection," loc. cit. Section 12.

(37a) and $\delta = 0$, then add (11) to the generating solutions to form (8) substituting $t - T$ for τ and for e the series (42).

PART II. APPLICATION TO THE *Hilda* TYPE

The aim throughout the general investigation was to develop a periodic orbit for small planets of the *Hilda* type, that is, for small planets whose mean motion is

$$\omega = \frac{3}{2}.$$

POINCARÉ suggested analytic continuation* for *Hilda*'s motion but the general development given here does not apply. The difficulty is essentially the same that G. W.

HILL encountered in his Memoir No. 72† in which he attempted to determine the eccentricity of the generating ellipse in the cases 2/1 and 3/1 from the velocity of the longitude of perihelion under perturbations of the first order. HILL finally resorted to mechanical quadrature in both cases. In the case 3/2 the expansion of the perturbative function converges very slowly‡ due to the large ratio of the mean distances but in the case of the periodic orbit the value of the eccentricity is so large that the orbit crosses the circle of *Jupiter*'s motion and consequently the series in the ratio of the distances diverges. The difficulty lay in determining e_0 (40) from the series (39) which in the case $\omega = 3/2$ to the fifth power of e , except for a constant factor, is

$$(43) \quad 0 = \frac{1}{4} \sum_m^{3,5,\dots,\infty} p_m M_m^{(0)} a_0^{m-2} - \frac{e}{4} \sum_m^{2,4,\dots,\infty} p_m M_m^{(1)} a_0^{m-2} + \frac{e^2}{32} \sum_m^{3,5,\dots,\infty} p_m M_m^{(2)} a_0^{m-2} - \frac{e^3}{32} \sum_m^{2,4,\dots,\infty} p_m M_m^{(3)} a_0^{m-2} \\ + \frac{e^4}{768} \sum_m^{3,5,\dots,\infty} p_m M_m^{(4)} a_0^{m-2} - \frac{e^5}{768} \sum_m^{2,4,\dots,\infty} p_m M_m^{(5)} a_0^{m-2} + \dots,$$

where

$$(44) \quad M_m^{(1)} = m(m+1), \quad M_m^{(3)} = m(m^3 - 2m^2 + 18m - 71), \quad M_m^{(5)} = m(m^5 - 9m^4 + 55m^3 - 195m^2 - 854m + 3162), \\ M_m^{(0)} = m + 6, \quad M_m^{(2)} = 3m^3 + 15m^2 - 53m - 420, \quad M_m^{(4)} = 5m^5 - 215m^3 + 358m^2 - 8358m + 15504,$$

$$p_3 = \frac{5}{2}, \quad a_0 = .76290, \quad p_{m=\text{odd}} = \frac{1 \cdot 3 \cdot 5 \cdots (m+2) \cdot 1 \cdot 3 \cdot 5 \cdots (m-4)}{\frac{m+3}{2}! \frac{m-3}{2}! 2^{m-1}}, \quad p_{m=\text{even}} = \left(\frac{1 \cdot 3 \cdot 5 \cdots (m-1)}{2 \cdot 4 \cdot 6 \cdots m} \right)^2.$$

On substituting (44) in (43) and computing in all some one hundred and twenty separate terms so that the coefficients of e up to the fifth power would be correct to five significant figures, it was found that

$$0 = 1.7374 - 1.9830 e + 4.2183 e^2 - 10.885 e^3 \\ + 76.234 e^4 - 133.62 e^5 + \dots$$

The sequel will show this series to be divergent.

PART III. PERIODIC ORBIT FOR THE *Hilda* TYPE

To obtain a periodic orbit for the case 3/2, we adopt mechanical quadrature. The eccentricities of the generating ellipses chosen for the first two investigations were $e = .40$ and $e = .45$. These values were selected after following the character of the perturbations by

rough geometrical constructions in fixed axes. That the correct value was so nearly guessed was somewhat accidental. The investigation is made from the standpoint of variations about the elliptic orbit in which the small planet is attracted by the *Sun* alone and, as will be seen later, only the terms in the first power of the variations and the terms independent of the variations involving the mass of *Jupiter* are included. However as the variations from the elliptic orbit are so very small, it seems that the complete solutions can not be materially different

**Mécanique Céleste*, loc. cit., Vol. I, p. 154.

† *Collected Works*, Vol. IV, p. 254 (Carnegie Institution of Washington).

‡ PROFESSOR LAVES discussed this slow convergence in his article on the "Orbit of the Minor Planet" (334) in the *Chicago Decennial Publications*, First Series, Vol. VIII (University of Chicago Press).

from the solutions set forth. For many valuable suggestions in this whole investigation, the writer is much indebted to PROFESSOR MOULTON. In his lectures on Analytic Differential Equations given in the winter and spring of 1914-1915, he developed the theoretical basis

for the formula of mechanical quadrature used here and the proof that starting with given initial conditions and proceeding by a finite number of steps sufficiently small, the solutions of the differential equations are obtained.

The formula for mechanical quadrature

$$\int_a^{a+\omega} f(x) dx = \omega \left\{ f(a + \omega) - \frac{1}{2} f_1(a + \frac{1}{2}\omega) - \frac{1}{12} f_2(a) - \frac{1}{24} f_3(a - \frac{1}{2}\omega) - \frac{1}{720} f_4(a - \omega) - \frac{3}{160} f_5(a - \frac{3}{2}\omega) - \dots \right\},$$

is, except for the interval ω instead of the unit interval, the same formula which SIR G. W. DARWIN used in his famous Memoir on *Periodic Orbits** in which he distributed the masses of the two finite bodies in the ratio 1 : 10. In this paper the masses are in ratio 1 : 999, that of *Jupiter* $\mu = .001$ and the *Sun* $\sigma = .999$. In the formula, $f(a)$ and $f(a + \omega)$ represent the values of the function at the beginning and end of the interval ω over which the integration takes place. The expressions with subscripts denote the successive differences as exhibited in the following table; each difference is obtained by subtracting the two quantities to the left, the one above from the one below.

$f(a - 2\omega)$	$f_1(a - \frac{3}{2}\omega)$	$f_2(a - \omega)$	$f_3(a - \frac{1}{2}\omega)$	$f_4(a)$
$f(a - \omega)$	$f_1(a - \frac{1}{2}\omega)$	$f_2(a)$	$f_3(a + \frac{1}{2}\omega)$	
$f(a)$	$f_1(a + \frac{1}{2}\omega)$	$f_2(a + \omega)$	$f_3(a + \frac{3}{2}\omega)$	
$f(a + \omega)$	$f_1(a + \frac{3}{2}\omega)$			
$f(a + 2\omega)$				

In the application of the formula the first three terms were sufficient except while the small planet was near perihelion passages and then, though the interval ω was taken one-half as large as in the rest of the orbit, the fourth terms sometimes played a part. For interpolation to determine the time and angle at which the small planet passes aphelion at the midperiod, NEWTON's well known formula

$$f(x) = f(a) + xf_1(a + \frac{1}{2}) + \frac{x(x-1)}{2!} f_2(a + 1) + \frac{x(x-1)(x-2)}{3!} f_3(a + \frac{3}{2}) + \dots,$$

was adopted. To simplify the numerical computations the interval ω was temporarily replaced by unity. The terms written were sufficient.

THE DIFFERENTIAL EQUATIONS

The differential equations (4) referred to rectangular coördinates rotating with period 2π , for the motion of the small planet, become

$$\begin{aligned} \ddot{x} &= 2\dot{y} + (x - \mu) - \frac{\sigma x}{r^3} - \frac{\mu(x-1)}{R^3}, \\ \ddot{y} &= -2\dot{x} + y - \frac{\sigma y}{r^3} - \frac{\mu y}{R^3}, \end{aligned} \quad (45)$$

r and R represent the distances from the small planet to *Jupiter* and the *Sun* respectively. Let

$$\begin{aligned} x &= x_0 + \xi, \\ y &= y_0 + \eta, \end{aligned} \quad (46)$$

where x_0 and y_0 are the solutions of the two body problem, i.e., with $\mu = 0$. Thus

$$\begin{aligned} \ddot{x}_0 &= 2\dot{y}_0 + x_0 - \frac{\sigma x_0}{r_0^3}, \\ \ddot{y}_0 &= -2\dot{x}_0 + y_0 - \frac{\sigma y_0}{r_0^3}. \end{aligned} \quad (47)$$

On substituting (46) in (45) and subtracting (47) the equations in the variations ξ, η are

$$\begin{aligned} \ddot{\xi} &= 2\dot{\eta} + \xi - \mu - \sigma \left\{ \frac{x_0 + \xi}{[(x_0 + \xi)^2 + (y_0 + \eta)^2]^{3/2}} - \frac{x_0}{r_0^3} \right\} - \mu \left\{ \frac{x_0 + \xi - 1}{[(x_0 + \xi)^2 + (y_0 + \eta)^2 - 2(x_0 + \xi) + 1]^{3/2}} \right\}, \\ \ddot{\eta} &= -2\dot{\xi} + \eta - \sigma \left\{ \frac{y_0 + \eta}{[(x_0 + \xi)^2 + (y_0 + \eta)^2]^{3/2}} - \frac{y_0}{r_0^3} \right\} - \mu \left\{ \frac{y_0 + \eta}{[(x_0 + \xi)^2 + (y_0 + \eta)^2 - 2(x_0 + \xi) + 1]^{3/2}} \right\}. \end{aligned} \quad (48)$$

**Acta Mathematica*, Vol. XXI, p. 99 and *Scientific Papers*, Vol. IV, Part 1.

If equations (48) are now expanded in powers of ξ and η , and the products of ξ , η and μ are neglected, they become

$$(49) \quad \begin{aligned} \ddot{\xi} &= 2\dot{\eta} + \left[1 + \sigma \left(\frac{2x_0^2 - y_0^2}{r_0^5} \right) \right] \xi + \sigma \frac{3x_0 y_0}{r_0^5} \eta - \mu \left[\frac{x_0 - 1}{R_0^3} + 1 \right], \\ \ddot{\eta} &= -2\dot{\xi} + \left[1 + \sigma \left(\frac{2y_0^2 - x_0^2}{r_0^5} \right) \right] \eta + \sigma \frac{3x_0 y_0}{r_0^5} \xi - \mu \left[\frac{y_0}{R_0^3} \right]. \end{aligned}$$

For the purpose of simplifying the numerical computations the scale of ξ and η is increased by the factor 1000, and the following abbreviations made,

$$\begin{aligned} P_1 &= 1 + \sigma \left(\frac{2x_0^2 - y_0^2}{r_0^5} \right), \\ P_3 &= 1 + \sigma \left(\frac{2y_0^2 - x_0^2}{r_0^5} \right), \\ B_2 &= 1000\mu \left(\frac{x_0 - 1}{R_0^3} + 1 \right) = \frac{x_0 - 1}{R_0^3} + 1, \\ B_4 &= 1000\mu \frac{y_0}{R_0^3} = \frac{y_0}{R_0^3}, \\ Q &= \frac{3\sigma x_0 y_0}{r_0^5}. \end{aligned}$$

The differential equations (49) then become,

$$(50) \quad \begin{aligned} \ddot{\xi} &= 2\dot{\eta} + P_1 \xi + Q \eta - B_2, \\ \ddot{\eta} &= -2\dot{\xi} + P_3 \eta + Q \xi - B_4. \end{aligned}$$

The problem is to determine the solutions of these equations such that [see (36)]

$$\dot{\xi}(2\pi) = \eta(2\pi) = 0,$$

that is, such that the disturbed small planet crosses the line joining *Jupiter* and the *Sun* at the midperiod perpendicularly and at the same time as the undisturbed small planet.

THE JACOBI INTEGRAL

The Jacobi Integral of equations (45) is,

$$(51) \quad \dot{x}^2 + \dot{y}^2 - (x - \mu)^2 - y^2 - \frac{2\sigma}{r} - \frac{2\mu}{R} + C = 0.$$

Again substitute

$$\begin{aligned} x &= x_0 + \xi, \\ y &= y_0 + \eta, \end{aligned}$$

and neglect products of ξ , $\dot{\xi}$, η , $\dot{\eta}$ and μ . The integral then is

$$(52) \quad \begin{aligned} \dot{x}_0^2 + \dot{y}_0^2 + 2(\dot{x}_0 \dot{\xi} + \dot{y}_0 \dot{\eta}) - r_0^2 - 2(x_0 \xi + y_0 \eta) \\ + 2x_0 \mu - \frac{2\sigma}{r_0} + \frac{2\sigma}{r_0^3} (x_0 \xi + y_0 \eta) - \frac{2\mu}{R} + C = 0. \end{aligned}$$

From the relations

$$(\dot{x}_0 - y_0)^2 + (\dot{x}_0 + y_0)^2 = \sigma \left(\frac{2}{r_0} - \frac{1}{a} \right),$$

$$x_0 \dot{y}_0 - y_0 \dot{x}_0 = r_0^2 \dot{\varphi}_0 = r_0^2 \dot{\nu}_0 - r_0^2 = \sqrt{\sigma a (1 - e^2)} - r_0^2,$$

in the two body problem, the integral (52) simplifies to

$$(53) \quad \dot{x}_0 \dot{\xi} + \dot{y}_0 \dot{\eta} + \left(x_0 - \frac{1}{R_0} \right) \mu - \left(1 - \frac{\sigma}{r_0^3} \right) (x_0 \xi + y_0 \eta) + K = 0,$$

in which

$$2K = C - 2\sqrt{\sigma a (1 - e^2)} - \sigma/a.$$

As the small planet is always started at perihelion and on the line joining the *Sun* and *Jupiter*,

$$\dot{x}_0 = y_0 = \dot{\xi} = \xi = \eta = 0 \quad (t = 0),$$

but

$$\dot{\eta}(0),$$

is left at choice. On aphelion passage at the midperiod

$$\dot{x}_0 = y_0 = 0 \quad (t = 2\pi).$$

Equating the expressions for the Jacobi Integral (53) at these two epochs there results

(54)

$$\left[\dot{y}_0 \dot{\eta} - \left(1 - \frac{\sigma}{r_0^3} \right) x_0 \dot{\xi} + \left(x_0 - \frac{1}{R_0} \right) \mu \right]_{t=2\pi} = \left[\dot{y}_0 \dot{\eta} + \left(x_0 - \frac{1}{R_0} \right) \mu \right]_{t=0}.$$

This relation (54) serves as a check formula for all the computations.

THE TABLES EMPLOYED

The interval ω used in the following orbits is

$$\omega = 2^\circ = .0349,$$

when the small planet was near perihelion, in other parts of the orbits

$$\omega = 4^\circ = .0698.$$

A table for the auxiliary functions P_1, P_3, Q, B_2, B_4 was first made Åstrand, *Hülfstafeln zur Auflösung des Kepler'schen Problems* were used to find r_0 from the relation,

$$r_0 = a(1 - e \cos E),$$

and the SCHLESINGER and UDICK, *Tables for the True Anomaly in Elliptic Orbits* were employed in tabulating the values of x_0 and y_0 from the equations,

$$x_0 = r_0 \cos \varphi_0 = r_0 \cos (v_0 - t),$$

$$y_0 = r_0 \sin \varphi_0 = r_0 \sin (v_0 - t).$$

The computations for the auxiliary functions are then obvious. To utilize the check formula (54) and to determine the angle at which the small planet crosses the axis Ox at the midperiod, the values of $\dot{x}_0$ and $\dot{y}_0$ were found from,

$$\dot{x}_0 = \dot{r}_0 \cos \varphi_0 - r_0 \dot{\varphi}_0 \sin \varphi_0,$$

$$\dot{y}_0 = \dot{r}_0 \sin \varphi_0 + r_0 \dot{\varphi}_0 \cos \varphi_0,$$

with

$$\dot{r}_0 = \frac{3a}{2r_0} \sqrt{a^2 e^2 - (a - r)^2},$$

and

$$\dot{\varphi}_0 = \dot{v}_0 - 1 = \frac{1}{r_0^2} \sqrt{\sigma a(1 - e^2)} - 1.$$

In the tables below, the values of $y_0, \eta, y, \dot{x}_0, \dot{\xi}, \dot{x}, \dot{y}_0, \dot{\eta}, \dot{y}$, are tabulated at the last five epochs computed for the three families of orbits about $e = .40, e = .45$, and $e = .47$. The results are given and numbered in the order found.

$e = .40$

(1)	$\dot{\eta}(0) = +.000000$				
t	352	356	360	364	368
y_0	-.0443	-.0220	+.0000	+.0220	+.0443
η	-.0100	-.0099	-.0099	-.0099	-.0099
y	-.0543	-.0319	-.0099	+.0121	+.0344
$\dot{x}_0$	-.1088	-.0621	+.0000	+.0621	+.1088
$\dot{\xi}$	+.0026	+.0026	+.0026	+.0026	+.0026
$\dot{x}$	-.1062	-.0595	+.0026	+.0647	+.1114
$\dot{y}_0$	+.3087	+.3164	+.3138	+.3164	+.3087
$\dot{\eta}$	+.0013	+.0008	+.0003	-.0002	-.0007
$\dot{y}$	+.3100	+.3172	+.3191	+.3162	+.3080

The time T and the angle φ at which the small planet passes aphelion are now obtained as follows. T is the time for which $y = 0$. This is found by applying NEWTON'S formula

$$y = 0 = -.0543 + .0224t - \frac{.0004}{2!}t(t-1) + \frac{.0004}{3!}t(t-1)(t-2).$$

The root of this equation is 2.4534 and therefore the planet passes across the axis Ox at $.4534 \times .0698$ greater than 2π , thus

$$T = 2\pi + .0316.$$

The x and y components of the velocity at $t = 2.4534$ are also found by means of NEWTON'S formula, we have

$$v_x = -.1062 + .0467t + \frac{.0054}{2!}t(t-1) - \frac{.0054}{3!}t(t-1)(t-2) = +.0165.$$

$$v_y = +.3100 + .0072t - \frac{.0053}{2!}t(t-1) + \frac{.0005}{3!}t(t-1)(t-2) = +.3183.$$

As

$$\tan \psi = v_y/v_x \\ \psi = 87^\circ 2'.$$

The check formula for $e = .40$ is

$$[.3188\dot{\eta} + .1924\dot{\xi} - .00155]_{t=2\pi} = [1.2897\dot{\eta} - .00139]_{t=0}.$$

In the orbit above, the left and right members have the values $-.00143$ and $-.00139$ respectively.

(2)	$\eta(0) = +.001000$				
t	352	356	360	364	368
η	+.0079	+.0084	+.0089	+.0094	+.0099
y	-.0364	-.0136	+.0089	+.0314	+.0542
ξ	-.0009	-.0007	-.0005	-.0003	-.0001
x	-.1097	-.0628	-.0005	+.0618	+.1087
η	+.0067	+.0069	+.0071	+.0073	+.0074
y	+.3154	+.3233	+.3259	+.3237	+.3161

T and ψ are found as before.

$$y = 0 = -.0364 + .0228t - \frac{.0003}{2!}t(t-1) \\ + \frac{.0003}{3!}t(t-1)(t-2), \quad t = 1.6037.$$

$$T = 2\pi - .0277.$$

$$v_x = -.1097 + .0469t + \frac{.0154}{2!}t(t-1) \\ - \frac{.0154}{3!}t(t-1)(t-2) = -.0260.$$

$$v_y = +.3154 + .0079t - \frac{.0053}{2!}t(t-1) \\ + \frac{.0005}{3!}t(t-1)(t-2) = +.3255.$$

$$\psi = 94^\circ 34'.$$

The check formula has the values $-.00006$ and $-.00010$ on the left and right.

(3)	$\eta(0) = +.000500$				
t	352	356	360	364	368
η	-.0014	-.0011	-.0009	-.0007	-.0005
y	-.0457	-.0231	-.0009	+.0213	+.0438
ξ	+.0009	+.0010	+.0011	+.0012	+.0013
x	-.1079	-.0611	+.0011	+.0633	+.1101
η	+.0037	+.0036	+.0034	+.0032	+.0030
y	+.3124	+.3200	+.3222	+.3196	+.3117

$$y = 0 = -.0457 + .0226t - \frac{.0004}{2!}t(t-1) \\ + \frac{.0004}{3!}t(t-1)(t-2), \quad t = 2.0407.$$

$$T = 2\pi + .0028.$$

$$v_x = -.1079 + .0468t + \frac{.0154}{2!}t(t-1) \\ - \frac{.0154}{3!}t(t-1)(t-2) = +.0038.$$

$$v_y = +.3124 + .0076t - \frac{.0054}{2!}t(t-1) \\ + \frac{.0006}{3!}t(t-1)(t-2) = +.3222.$$

$$\psi = 89^\circ 19'.$$

The values of the check formula on the left and right are $-.00080$ and $-.00075$.

(4)	$\eta(0) = +.000600$				
t	352	356	360	364	368
η	+.0006	+.0009	+.0012	+.0015	+.0018
y	-.0437	-.0211	+.0012	+.0235	+.0461
ξ	+.0007	+.0008	+.0009	+.0010	+.0011
x	-.1081	-.0613	+.0009	+.0631	+.1099
η	+.0043	+.0042	+.0041	+.0040	+.0039
y	+.3130	+.3206	+.3229	+.3204	+.3127

$$y = 0 = -.0437 + .0226t - \frac{.0003}{2!}t(t-1) \\ + \frac{.0003}{3!}t(t-1)(t-2), \quad t = 1.9161.$$

$$T = 2\pi - .0038.$$

$$v_x = -.1081 + .0468t + \frac{.0154}{2!}t(t-1) \\ - \frac{.0154}{3!}t(t-1)(t-2) = -.0027.$$

$$v_y = +.3130 + .0076t - \frac{.0053}{2!}t(t-1) \\ + \frac{.0005}{3!}t(t-1)(t-2) = +.3229.$$

$$\psi = 90^\circ 29'.$$

The left and right members of the check formula are $-.00065$ and $-.00062$.

$$(5) \quad \eta(0) = +.000560$$

t	352	356	360	364	368
η	+.0000	+.0003	+.0006	+.0009	+.0012
y	-.0443	-.0217	+.0006	+.0229	+.0455
ξ	+.0006	+.0008	+.0009	+.0010	+.0011
x	-.1082	-.0613	+.0009	+.0631	+.1099
η	+.0044	+.0042	+.0041	+.0040	+.0039
y	+.3131	+.3206	+.3229	+.3204	+.3126

$$y = 0 = -.0443 + .0226 t - \frac{.0003}{2!} t(t-1) \\ + \frac{.0003}{3!} t(t-1)(t-2), \quad t = 1.9730.$$

$$T = 2\pi - .0019.$$

$$v_x = -.1082 + .0469 t + \frac{.0153}{2!} t(t-1) \\ - \frac{.0153}{3!} t(t-1)(t-2) = -.0008.$$

$$v_y = .3131 + .0075 t - \frac{.0052}{2!} t(t-1) \\ + \frac{.0004}{3!} t(t-1)(t-2) = +.3229.$$

$$\psi = 90^\circ 8'.5.$$

Both members of the check formula have the same value to five decimal places, viz., $-.00067$.

$$(6) \quad \eta(0) = +.000547$$

t	352	356	360	364	368
η	-.0004	-.0001	+.0002	+.0005	+.0007
y	-.0447	-.0221	+.0002	+.0225	+.0450
ξ	+.0007	+.0008	+.0010	+.0011	+.0012
x	-.1081	-.0613	+.0010	+.0632	+.1100
η	+.0042	+.0041	+.0040	+.0038	+.0037
y	+.3129	+.3205	+.3228	+.3202	+.3124

$$y = 0 = -.0447 + .0226 t - \frac{.0003}{2!} t(t-1) \\ + \frac{.0003}{3!} t(t-1)(t-2), \quad t = 1.9901.$$

$$T = 2\pi - .0007.$$

$$v_x = -.1081 + .0468 t + \frac{.0155}{2!} t(t-1) \\ - \frac{.0156}{3!} t(t-1)(t-2) = +.00036.$$

$$v_y = +.3129 + .0076 t - \frac{.0053}{2!} t(t-1) \\ + \frac{.0004}{3!} t(t-1)(t-2) = +.32283.$$

$$\psi = 89^\circ 56'.2.$$

Again both members of the check formula agree to five decimal places, viz., $-.00068$.

Interpolation from the six orbits computed about $e = .40$ indicates that the initial conditions and results for a periodic orbit would be,

$$I. \quad \eta(0) = +.000551, \quad T = 2\pi - .0011, \quad \psi = 90^\circ, \\ e = .45$$

$$(1) \quad \eta(0) = +.000550$$

t	352	356	360	364	368
y_0	-.0555	-.0280	+.0000	+.0280	+.0555
η	+.0002	+.0004	+.0007	+.0010	+.0013
y	-.0553	-.0276	+.0007	+.0290	+.0568
x_0	-.1281	-.0632	+.0000	+.0632	+.1281
ξ	-.0004	-.0003	-.0001	+.0001	+.0002
x	-.1285	-.0635	-.0001	+.0633	+.1283
y_0	+.3890	+.3984	+.4013	+.3984	+.3890
η	+.0040	+.0040	+.0041	+.0041	+.0041
y	+.3930	+.4024	+.4054	+.4025	+.3931

$$y = 0 = -.0553 + .0277 t + \frac{.0006}{2!} t(t-1) \\ - \frac{.0006}{3!} t(t-1)(t-2), \quad t = 1.9783.$$

$$T = 2\pi - .0015.$$

$$v_x = - .1285 + .0650t - \frac{.0016}{2!}t(t-1) \\ + \frac{.0016}{3!}t(t-1)(t-2) = - .0015 .$$

$$v_y = + .4054 .$$

$$\psi = 90^\circ 12'.5 .$$

The check formula for $e = .45$, is,

$$[.4014\overset{1}{\eta} + .2839\xi - .00158]_{t=2\pi} = [1.4388\overset{1}{\eta} - .00130]_{t=0} ,$$

which for the orbit above has the values $-.00054$ and $-.00051$ on the left and right.

$$(2) \quad \overset{1}{\eta}(0) = +.000535$$

t	352	356	360	364	368
η	-.0001	+.0002	+.0005	+.0008	+.0011
y	-.0556	-.0278	+.0005	+.0288	+.0566
ξ	-.0004	-.0003	-.0001	+.0000	+.0002
x	-.1285	-.0635	-.0001	+.0632	+.1283
η	+.0040	+.0040	+.0041	+.0041	+.0041
y	+.3930	+.4024	+.4054	+.4025	+.3931

$$y = 0 = - .0556 + .0278t + \frac{.0005}{2!}t(t-1) \\ - \frac{.0005}{3!}t(t-1)(t-2) , \quad t = 1.9824 .$$

$$T = 2\pi - .0012 .$$

$$v_x = - .1285 + .0650t - \frac{.0016}{2!}t(t-1) \\ + \frac{.0016}{3!}t(t-1)(t-2) = - .0012 .$$

$$v_y = + .4054$$

$$\psi = 90^\circ 10'.3 .$$

The check formula has the values $-.00055$ and $-.00053$ on the left and right.

$$(3) \quad \overset{1}{\eta}(0) = +.000180$$

t	352	356	360	364	368
η	-.0012	-.0009	-.0007	-.0004	-.0002
y	-.0567	-.0289	-.0007	+.0276	+.0553
ξ	-.0002	-.0001	+.0001	+.0002	+.0004

t	352	356	360	364	368
x	-.1283	-.0633	+.0001	+.0634	+.1285
η	+.0037	+.0037	+.0037	+.0036	+.0036
y	+.3927	+.4021	+.4050	+.4020	+.3926

$$y = 0 = - .0567 + .0278t + \frac{.0006}{2!}t(t-1) \\ - \frac{.0005}{3!}t(t-1)(t-2) , \quad t = 2.0175 .$$

$$T = 2\pi + .0012 .$$

$$v_x = - .1283 + .0650t - \frac{.0016}{2!}t(t-1) \\ + \frac{.0015}{3!}t(t-1)(t-2) = + .0012 .$$

$$v_y = + .4050 .$$

$$\psi = 89^\circ 49'.6 .$$

The check formula has the values $-.00064$ and $-.00061$ on the left and right.

$$(4) \quad \overset{1}{\eta}(0) = +.000508$$

t	352	356	360	364	368
η	-.0004	-.0001	+.0002	+.0005	+.0008
y	-.0559	-.0281	+.0002	+.0285	+.0563
ξ	-.0003	-.0001	+.0000	+.0002	+.0003
x	-.1284	-.0633	+.0000	+.0634	+.1284
η	+.0042	+.0042	+.0042	+.0042	+.0042
y	+.3932	+.4026	+.4055	+.4026	+.3932

$$y = 0 = - .0559 + .0278t + \frac{.0005}{2!}t(t-1) \\ - \frac{.0005}{3!}t(t-1)(t-2) , \quad t = 1.9929 .$$

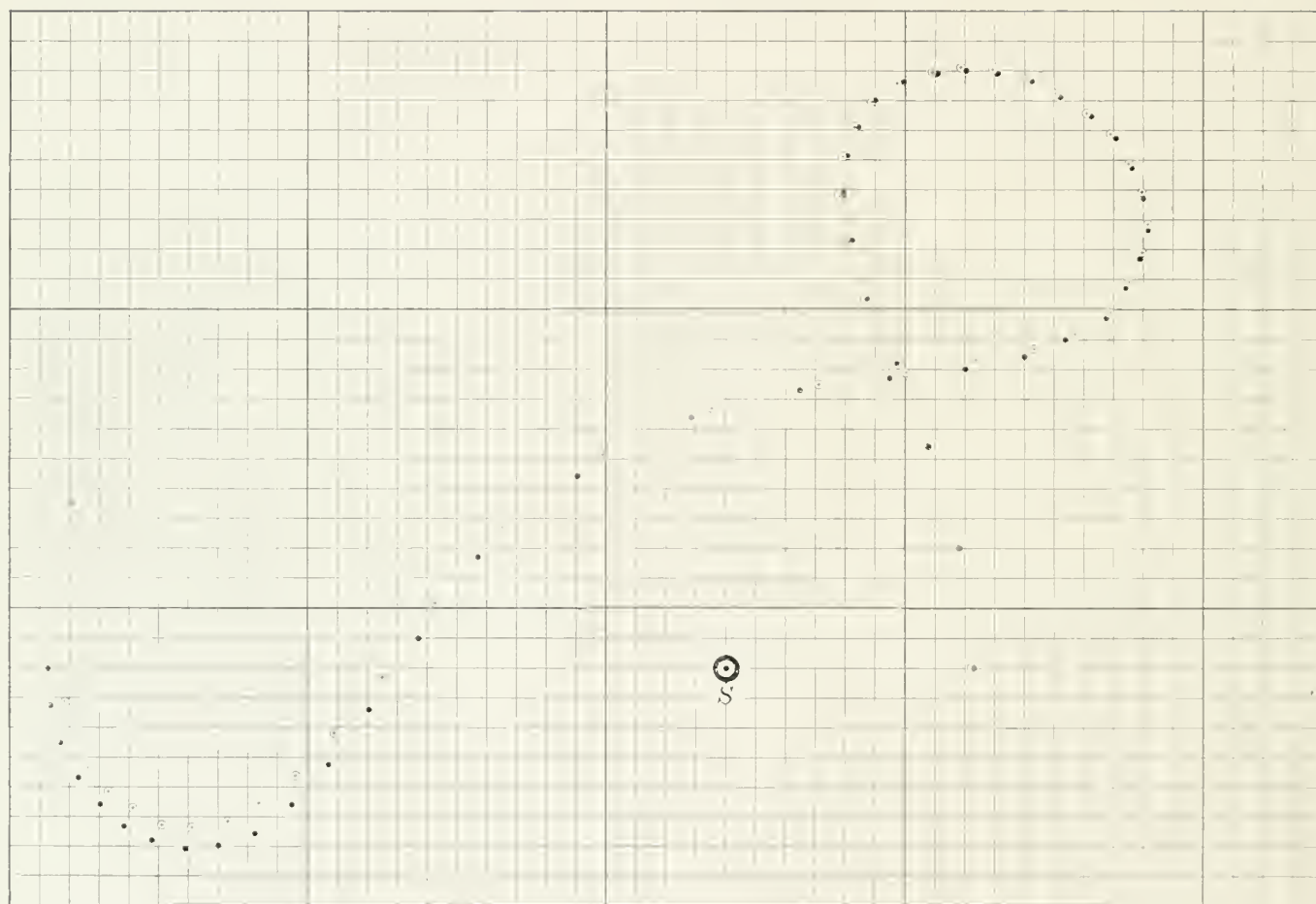
$$T = 2\pi - .0005 .$$

$$v_x = - .1284 + .0651t - \frac{.0018}{2!}t(t-1) \\ + \frac{.0019}{3!}t(t-1)(t-2) = - .0004 .$$

$$v_y = + .4055$$

$$\psi = 90^\circ 3'.7 .$$

PERIODIC ORBIT FOR THE *HILDA* TYPE



(*The Astronomical Journal*, Nos. 683-684.)

-.00053 and -.00057 are the values of the check formula on the left and right.

Interpolation from the four orbits computed about $e = .45$ leads to the periodic orbit,

$$\text{II. } \dot{\eta}(0) = +.000500, \quad T = 2\pi - .0001, \quad \psi = 90^\circ.$$

$$e = .47$$

$$(1) \quad \dot{\eta}(0) = +.000480$$

t	352	356	360	364	368
y_0	-.0601	-.0303	+.0000	+.0303	+.0601
η	-.0007	-.0004	-.0001	+.0001	+.0004
y	-.0608	-.0307	-.0001	+.0304	+.0605
$\dot{x}_0$	-.1127	-.0547	+.0000	+.0547	+.1127
$\dot{\xi}$	-.0006	-.0005	-.0003	-.0002	-.0000
$\dot{x}$	-.1133	-.0552	-.0003	+.0545	+.1127
y_0	+.4233	+.4318	+.4345	+.4318	+.4233
$\dot{\eta}$	+.0039	+.0039	+.0040	+.0040	+.0040
$\dot{y}$	+.4272	+.4357	+.4385	+.4358	+.4273

$$y = 0 = -.0608 + .0301t + \frac{.0005}{2!}t(t-1) - \frac{.0006}{3!}t(t-1)(t-2), \quad t = 2.0033.$$

$$v_x = -.1133 + .0581t - \frac{.0032}{2!}t(t-1) + \frac{.0003}{3!}t(t-1)(t-2) = -.0001.$$

$$v_y = +.4385$$

$$\text{III. } T = 2\pi + .0002, \quad \psi = 90^\circ 00'.8.$$

The check formula is

$$[.4345\dot{\eta} + .3271\xi - .00159]_{t=2\pi} = [1.5015\dot{\eta} - .00127]_{t=0},$$

which has the values -.00052 and -.00055. This orbit III is almost periodic and is taken as such.

ORBIT OF TYPE 3/2

As the periods of the orbits I and II are too short and that of III is too long, the existence of a periodic orbit of period 4π is demonstrated. The eccentricity of the undisturbed ellipse is between .45 and .47 and the initial velocity relative to this ellipse in the rotating axes is between +.000500 and +.000480. As the tables used tabulate the eccentricity to the second decimal place only, the values of e and $\eta(0)$ chosen are

$$e = .46, \quad \dot{\eta}(0) = +.00049.$$

The accompanying figure presents this orbit. The $\bigcirc$ and $\bullet$ indicate at intervals $\omega = 8^\circ$, the positions of the undisturbed and disturbed small planet. The scale of the perturbations is increased by a factor 10. To the degree of approximation so far employed, the results are,

$$\text{IV. } T = 2\pi, \quad \psi = 90^\circ 00'.8.$$

The check formula for $e = .46$ is,

$$[.4177\dot{\eta} + .3084\xi - .00159]_{t=2\pi} = [1.4702\dot{\eta} - .00129]_{t=0},$$

which for the orbit computed has the value -.00058 on the left and -.00057 on the right.

On interpolation from the periodic orbits I, II, III and IV more accurate initial conditions for the motion of type 3/2 are

$$e = .457, \quad \dot{\eta}(0) = +.00049.$$

TABLES FOR TYPE 3/2 UNDER FIRST ORDER PERTURBATIONS

t	$\ddot{\xi}$	$\dot{\xi}$	ξ	$\ddot{\eta}$	$\dot{\eta}$	η	x_0	y_0	x	y
0	+ 2.9	+ .00	+ .00	- 0.0	+ .49	+ .00	+ .4119	+ .0000	+ .4119	+ .0000
2	+ 2.9	+ .10	+ .00	- .7	+ .48	+ .02	+ .4103	+ .0512	+ .4103	+ .0512
4	+ 3.1	+ .21	+ .01	- 1.1	+ .45	+ .03	+ .4058	+ .1018	+ .4058	+ .1018
6	+ 3.3	+ .32	+ .02	- 1.4	+ .40	+ .05	+ .3986	+ .1513	+ .3986	+ .1514
8	+ 3.4	+ .43	+ .03	- 1.4	+ .35	+ .06	+ .3890	+ .1992	+ .3890	+ .1993
10	+ 3.4	+ .55	+ .05	- 1.2	+ .31	+ .07	+ .3774	+ .2451	+ .3775	+ .2452
12	+ 3.2	+ .67	+ .07	- 1.0	+ .27	+ .08	+ .3643	+ .2890	+ .3644	+ .2891
14	+ 3.0	+ .78	+ .09	- .7	+ .24	+ .09	+ .3505	+ .3307	+ .3506	+ .3308
16	+ 2.6	+ .88	+ .12	- .5	+ .21	+ .10	+ .3360	+ .3701	+ .3361	+ .3702
20	+ 1.8	+ 1.03	+ .19	- .3	+ .19	+ .11	+ .3066	+ .4428	+ .3068	+ .4429

TABLES FOR TYPE 3/2 UNDER FIRST ORDER PERTURBATIONS—*Continued*

t	$\overset{u}{\xi}$	$\overset{i}{\xi}$	ξ	$\overset{u}{\eta}$	$\overset{i}{\eta}$	η	x_0	y_0	x	y
24	+ 1.1	+1.13	+ .26	- .3	+ .17	+ .13	+ .2788	+ .5078	+ .2791	+ .5079
28	+ .5	+1.18	+ .35	- .4	+ .14	+ .14	+ .2536	+ .5665	+ .2540	+ .5666
32	+ .0	+1.20	+ .43	- .6	+ .10	+ .15	+ .2326	+ .6195	+ .2330	+ .6197
36	- .3	+1.19	+ .51	- .8	+ .06	+ .15	+ .2159	+ .6677	+ .2164	+ .6679
40	- .6	+1.15	+ .59	- .9	- .00	+ .15	+ .2034	+ .7117	+ .2040	+ .7119
44	- .8	+1.10	+ .67	- 1.0	- .07	+ .15	+ .1955	+ .7521	+ .1962	+ .7523
48	- 1.0	+1.04	+ .75	- 1.0	- .14	+ .14	+ .1917	+ .7894	+ .1924	+ .7895
52	- 1.1	+ .97	+ .82	- 1.1	- .21	+ .13	+ .1921	+ .8234	+ .1929	+ .8235
56	- 1.2	+ .89	+ .88	- 1.0	- .29	+ .11	+ .1967	+ .8545	+ .1976	+ .8546
60	- 1.3	+ .80	+ .94	- 1.0	- .36	+ .09	+ .2046	+ .8827	+ .2055	+ .8828
64	- 1.4	+ .71	+ .99	- .9	- .42	+ .06	+ .2160	+ .9078	+ .2170	+ .9079
68	- 1.4	+ .61	+1.04	- .8	- .48	+ .03	+ .2305	+ .9305	+ .2315	+ .9305
72	- 1.5	+ .51	+1.08	- .7	- .54	- .00	+ .2481	+ .9502	+ .2492	+ .9502
76	- 1.5	+ .40	+1.11	- .6	- .58	- .04	+ .2678	+ .9667	+ .2689	+ .9667
80	- 1.6	+ .30	+1.14	- .5	- .62	- .08	+ .2899	+ .9806	+ .2910	+ .9805
84	- 1.6	+ .18	+1.15	- .4	- .65	- .13	+ .3136	+ .9918	+ .3148	+ .9917
88	- 1.6	+ .07	+1.16	- .3	- .68	- .18	+ .3386	+ .9993	+ .3398	+ .9991
92	- 1.6	- .04	+1.16	- .1	- .69	- .22	+ .3656	+1.0053	+ .3668	+1.0051
96	- 1.6	- .16	+1.16	+ .0	- .69	- .27	+ .3927	+1.0075	+ .3939	+1.0072
100	- 1.6	- .27	+1.14	+ .1	- .69	- .32	+ .4205	+1.0070	+ .4216	+1.0067
104	- 1.6	- .38	+1.12	+ .3	- .67	- .37	+ .4486	+1.0037	+ .4497	+1.0033
108	- 1.6	- .49	+1.09	+ .4	- .65	- .41	+ .4765	+ .9977	+ .4776	+ .9977
112	- 1.5	- .60	+1.05	+ .5	- .62	- .46	+ .5055	+ .9886	+ .5065	+ .9881
116	- 1.5	- .70	+1.00	+ .6	- .59	- .50	+ .5311	+ .9781	+ .5321	+ .9776
120	- 1.4	- .81	+ .95	+ .7	- .54	- .54	+ .5569	+ .9645	+ .5579	+ .9640
124	- 1.4	- .91	+ .89	+ .8	- .49	- .58	+ .5816	+ .9491	+ .5825	+ .9485
128	- 1.3	-1.00	+ .82	+ .8	- .44	- .61	+ .6033	+ .9320	+ .6041	+ .9314
132	- 1.3	-1.09	+ .75	+ .9	- .38	- .64	+ .6257	+ .9116	+ .6265	+ .9110
136	- 1.2	-1.18	+ .67	+ .9	- .31	- .66	+ .6449	+ .8904	+ .6456	+ .8897
140	- 1.2	-1.27	+ .59	+ 1.0	- .25	- .68	+ .6618	+ .8676	+ .6624	+ .8669
144	- 1.2	-1.35	+ .50	+ 1.0	- .18	- .69	+ .6759	+ .8437	+ .6764	+ .8430
148	- 1.1	-1.43	+ .40	+ 1.0	- .11	- .70	+ .6879	+ .8192	+ .6883	+ .8185
152	- 1.1	-1.51	+ .30	+ 1.0	- .04	- .71	+ .6962	+ .7929	+ .6965	+ .7922
156	- 1.1	-1.58	+ .19	+ .9	+ .02	- .71	+ .7021	+ .7675	+ .7023	+ .7668
160	- 1.1	-1.66	+ .08	+ .9	+ .09	- .71	+ .7044	+ .7413	+ .7045	+ .7406
164	- 1.2	-1.74	- .04	+ .8	+ .15	- .70	+ .7034	+ .7153	+ .7034	+ .7146
168	- 1.2	-1.83	- .17	+ .7	+ .20	- .69	+ .6968	+ .6899	+ .6966	+ .6892
172	- 1.4	-1.92	- .30	+ .6	+ .25	- .67	+ .6904	+ .6648	+ .6901	+ .6641
176	- 1.5	-2.02	- .44	+ .5	+ .30	- .65	+ .6783	+ .6411	+ .6779	+ .6404
180	- 1.7	-2.13	- .58	+ .4	+ .32	- .63	+ .6622	+ .6186	+ .6616	+ .6180
184	- 2.0	-2.26	- .73	+ .2	+ .34	- .61	+ .6417	+ .5976	+ .6410	+ .5970
188	- 2.4	-2.41	- .90	- .1	+ .34	- .58	+ .6170	+ .5781	+ .6161	+ .5775
192	- 2.9	-2.59	-1.07	- .5	+ .33	- .56	+ .5878	+ .5607	+ .5867	+ .5601
196	- 3.5	-2.82	-1.26	- 1.0	+ .28	- .54	+ .5536	+ .5454	+ .5523	+ .5449
200	- 4.4	-3.09	-1.47	- 1.9	+ .18	- .52	+ .5146	+ .5320	+ .5131	+ .5315
204	- 5.3	-3.43	-1.69	- 3.2	+ .00	- .52	+ .4703	+ .5208	+ .4686	+ .5203
206	- 5.8	-3.62	-1.82	- 4.0	- .13	- .52	+ .4459	+ .5159	+ .4441	+ .5154
208	- 6.3	-3.84	-1.95	- 5.2	- .28	- .52	+ .4201	+ .5111	+ .4181	+ .5106
210	- 6.8	-4.07	-2.08	- 6.7	- .49	- .54	+ .3927	+ .5069	+ .3906	+ .5064
212	- 7.1	-4.31	-2.23	- 8.4	- .75	- .56	+ .3637	+ .5030	+ .3615	+ .5024
214	- 7.3	-4.56	-2.38	-10.5	- 1.08	- .59	+ .3329	+ .4991	+ .3305	+ .4985
216	- 7.2	-4.86	-2.55	-13.1	- 1.49	- .64	+ .3004	+ .4953	+ .2978	+ .4947
218	- 6.7	-5.06	-2.72	-16.1	- 2.00	- .70	+ .2662	+ .4912	+ .2635	+ .4905
220	- 5.5	-5.27	-2.90	-19.6	- 2.62	- .78	+ .2302	+ .4870	+ .2273	+ .4862
222	- 3.4	-5.43	-3.09	-23.1	- 3.37	- .88	+ .1923	+ .4818	+ .1892	+ .4809
224	- .1	-5.49	-3.28	-27.4	- 4.26	-1.01	+ .1525	+ .4760	+ .1492	+ .4750
226	+ 4.8	-5.42	-3.47	-31.0	- 5.28	-1.18	+ .1111	+ .4688	+ .1076	+ .4676
228	+11.4	-5.11	-3.65	-33.9	- 6.41	-1.38	+ .0681	+ .4600	+ .0644	+ .4586
230	+19.5	-4.61	-3.83	-34.9	- 7.62	-1.63	+ .0236	+ .4494	+ .0198	+ .4478
232	+28.8	-3.77	-3.97	-33.2	- 8.82	-1.92	- .0220	+ .4364	- .0260	+ .4345

TABLES FOR TYPE 3/2 UNDER FIRST ORDER PERTURBATIONS—*Continued*

<i>t</i>	$\overset{u}{\xi}$	$\overset{l}{\xi}$	ξ	$\overset{u}{\eta}$	$\overset{l}{\eta}$	η	x_3	y_3	x	y
234	+38.2	-2.60	-4.08	-27.9	-9.89	-2.24	-.0683	+.4208	-.0724	+.4186
236	+46.0	-1.12	-4.15	-18.8	-10.73	-2.60	-.1147	+.4023	-.1189	+.3997
238	+50.5	+.57	-4.16	-6.4	-11.18	-2.99	-.1608	+.3810	-.1650	+.3780
240	+50.6	+2.35	-4.11	+7.7	-11.16	-3.38	-.2060	+.3567	-.2101	+.3533
242	+45.7	+4.05	-4.00	+21.3	-10.65	-3.76	-.2495	+.3298	-.2535	+.3260
244	+36.9	+5.50	-3.83	+32.3	-9.70	-4.12	-.2911	+.3005	-.2949	+.2964
246	+25.7	+6.60	-3.62	+39.5	-8.44	-4.43	-.3303	+.2695	-.3339	+.2651
248	+14.1	+7.29	-3.37	+42.4	-7.00	-4.70	-.3670	+.2372	-.3704	+.2325
250	+3.6	+7.60	-3.11	+41.9	-5.52	-4.92	-.4010	+.2043	-.4041	+.1994
252	-4.8	+7.57	-2.85	+38.7	-4.10	-5.09	-.4324	+.1711	-.4352	+.1660
254	-10.9	+7.29	-2.59	+34.3	-2.83	-5.21	-.4616	+.1382	-.4642	+.1330
256	-14.9	+6.83	-2.34	+29.3	-1.72	-5.29	-.4884	+.1061	-.4907	+.1008
258	-17.3	+6.27	-2.11	+24.3	-.78	-5.33	-.5135	+.0744	-.5156	+.0691
260	-18.2	+5.64	-1.90	+19.8	-.01	-5.35	-.5368	+.0441	-.5387	+.0388
262	-18.4	+5.00	-1.72	+16.0	+.61	-5.33	-.5585	+.0151	-.5602	+.0098
264	-17.9	+4.36	-1.56	+12.7	+.11	-5.30	-.5792	-.0125	-.5808	-.0178
266	-16.7	+3.76	-1.41	+10.0	+1.50	-5.26	-.5987	-.0387	-.6001	-.0440
238	-15.5	+3.20	-1.29	+7.8	+1.81	-5.20	-.6174	-.0634	-.6187	-.0686
270	-14.3	+2.67	-1.19	+6.1	+2.05	-5.13	-.6353	-.0867	-.6365	-.0918
272	-13.0	+2.20	-1.10	+4.8	+2.24	-5.06	-.6527	-.1082	-.6538	-.1133
274	-11.8	+1.77	-1.04	+3.7	+2.39	-4.98	-.6697	-.1282	-.6707	-.1332
276	-10.6	+1.38	-.98	+2.9	+2.50	-4.89	-.6863	-.1469	-.6873	-.1518
280	-8.5	+.72	-.91	+1.9	+2.67	-4.71	-.7180	-.1797	-.7189	-.1844
284	-6.7	+.19	-.88	+1.3	+2.78	-4.52	-.7492	-.2068	-.7501	-.2113
288	-5.3	-.23	-.88	+1.0	+2.86	-4.32	-.7795	-.2287	-.7804	-.2330
292	-4.0	-.55	-.91	+.9	+2.92	-4.12	-.8091	-.2453	-.8100	-.2494
296	-3.0	-.80	-.95	+.8	+2.98	-3.91	-.8383	-.2569	-.8393	-.2608
300	-2.2	-.98	-1.02	+.9	+3.04	-3.70	-.8668	-.2642	-.8678	-.2679
304	-1.5	-1.11	-1.09	+.9	+3.10	-3.49	-.8943	-.2659	-.8954	-.2704
308	-.9	-1.19	-1.17	+.9	+3.16	-3.27	-.9211	-.2665	-.9223	-.2688
312	-.4	-1.23	-1.25	+1.0	+3.23	-3.05	-.9469	-.2602	-.9482	-.2633
316	+.1	-1.24	-1.34	+1.0	+3.30	-2.82	-.9712	-.2515	-.9725	-.2543
320	+.5	-1.22	-1.43	+1.0	+3.37	-2.59	-.9950	-.2364	-.9964	-.2390
324	+.8	-1.17	-1.51	+1.0	+3.44	-2.35	-1.0159	-.2242	-1.0174	-.2266
328	+1.1	-1.10	-1.59	+.9	+3.51	-2.11	-1.0348	-.2064	-1.0364	-.2085
332	+1.4	-1.02	-1.67	+.9	+3.58	-1.86	-1.0535	-.1861	-1.0552	-.1880
336	+1.6	-.91	-1.73	+.8	+3.64	-1.61	-1.0688	-.1635	-1.0705	-.1651
340	+1.7	-.80	-1.79	+.7	+3.69	-1.35	-1.0820	-.1393	-1.0838	-.1407
344	+1.9	-.67	-1.84	+.6	+3.74	-1.09	-1.0938	-.1133	-1.0956	-.1144
348	+2.0	-.54	-1.89	+.5	+3.78	-.83	-1.1025	-.0861	-1.1044	-.0869
352	+2.1	-.40	-1.92	+.4	+3.81	-.56	-1.1087	-.0565	-1.1106	-.0571
356	+2.1	-.25	-1.94	+.3	+3.83	-.30	-1.1127	-.0291	-1.1146	-.0294
360	+2.1	-.10	-1.96	+.2	+3.85	-.03	-1.1140	-.0000	-1.1160	-.0000
364	+2.1	+.05	-1.96	+.1	+3.86	+.24	-1.1127	+.0291	-1.1147	+.0293
368	+2.1	+.19	-1.95	+.0	+3.86	+.51	-1.1087	+.0565	-1.1107	+.0570

*University of Michigan, April, 1915.*THE PERIOD OF *O HERCULIS*,

BY ELIAS BRESOX.

I have observed the star *o Herculis* R. A. 18^h 1^m 53^s, Decl. +28° 44' 7" (1855.0) from July 4, 1912 to July 14, 1914, on 187 evenings. The ARGELANDER step-method was used with the following comparison stars from the *Potsdamer Durchmusterung*:

ζ <i>Lyra</i>	= 4 ^m .73 = <i>a</i>
κ <i>Lyra</i>	= 4 .51 = <i>b</i>
μ <i>Herculis</i>	= 3 .62 = <i>c</i>

ξ <i>Herculis</i>	= 3 ^m .88 = <i>d</i>
ν <i>Herculis</i>	= 4 .72 = <i>e</i>
θ <i>Herculis</i>	= 3 .95 = <i>f</i>

Every evening two or three estimates were made, each observation being weighted in the reduction. The observations were taken with a binocular 4 cm. objective and power 2.5. The dates are in Julian days and Greenwich mean time.

MAXIMA	MINIMA
2419869.0	2419863.392
882.364	875.0
890.0	885.361
900.462	896.364
911.406	904.392
922.410	918.396
936.434	931.437

$$\text{Minimum} = 2419863.392 + 11.985 \times E$$

The maximum light occurred on the Julian day 241-9900.462 G.M.T., when the star's magnitude was 4^m.09, on the scale of the *Potsdamer Durchmusterung*, and the minimum light occurred on the Julian day 2419863.392 G.M.T., with the magnitude 4^m.60. Thus the light range amounts to 0^m.51.

The following elements were deduced:

The light curve seems to indicate that the star is of the type of ζ *Geminorum*.

OBSERVATIONS

1913	Gr. M.T.	J. D. 2419+	COMPARISONS	M	
March 26	9 ^h 30 ^m	852	<i>d 4 v; c 6 v; v 0 a</i>	4.40 P.D.	
April 4	9 30	861	<i>v 1 a; d 5 v; f 4 v 4 e</i>	4.42	
5	9 10	862	<i>a 1 v 2 e</i>	4.67	
6	9 25	863	<i>a 2 v 2 e</i>	4.72	
7	9 25	864	<i>c 4 v; d 2 v</i>	4.35	some haze.
8	8 35	865	<i>v 2 a; v 5 e</i>	4.37	do
13	8 30	870	<i>v 4 e; v 1 a</i>	4.47	
14	8 35	871	<i>v 0 a; v 3 e</i>	4.57	
16	9 30	873	<i>v 2 a; v 4 e</i>	4.42	
21	8 25	878	<i>v 3 e; v 0 a</i>	4.57	
22	8 15	879	<i>v 2 a; v 4 e</i>	4.42	
24	8 45	881	<i>v 6 e; d*3 v 3 a</i>	4.24	
25	8 45	882	<i>v 6 e; d 2 v</i>	4.10	some haze.
26	8 45	883	<i>v 3 e; v 1 b</i>	4.41	
26	8 55	883	<i>v 4 e; v 2 b; f 3 v 1 a</i>	4.38	
27	8 40	884	<i>f 5 v 3 e; v 0 a; v 1.5 b</i>	4.48	
28	8 40	885	<i>v 2.5 e; v 1 a; v 1.5 b</i>	4.48	
29	8 35	886	<i>v 3 e; v 1 a; v 2 b</i>	4.45	
30	9 35	887	<i>v 4 e; v 1 a; v 3 b</i>	4.38	
May 1	8 35	888	<i>v 4.5 e; v 2 a; f 2 v</i>	4.31	
5	8 30	892	<i>v 5 e; v 2.5 a; v 3 b; f 3 v</i>	4.28	
6	8 50	893	<i>f 3.5 v; v 4 e; v 2 b; v 1 a</i>	4.38	
7	9 30	894	<i>v 3 e; v 1.5 b; v 0 a</i>	4.50	
8	8 45	895	<i>v 3 e</i>	4.42	some clouds.
8	9 15	895	<i>v 3 e; v 2 b; v 1 a</i>	4.43	
9	9 15	896	<i>v 2.5 e; v 1 a</i>	4.54	
9	9 35	896	<i>v 3 e; v 1 b</i>	4.41	
10	9 10	897	<i>v 3.5 e; f 3.5 v; v 1.5 b</i>	4.36	
11	9 15	898	<i>v 4 e; b 2 v; v 2 a</i>	4.38	
12	9 05	899	<i>v 3 b; v 2 a; f 3 v</i>	4.32	
13	9 10	900	<i>v 3.5 b*; v 2.5 a; v 5 e</i>	4.29	
13	9 20	900	<i>d 3 v; f 3 v (!)</i>	4.16	
13	11 05	900	<i>d 2 v; f 1.5 v</i>	4.09	
14	10 10	901	<i>d 3 v</i>	4.18	some clouds.
15	9 45	902	<i>d 3.5 v; f 2 v</i>	4.19	moon.
16	9 10	903	<i>v 4 e; v 2 b; v 1 a</i>	4.22	
17	9 25	904	<i>v 3 e; v 2 b; v 0 a</i>	4.49	
18	9 10	905	<i>v 3.5 e; v 1.5 b</i>	4.36	full moon.
22	9 15	909	<i>v 4 e; v 2 b*</i>	4.32	some haze.
24	9 45	911	<i>f 1.5 v; d 2 v</i>	4.09	
25	9 15	912	<i>d 3.5 v; f* 2.5 v</i>	4.22	some clouds.
26	9 45	913	<i>d 3 v; f 2.5 v</i>	4.19	
28	9 50	915	<i>d 3.5 v; f 3 v 3 e</i>	4.30	
30	9 15	917	<i>v 2.5 e; v 0 a</i>	4.60	some haze.
31	9 30	918	<i>v 3 e; v 1.5 a</i>	4.50	
June 2	9 20	920	<i>v 2 e; v 1 b; v 0 a</i>	4.55	some haze.
3	9 45	921	<i>v 4 e; v 2 b; v 1 a</i>	4.42	
4	9 35	922	<i>v 3.5 e; v 2 b; f 3 v</i>	4.31	
4	9 50	922	<i>v 2 a; v 2.5 b; v 3.5 e; f 2.5 v</i>	4.34	

			OBSERVATIONS — <i>Continued</i>		
1913	Gr. M. T.	J. D. 2419 +	COMPARISONS	<i>M</i>	
June 13	10 30	931	<i>v</i> 2 <i>e</i> ; <i>v</i> 1 <i>b</i>	4.46	
14	9 30	932	<i>v</i> 2.5 <i>e</i> ; <i>v</i> 1.5 <i>b</i>	4.41	Zeiss Binocular used, magnifying 6x.
15	10 10	933	<i>v</i> 3 <i>e</i> ; <i>f</i> 2.5 <i>v</i>	4.31	
16	10 05	934	<i>v</i> 4 <i>e</i> ; <i>d</i> 3 <i>v</i>	4.25	
18	9 50	936	<i>f</i> 3 <i>v</i> ; <i>v</i> 3 <i>e</i>	4.33	
18	10 25	936	<i>v</i> 4 <i>e</i> ; <i>d</i> 2 <i>v</i>	4.20	
19	10 25	937	<i>v</i> 2.5 <i>e</i> ; <i>v</i> 1.5 <i>b</i>	4.41	
20	10 00	938	<i>v</i> 2 <i>e</i> ; <i>v</i> 1 <i>b</i>	4.46	do

* Observation given half weight.

† Correction for extinction.

v *o* *Herculis*.*Helsingør, Denmark, July 28, 1915.*

OBSERVATIONS OF VARIABLE STARS,

By PAUL S. YENDELL.

SW Andromeda

Twelve observations of this short-period star, made in the Autumn of 1913 and forty-eight in September and October 1914, indicate maxima as follows:

$o - c$				
1913 Sept. 27	7 ^h 20 ^m	Local M.T.	0 ^m	Wt. 1½
Oct. 27	8 14		- 11	1½
1914 Sept. 19	7 52		+ 9	1
22 10 0			+ 5	1

The comparisons are with HARTWIG's elements.

X Camelopardalis

I have twenty-four observations of this star, from 1915, January 20 to June 23. At the first-mentioned date, the star's brightness was estimated at 6^m.7, from which it decreased rather rapidly to a minimum of 11^m.1, which was indicated on March 20; on April 8 it had increased to 10^m.1, and when next observed on May 10 was still 10^m.1. From this it increased steadily until at the last observation June 28, it was estimated as 7^m.5.

RU Camelopardalis

Seventeen observations from 1909, November 6 to 1910, February 10, and eleven from 1915, February 20 to May 10, indicate phases as follows:

Max. 1909 Dec.	5.312	Min. Dec.	10.312
1910 Feb.	2.340		
1915 Feb.	21.3	Mar.	4.3
Mar.	9.3	Apr.	9.3

V Leonis

Six observations, made in the Spring of 1894, show a maximum of *V Leonis* on March 27, 1894. The star's brightness at the first observation is estimated as 9^m.6, at the maximum as 8^m.8, and when last observed as 9^m.0.

Compare H. M. PARKHURST, *A.J.* 14, 108, and GRUSS & LASKA, 15, 91.

R Ursæ Majoris

Fourteen observations of *R Ursæ Majoris*, from 1915, February 12 to April 8, show a maximum of 7^m.6 on February 21; when last seen it had decreased to 8^m.9.

V Vulpeculæ

During the last three years I have observed this star as follows: 1913, June 11 to December 19, fifty times; 1914, June 16 to December 20, thirty-one times, and 1915, January 3 to October 27, twenty-nine times.

From these observations I have deduced the phases as follows:

MAXIMA				MINIMA			
1913 July	7.7	8 ^m .5		1913 July	21.3	9 ^m .4	
Sept.	27.1	8 .2		Sept.	3.0	9 .4	
1914 July	14.1	8 .4		1914 July	25.6	9 .4	
Aug	23.2	8 .6					
Sept.	25.7	8 .5		1914 Sept.	9.0	9 .4	
1915 Oct.	10.4	8 .5		1915 July	7.0	9 .4	

SS Cygni

Since the last observations published in this *Journal*, Vol. XXIX, p. 38, as close a watch as possible has been kept on the star until 1915, October 28.

The observations are as follows:

1914 Nov.	23.337	not seen; saw d (10 ^m .8).
Dec.	10.302	normal
	11.323	normal
	12.444	not seen; saw d
	14.312	normal
	15.312	normal; glimpsed
	16.306	normal
	17.315	normal
	18.315	normal
	20.270	normal
1915 Jan.	3.250	8 ^m .40

1915 Jan.	4.250	9 ^m .15	
	5.257	9 .15	
	7.260	9 .53	
	8.257	9 .50	
	9.250	9 .5	
	10.260	10 .2	
June	20.418	normal.	Moon 8 da.
	23.403	10 ^m .5	Moon bright (checked later)
	24.409	9 .76	Moon bright
	26.402	9 .14	Moon full
	27.389	9 .00	Moon bright
	28.415	8 .25	Moon bright
	29.437	8 .40	Moon bright
1914 July	6.392	normal	
	9.375	normal	
	12.329	normal	
	18.359	normal.	Moon 6 da.
	23.368	normal.	Moon bright
Aug.	17.348	10 ^m .5	Moon 1 qr.
Sept.	18.340	normal.	Moon bright
	19.302	normal.	Moon bright
	22.309	normal.	Moon bright
	27.319	normal	
	30.319	normal	
Oct.	4.313	normal	
	6.347	not 10 ^m	(milky sky)
	10.368	not seen	
	11.309	normal	
	12.312	normal	
	13.315	normal.	Moon 6 da.
	16.323	normal.	Moon 1 qr.
	24.309	9 ^m .10	Moon full
	27.391	9 .15	Moon bright
	28.368	9 .0	

U Geminorum

I observed *U Geminorum* upon all the available evenings from 1914, December 13 until 1915, May 10, the observations numbering in all thirty-five. The observations from 1914, December 10.415 to 16.423 indicate a maximum of 9^m.30 on December 11.8. The curve appears to have been of the "short" type.

The observations in detail are as follows:

1914. Dec.	10.415	e2v, v1b	9 ^m .61
	11.427	a3v, v1b	9 .30
	12.417	v(be)	9 .62
	14.117	b3v, v2d	9 .84
1914. Dec.	15.403	d2v	10 ^m .30
	16.427	ve	10 .8 seeing dull
	18.417	not seen; saw e	<11.0
1915 Jan.	5.347	not seen	<10.5
	7.333	not seen	<10.5
	8.365	not seen; saw d	
	9.333	not seen; saw e	
	20.333	not seen; saw d and e.	Moon 5 da.
	21.333	not seen	<10
Feb.	10.312	not seen	<10
	12.312	not seen; saw d	
	13.312	not seen; saw d	
	17.319	not seen; saw e.	Moon 4 da.
	18.333	not seen; seen d.	Moon 5 da.
	19.312	not seen	<10. Moon 6 da.
	20.312	not seen; saw d.	Moon 7 da.
	.372	not seen; saw e	
	21.315	not seen; saw d.	Moon bright
Mar.	4.338	not seen; saw d, e	
	11.338	not seen; saw d	
	14.337	not seen; saw d	
	16.333	not seen; saw d	
	18.326	not seen; saw e	
	20.354	not seen; saw d	
Apr.	4.326	not seen; saw d	
	8.323	not seen; saw d	
	12.316	not seen	<10.5
	13.352	not seen; saw d	
May	6.340	not seen; saw d	
	9.378	not seen; saw d	
	10.355	not seen; saw d	

These observations were made with a power of 60 until after February 12, after which a power of 30 was used.

Dorchester, Mass., 1916, January 19.

CORRIGENDUM.

In A. J. 682, p. 77, the observations reduced to October 21 are given in G. M. T. To correspond with the rest of the paper, the hours should be 11 instead of 17.

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AN INVESTIGATION OF THE CLUSTER *M* 37 (*N.G.C.* 2099) FOR PROPER-MOTION,

BY ALFRED H. JOY.

The star cluster *M* 37 in *Auriga* ($5^h 46^m, +32^\circ 32'$) is one of the better known clusters. The positions of 1300 stars were measured by GIEBELER (*Veröff. Sternw. zu Bonn*, No. 12) from two plates taken at Bonn in 1904. NORDLUND (*Arkiv. för Mat. Astr. och Fysik*, Bd. 5, No. 17) studied extensively the magnitudes and positions of 854 stars of the region from four plates, three of which were taken in 1907 and 1908 at Upsala with the photographic refractor of 4.33 meters focal length and one in 1899 at Stockholm with the refractor of 2.35 meters focal length. The magnitudes were deduced from the diameters of the images and based on photometric measures of certain of the stars by ZEIPPEL. The positions given by the two series of plates agree closely but the short focal length of the instruments used give a scale-value which makes it impossible to draw any conclusions in regard to the proper-motions of the stars in the cluster.

THE PLATES

In looking over the plates taken by SCHLESINGER with the 40-inch telescope of the Yerkes Observatory in 1903 and 1904, Mr. O. J. LEE found several plates of *M* 37 which showed round images and fair definition for the stars of the cluster down to the 13.5 magnitude. On account of the scale-value given by the 40-inch telescope it was thought that eleven years would be a sufficient interval of time to show with considerable accuracy the proper-motion of the cluster, or at least to give an upper limit to the value.

Two of the best SCHLESINGER plates were chosen. These two plates were taken without the color-filter on isochromatic plates in the same manner as the plates for parallax described by SCHLESINGER. (*Astrophysical Journal*, 32, 372, 1910). The images are round but fuzzy, making measures difficult.

An additional plate for the second epoch was taken by LEE and SULLIVAN in January, 1915, with the color-filter now used for all parallax plates. It is an excellent plate. The images are sharp and round. Although the exposure

time was nearly the same as that of the early plates it shows more than twice as many stars.

The plates, which were measured and reduced by the writer, are:

Plate	Date	Exp.	G.M.T.	H.A.	Par. Fact.	Quality
S166	Nov. 17, 1903	60 ^m	19 ^h 37 ^m	E 0 ^h 26 ^m	+ .54	Poor
S212	Feb. 4, 1904	45	16 12	W 1 29	— .72	Fair
π 1909	Jan. 14, 1915	60	15 28	E 0 40	— .44	Good

Owing to the inferior quality of the early plates it was thought that the mean of the two could fairly be given a weight equal to that of the 1915 plate.

MEASUREMENT OF THE PLATES

The measuring machine used was the same as is regularly used in the parallax measures. The plates were oriented in the machine by means of a star trail on π 1909 and were measured in the manner described by SCHLESINGER, except that three settings were made on the star and one on the scale, whereas in the parallax measures four settings were made on the star and two on the scale. The plates were measured in direct and reverse positions in both right ascension and declination. Corrections to the measures were applied for runs and errors in the division of the scale.

Three hundred twenty stars which had good images on the three plates were at first chosen, but later this number was reduced to 292 so that all stars used might be within 16' of the center of the cluster. This avoided any possible danger of using stars whose images were deformed on account of being too near the edge of the plates and allowed the second order refraction terms to be neglected in the reduction.

MAGNITUDES

Magnitudes were determined from rough measures of diameters of the images and should be equivalent to the visual on account of the visual luminosity filter. The magnitudes of seventy-seven stars which were determined

by ZEIPPEL by visual methods with the Zöllner photometer were used as standards. They were divided into eleven groups according to magnitude, and the mean of each group, weighted according to the number of the stars in the group, used in a least-squares solution with eleven equations of the form

$$a + bD = M,$$

which PARKHURST has found to most nearly represent measures of diameters on plates taken with the 40-inch telescope.

The constants for $\pi 1909$ were found to be

$$a = +14.82 \quad b = -2.48$$

A comparison of these visual magnitudes with the photographic magnitudes given by NORDLUND shows

0 stars with color-index over 2.0 magnitudes			
12	"	"	1.5
36	"	"	1.0
54	"	"	0.5

The colored stars are about evenly distributed among the cluster and the reference stars:

REDUCTION

As differential proper-motions were sought, the differences between the 1915 positions and the mean of the two plates of the early epoch were made the basis of a reduction in each coördinate by TURNER's well-known formula. Second-order refraction terms were found to be negligible on plates of such small area taken near the zenith and meridian. The plate constants were determined from a least-squares solution where the equations of condition were of the form

$$ax + by + c = \Delta x$$

$$dx + ey + f = \Delta y$$

and were found to be, in units of a quarter-millimeter,

$$\begin{array}{ll} a = -.000583 & d = -.000315 \\ b = +.000159 & e = -.000515 \\ c = -.0033 & f = -.0244 \end{array}$$

The residuals give a direct comparison of the difference of star positions at the two epochs referred to the mean of all the stars, and these differences are freed by the solution from all errors which may be of a linear form due to orientation, scale-value and zero of the plates or to refraction, aberration, precession and nutation. Errors due to any shift in the centering of the objective, which may have been changed in eleven years, or to the use of the color-filter at the last epoch are accounted for, only in so far as they affect the orientation and scale-value; but they are, without doubt, of an order of magnitude which may safely be neglected.

RESULTS

The residuals may be taken as the resulting proper-motions of the individual stars for the period of 11.05 years which elapsed between the two epochs. They are referred to the mean of all the stars and are derived without any assumption whatsoever as to which stars belong to the cluster. In order to find an indication of common proper-motion of the central stars of the plate, circles with radii of 4', 8' and 16' were drawn about the center of the cluster and the residuals divided into three groups according to the zone in which the star was found. The mean residuals for the zones are:

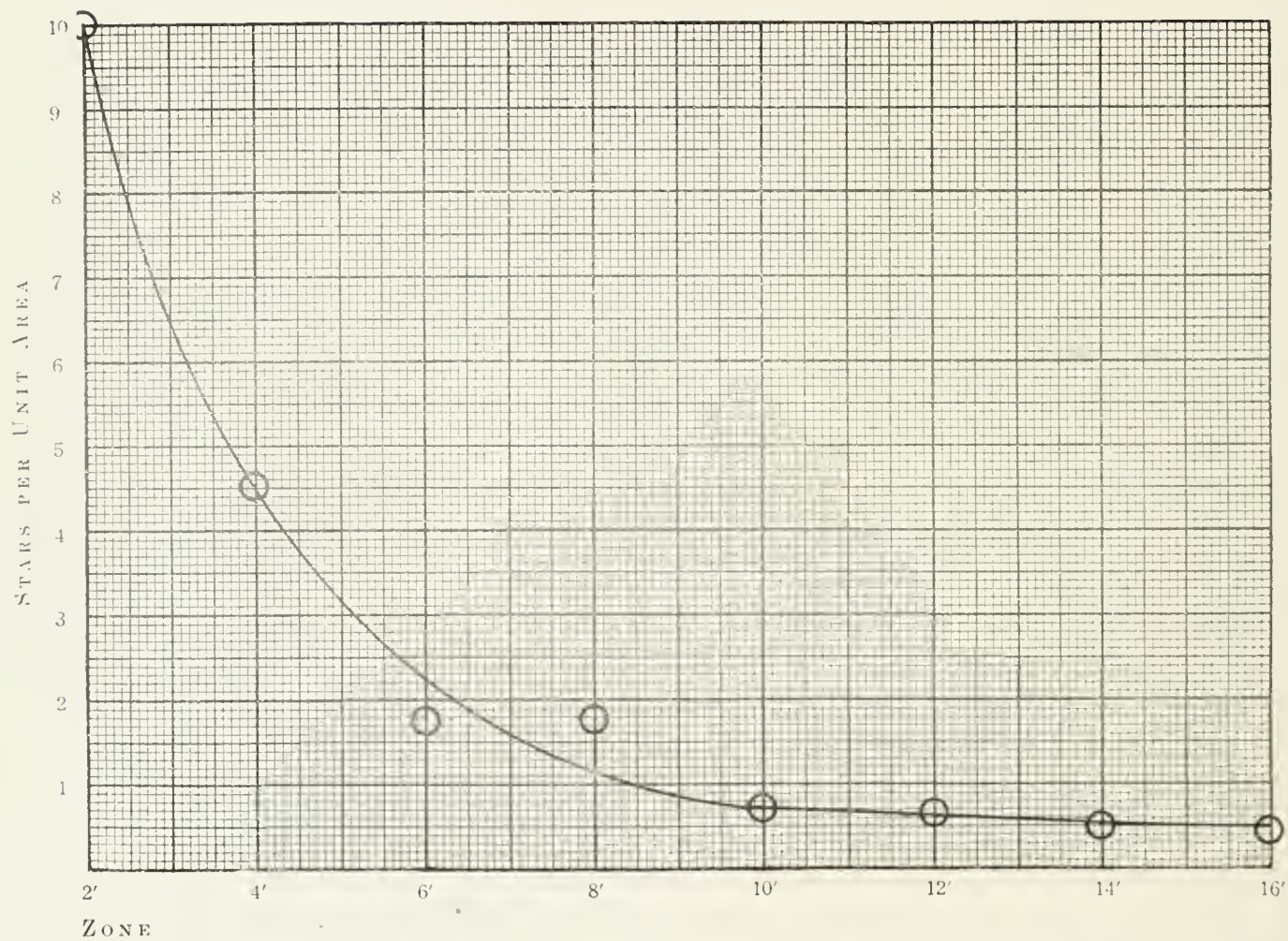
	No. of Stars	μ_{α}	μ_{δ}
Circle 4'	94	+ ".021	+ ".016
Zone 4' - 8'	87	+ .003	+ .003
Zone 8' - 16'	111	- .019	- .016

The difference between the mean of the outer zone and the inner circle is 0".040 in α and 0".032 in δ . This would be the relative proper-motion of the cluster if all the stars of the central circle were known to belong to the cluster and all the outer zone were outside the cluster.

The same grouping with each magnitude was tried to see if the proper-motions were limited to certain stellar magnitudes.

Mag. 13.0 - 13.5	No. of Stars	μ_{α}	μ_{δ}
Circle 4'	9	- ".027	- ".016
Zone 4' - 8'	15	- .048	- .072
Zone 8' - 16'	12	- .059	- .040
Total	36 Mean	- ".045	- ".043
Mag. 12.5 - 13.0			
Circle 4'	25	+ ".016	- ".016
Zone 4' - 8'	37	- .003	- .037
Zone 8' - 16'	16	- .019	- .045
Total	78 Mean	- ".002	- ".033
Mag. 12.0 - 12.5			
Circle 4'	15	+ ".021	+ ".008
Zone 4' - 8'	27	- .019	+ .011
Zone 8' - 16'	23	- .016	- .029
Total	65 Mean	- ".005	- ".003
Mag. 11.5 - 12.0			
Circle 4'	20	+ ".032	+ ".048
Zone 4' - 8'	19	+ .013	+ .029
Zone 8' - 16'	27	+ .003	+ .043
Total	66	+ ".016	+ ".040

M 37 — STAR DENSITY



(The Astronomical Journal, No. 685.)

Mag. 10.0 - 11.5	No. of Stars	ν_a	ν_s
Circle 4'	18	+ ".024	+ ".040
Zone 4' - 8'	13	+ .040	+ .066
Zone 8' - 16'	16	+ .005	+ .037
Total	47 Mean	+ ".024	+ ".048

It will be seen that there is about the same difference between the inner circle and the outer zone in all the magnitudes. The residuals of the outer zone are negative and the residuals of the faint stars are in the same sense, which is as would be expected if the cluster were moving over a background of fainter stars.

STAR COUNTS

Counts of stars of the different magnitudes in the different zones also indicate that the cluster contains stars of the fainter magnitudes as well as the brighter, though the condensation of the faint stars is not so great as for the brighter stars.

No. of Stars	$\overset{M}{10-11.5}$	$\overset{M}{11.5-12}$	$\overset{M}{12-12.5}$	$\overset{M}{12.5-13}$	$\overset{M}{13-13.5}$
Circle 4'	17	27	25	18	12
Zone 4'-8'	17	20	15	25	10
Zone 8'-16'	13	19	24	36	14

Condensation over outer ring per unit area:

Circle 4'	15	17	12	6	10
Zone 4' 8'	5	4	2	3	2

A curve showing the condensation of all the stars of the region taken together was drawn from the counts of eight zones of 2' each.

It is clear from the curve that the limit of the cluster is about 12' from the center.

SECOND SOLUTION

An attempt to arbitrarily separate the stars belonging to the cluster from the other stars of the region and so to permit a solution based on the reference stars only was next undertaken.

First, twenty stars whose residuals in the first solution were greater than $0''.16$, or three times the probable error of a single observation, were excluded on the grounds of individual proper-motion and hence belonging neither to the cluster nor to the reference stars.

Second, considering the average distribution of the stars of different magnitudes in the 12 - 16' zone as standard for that region of the sky we may choose as reference stars from each unit of area covered by the cluster, stars from each magnitude equal in number to those found in the standard area. Stars of the standard areas outside the cluster have in general negative residu-

als, so that it is natural to choose as belonging to the reference stars those stars of the central zones which have the largest negative residuals.

In accordance with these arbitrary rules of choice, 117 reference stars were selected from the plate with which a second solution was made and new plate constants determined.

The grouping of stars in each zone is then,

Zone	Cluster	Reference	Individual P.-M.	Total
0' - 4'	83	7	4	94
4' - 8'	60	20	7	87
8' - 12'	12	44	3	59
12' - 16'	0	46	6	52
Total	155	117	20	292

The plate constants given by the second solution are,

$$\begin{aligned} a &= -.000569 & d &= -.000299 \\ b &= +.000159 & c &= -.000519 \\ c &= +.0076 & f &= -.0137 \end{aligned}$$

The probable error for a single star is $\pm 0''.048$ in a and $\pm 0''.045$ in δ , as determined from the residuals of the reference stars.

The new plate constants were then used in the equations for each of the 155 stars of the cluster proper and the residuals considered as proper-motions. The resulting mean proper-motion is $+0''.053 \pm 0''.0027$ in R. A. and $+0''.051 \pm 0''.0032$ in declination for the period of 11.05 years or $+0''.48'$ in R. A. and $+0''.46'$ in declination per century. The proper-motion of the cluster with respect to the reference stars chosen is then $0''.66$ per century in position angle $46^\circ.20$.

The inclusion of the twenty stars of individual proper-motion would not have changed the resulting proper-motion for the cluster, but would have increased the probable error somewhat.

Column one of the table gives the numbers of the stars used in the solution; column two gives the corresponding numbers of NORDLUND; column three gives the group to which each star was assigned in the second solution, i indicating individual proper-motion, r reference star, and c belonging to the cluster; the fourth column gives the magnitudes determined; columns five and six give the approximate x and y on the plates in units of a quarter millimeter; columns seven and eight give the residuals from the first solution and columns nine and ten from the second solution, all in units of thousandths of a quarter millimeter.

No.	Nord. No.	Group	Mag.	X	Y	v_a	v_s	v'_a	v'_s
1	13	r	12.21	-375	+ 8	- 57	- 7	- 51	- 2
2	16	r	11.23	371	+ 53	- 9	- 28	- 3	- 23
3	24	r	13.19	343	- 86	- 61	- 52	- 55	- 47
4	27	r	11.31	329	+194	+ 26	- 7	+ 32	- 3
5	29	i	13.40	325	- 11	-157	+ 77	-150	+ 83
6	34	i	13.01	318	+ 87	- 87	+ 17	- 80	+ 23
7	36	r	12.63	317	+ 19	- 1	- 6	+ 5	- 1
8	40	r	12.93	304	- 60	- 37	- 15	- 30	- 10
9	44	i	12.58	295	-110	+ 22	- 79	+ 29	- 73
10	49	r	12.70	287	- 17	- 21	+ 4	- 14	+ 10
11	60	r	13.33	287	+214	- 68	+ 19	- 61	+ 24
12	51	r	12.36	284	- 1	- 11	- 13	- 4	- 6
13	54	r	12.35	280	- 46	- 40	+ 9	- 33	+ 15
14	57	r	13.13	277	+158	- 20	+ 14	- 13	+ 20
15	58	r	11.82	277	+147	+ 12	- 5	+ 19	0
16	66	r	13.12	261	- 60	- 47	- 12	- 40	- 6
17	..	i	12.90	255	+273	+ 70	- 1	+ 77	+ 4
18	75	i	13.35	249	- 29	- 40	- 81	- 33	- 74
19	77	r	13.12	246	-178	- 39	- 14	- 32	- 6
20	78	r	13.09	243	+ 17	- 29	+ 3	- 22	+ 9
21	79	e	12.08	242	+ 34	+ 17	+ 17	+ 24	+ 24
22	81	r	12.15	242	-165	- 19	+ 19	- 12	+ 26
23	82	r	12.59	240	+ 67	- 3	- 43	+ 4	- 37
24	90	r	11.84	236	-244	+ 20	+ 22	+ 28	+ 30
25	93	r	12.07	234	-129	- 25	+ 12	- 17	+ 20
26	100	r	12.59	222	+143	+ 8	- 11	+ 16	- 6
27	101	r	12.94	221	-101	+ 16	- 26	+ 24	- 19
28	105	r	12.47	217	- 29	- 6	- 28	+ 2	- 21
29	107	r	11.93	216	+207	+ 48	+ 42	+ 56	+ 49
30	109	r	10.80	214	-127	- 13	+ 7	- 5	+ 16
31	110	e	12.11	213	- 94	+ 18	- 5	+ 26	+ 3
32	111	r	11.83	212	+100	- 7	+ 18	+ 1	+ 26
33	117	r	12.31	205	+237	+ 36	+ 1	+ 44	+ 7
34	128	r	11.61	198	-260	+ 34	+ 12	+ 43	+ 21
35	129	i	13.04	197	- 50	+ 29	-131	+ 38	-124
36	135	e	11.87	193	- 56	+ 41	+ 10	+ 49	+ 18
37	134	r	12.36	193	+167	+ 49	- 9	+ 57	- 3
38	137	r	11.33	192	-256	- 2	+ 24	+ 7	+ 32
39	139	e	12.73	188	- 24	+ 14	- 29	+ 22	- 21
40	141	e	12.13	187	+ 28	+ 15	- 4	+ 23	+ 3
41	140	r	13.19	187	+270	+ 7	+ 13	+ 15	+ 19
42	146	r	12.18	182	+160	0	- 16	+ 8	- 10
43	152	r	12.44	179	+163	+ 18	- 15	+ 26	- 8
44	156	i	13.32	177	- 61	- 79	- 42	- 71	- 34
45	157	e	11.69	177	- 55	+ 25	+ 20	+ 33	+ 27
46	158	r	13.21	176	-166	+ 4	- 61	+ 13	- 52
47	173	i	11.51	168	+ 58	+ 36	- 63	+ 45	- 55
48	178	e	11.82	165	-102	+ 10	+ 4	- 19	+ 11
49	181	e	12.24	163	-179	+ 15	+ 36	+ 24	+ 44
50	185	r	13.51	163	-100	- 34	- 59	- 25	- 51
51	186	r	12.90	162	-138	- 16	- 29	- 7	- 20
52	187	r	12.71	161	-258	+ 14	- 3	+ 23	+ 6
53	194	e	13.07	157	- 13	- 18	- 45	- 9	- 36
54	199	r	11.27	153	- 11	+ 31	- 7	- 9	+ 2
55	206	r	12.37	151	+ 3	- 18	- 17	- 9	- 9
56	202	r	12.83	151	+328	+ 12	+ 34	+ 21	+ 41
57	207	e	12.95	151	- 42	- 13	+ 12	- 4	+ 21
58	212	r	12.79	146	+276	+ 25	- 34	+ 34	- 26
59	224	e	12.57	137	-126	+ 36	- 18	+ 15	- 9
60	228	r	13.13	-134	- 26	- 17	- 30	- 8	- 22

No.	Nord. No.	Group	Mag.	X	Y	v_a	v_b	v'_a	v'_b
61	231	c	12.37	-132	+ 1	- 42	+ 47	- 33	+ 55
62	233	c	11.63	130	- 82	+ 13	+ 11	+ 22	+ 19
63	237	c	12.22	127	- 2	+ 20	- 34	+ 29	- 25
64	238	r	11.90	127	+148	- 15	- 18	- 6	- 10
65	240	c	11.81	123	+ 81	+ 37	- 7	+ 46	+ 1
66	242	r	13.23	122	+225	+ 5	+ 1	+ 14	+ 9
67	243	r	11.98	120	+266	+ 2	+ 15	+ 11	+ 22
68	245	c	11.81	118	+ 73	+ 15	0	+ 24	+ 9
69	264	r	13.34	103	-148	+ 4	- 59	+ 13	- 49
70	266	c	11.60	103	- 45	+ 30	+ 28	+ 39	+ 37
71	265	r	13.48	102	+223	- 36	- 30	- 27	- 23
72	269	e	12.23	102	+101	+ 15	- 24	+ 25	- 15
73	271	i	13.43	101	+ 93	- 69	- 25	- 59	- 16
74	274	c	12.97	99	-162	+ 7	- 4	+ 17	+ 8
75	279	c	12.23	97	+ 63	+ 0	+ 33	+ 10	+ 41
76	281	c	10.57	97	- 67	+ 7	+ 24	+ 17	+ 33
77	282	c	12.37	95	- 16	- 3	- 6	+ 7	+ 3
78	291	c	13.29	88	+130	- 10	+ 53	0	+ 60
79	294	i	13.28	87	- 14	- 64	0	- 54	+ 9
80	296	c	11.54	86	-155	+ 11	+ 3	+ 21	+ 15
81	300	c	11.73	85	- 9	- 5	+ 25	+ 5	+ 35
82	302	r	12.03	83	+148	- 11	- 9	- 1	- 1
83	306	c	11.15	81	-116	+ 10	+ 35	+ 20	+ 46
84	307	c	12.83	80	+ 63	+ 1	+ 20	+ 11	+ 29
85	312	c	11.43	78	-125	- 6	+ 33	+ 4	+ 44
86	...	r	13.42	75	+353	+ 7	- 35	+ 17	- 29
87	322	c	12.47	74	- 92	- 23	+ 33	- 13	+ 42
88	326	c	11.84	73	- 15	- 6	+ 6	+ 4	+ 16
89	330	c	11.09	71	+ 14	- 10	+ 27	0	+ 37
90	332	r	12.86	71	-171	- 12	- 12	- 2	0
91	333	c	13.52	71	-152	+ 17	- 43	+ 27	- 32
92	335	c	13.33	70	- 27	+ 2	- 5	+ 12	+ 4
93	337	c	10.99	70	+ 7	+ 2	- 3	+ 12	+ 6
94	339	c	11.38	69	- 71	- 6	+ 15	+ 4	+ 25
95	343	r	13.23	68	+ 31	- 14	- 25	- 4	- 16
96	344	c	11.98	68	- 39	+ 3	+ 16	+ 13	+ 26
97	352	c	11.82	65	- 23	+ 26	+ 30	+ 36	+ 40
98	353	c	12.73	64	- 96	- 5	+ 21	+ 5	+ 30
99	355	c	12.24	64	+ 62	- 7	+ 32	+ 2	+ 42
100	362	c	11.47	61	- 52	+ 4	+ 14	+ 14	+ 24
101	365	c	11.59	60	- 47	+ 3	+ 43	+ 13	+ 52
102	368	r	12.29	58	+330	+ 10	- 17	+ 1	- 8
103	369	c	11.39	58	- 65	+ 21	+ 25	+ 33	+ 33
104	374	c	12.93	56	- 92	+ 10	- 21	+ 21	- 11
105	377	r	12.75	53	- 18	- 14	- 26	- 3	- 16
106	381	c	12.78	52	+ 20	+ 38	+ 20	+ 49	+ 30
107	383	c	13.20	52	+ 32	+ 3	- 7	+ 14	+ 3
108	384	c	12.59	52	+ 16	+ 11	- 5	+ 22	+ 5
109	387	c	12.72	52	- 75	+ 13	- 6	+ 24	+ 4
110	388	r	12.81	52	-115	- 40	+ 3	- 29	+ 14
111	390	c	11.64	51	-178	- 9	+ 46	+ 2	+ 57
112	391	c	11.38	50	+113	+ 23	+ 26	+ 34	+ 35
113	394	i	12.08	49	+ 18	+ 80	0	+ 91	+ 10
114	392	i	11.48	49	+148	+ 29	+ 73	+ 40	+ 82
115	397	c	11.75	48	- 17	+ 16	+ 42	+ 27	+ 52
116	400	c	11.47	46	+ 13	+ 34	+ 2	+ 45	+ 12
117	402	c	11.75	46	+ 34	+ 28	+ 37	+ 39	+ 47
118	401	c	12.63	46	-124	+ 2	+ 25	+ 13	+ 36
119	403	c	11.72	46	-205	- 17	+ 39	- 6	+ 50
120	404	c	12.50	- 46	+ 8	+ 38	+ 15	+ 49	+ 25

No.	Nord. No.	Group	Mag.	X	Y	v_a	v_b	v'_a	v'_b
121	405	e	12.31	- 45	- 22	- 7	+ 20	+ 4	+ 30
122	406	e	11.87	44	+130	+ 7	+ 33	+ 18	+ 42
123	413	r	12.10	42	-334	- 35	+ 15	- 24	+ 27
124	416	r	12.64	40	-210	+ 21	+ 3	+ 32	+ 14
125	415	r	13.14	40	-274	+ 2	+ 16	+ 13	+ 27
126	419	e	11.73	39	+ 27	+ 12	- 10	+ 23	0
127	428	e	11.38	36	- 45	- 3	+ 1	+ 8	+ 11
128	429	e	12.03	36	- 70	0	+ 29	+ 11	+ 39
129	431	r	13.01	35	+122	- 37	- 33	- 26	- 24
130	433	e	13.12	34	- 24	+ 31	- 17	+ 42	- 8
131	436	e	12.31	33	- 58	+ 18	- 17	+ 29	- 8
132	435	e	12.29	33	+ 44	+ 40	+ 26	+ 51	+ 36
133	438	e	12.81	33	-188	+ 12	+ 30	+ 23	+ 41
134	439	e	12.03	32	- 79	+ 13	+ 33	+ 24	+ 42
135	440	e	11.73	31	+ 5	+ 7	+ 18	+ 18	+ 29
136	441	e	11.27	31	+ 60	+ 18	+ 18	+ 29	+ 27
137	442	e	11.81	31	+139	+ 18	+ 9	+ 29	+ 17
138	444	e	12.21	31	- 18	- 9	+ 30	+ 2	+ 39
139	446	e	11.42	30	+ 24	+ 29	+ 10	+ 40	+ 20
140	450	e	12.59	29	+ 70	+ 50	+ 2	+ 61	+ 12
141	452	e	11.71	29	- 45	+ 14	+ 23	+ 25	+ 33
142	454	r	12.56	28	-140	- 5	- 21	+ 6	- 9
143	455	e	12.96	27	+ 40	+ 40	- 42	+ 51	- 31
144	457	e	13.33	27	- 85	- 29	+ 8	- 18	+ 19
145	458	e	12.58	27	- 28	+ 13	+ 2	+ 24	+ 13
146	461	r	12.96	27	-283	+ 29	- 24	+ 39	- 13
147	460	e	10.49	27	-260	+ 12	+ 43	+ 23	+ 54
148	462	e	12.86	26	+ 32	+ 10	+ 15	+ 21	+ 26
149	463	e	11.49	26	- 16	+ 19	+ 10	+ 30	+ 21
150	468	i	11.11	24	+ 21	- 63	+ 19	- 52	+ 30
151	470	e	12.21	24	+ 53	+ 17	+ 31	+ 28	+ 42
152	469	e	12.20	24	+ 27	+ 9	+ 11	+ 20	+ 22
153	473	e	12.03	24	-132	+ 6	+ 49	+ 17	+ 60
154	476	e	12.93	23	+ 63	- 7	- 16	+ 4	- 5
155	477	i	13.19	22	- 79	+ 10	- 62	+ 21	- 51
156	481	e	11.83	21	- 54	- 28	+ 28	- 17	+ 39
157	482	e	12.37	21	- 19	+ 16	+ 18	+ 27	+ 29
158	483	e	12.89	19	- 33	- 18	+ 23	- 7	+ 34
159	484	e	9.69	19	- 11	+ 46	+ 17	+ 57	+ 27
160	485	e	11.78	18	- 31	+ 48	+ 10	+ 59	+ 21
161	489	e	13.03	16	- 10	+ 31	- 39	+ 42	- 28
162	491	r	11.62	15	+160	- 3	0	+ 8	+ 10
163	492	e	12.93	15	+140	+ 43	+ 17	+ 54	+ 27
164	491	e	11.95	14	- 41	+ 9	- 5	+ 20	+ 5
165	497	e	11.73	12	+ 3	+ 26	+ 27	+ 37	+ 38
166	498	e	10.96	11	+ 31	+ 17	+ 11	+ 28	+ 21
167	500	e	11.80	10	- 58	+ 6	+ 15	+ 17	+ 26
168	504	e	13.26	9	+ 39	+ 11	- 11	+ 22	0
169	506	e	11.98	9	- 5	+ 27	- 17	+ 38	- 6
170	508	e	13.33	8	- 61	- 16	- 5	- 5	+ 5
171	511	r	11.38	5	-232	+ 5	+ 53	+ 16	+ 65
172	514	e	12.39	5	- 45	+ 38	+ 9	+ 49	+ 20
173	515	e	12.63	4	+113	+ 34	- 26	+ 45	- 16
174	517	e	11.56	4	+ 5	+ 20	+ 10	+ 31	+ 21
175	519	e	12.00	3	- 57	+ 7	- 4	+ 18	+ 7
176	520	e	11.87	3	- 44	+ 17	+ 30	+ 28	+ 41
177	521	e	13.18	- 2	- 7	+ 44	- 4	+ 55	+ 6
178	523	e	12.96	0	+169	+ 29	- 3	+ 40	+ 7
179	524	r	11.15	0	+218	+ 11	+ 12	+ 22	+ 22
180	530	e	12.52	+ 3	+ 39	- 4	- 29	+ 7	- 18

No.	Nord. No.	Group	Mag.	X	Y	r_a	r_b	r'_a	r'_b
181	532	c	12.81	+ 4	- 19	- 25	+ 5	- 14	+ 16
182	537	r	11.92	5	+ 29	- 33	+ 3	- 22	+ 13
183	538	c	12.93	6	+ 115	+ 11	- 14	0	- 4
184	539	c	12.10	6	+ 87	+ 29	- 24	+ 40	- 14
185	540	c	12.30	6	- 7	- 2	+ 3	+ 9	+ 14
186	541	c	13.26	6	- 152	+ 5	- 34	+ 16	- 23
187	543	c	12.12	7	+ 45	- 16	+ 4	- 5	+ 14
188	546	c	11.08	8	+ 111	+ 18	+ 29	+ 29	+ 40
189	548	c	12.86	11	+ 139	+ 29	- 60	+ 40	- 48
190	550	c	12.11	12	- 174	+ 35	+ 38	+ 46	+ 49
191	551	r	13.07	14	- 233	+ 45	- 30	+ 56	- 18
192	554	r	11.47	16	+ 235	+ 9	+ 1	+ 20	+ 11
193	557	c	11.74	18	- 172	+ 2	+ 41	+ 13	+ 29
194	559	c	13.12	19	- 11	+ 9	+ 35	+ 20	+ 46
195	...	r	12.72	22	- 353	- 38	- 45	- 27	- 33
196	569	c	12.99	25	- 6	+ 37	- 18	+ 48	- 8
197	571	c	12.10	26	+ 4	+ 38	+ 26	+ 49	+ 36
198		c	11.97	26	+ 3	+ 6	+ 5	+ 17	+ 15
199	573	c	12.47	27	+ 122	- 6	+ 59	+ 5	+ 69
200	574	c	12.31	28	- 48	- 4	- 1	+ 7	+ 10
201	575	r	12.30	28	+ 52	- 8	- 27	+ 3	- 16
202	578	c	12.95	29	- 10	- 5	- 1	+ 6	+ 9
203	579	r	11.43	29	- 27	+ 8	+ 39	+ 19	+ 50
204	581	c	12.96	29	- 4	+ 2	- 14	+ 13	- 3
205	583	c	13.12	30	- 46	- 39	- 19	- 28	- 9
206	592	r	13.23	32	- 293	+ 25	- 36	+ 36	- 23
207	587	c	12.72	32	+ 23	- 10	+ 1	+ 1	+ 13
208	591	c	12.48	34	- 32	+ 18	- 20	+ 29	- 8
209	...	r	12.52	36	+ 345	+ 3	- 15	+ 14	- 5
210	599	c	11.67	37	- 38	+ 36	+ 10	+ 47	+ 22
211	601	c	11.98	37	+ 23	- 32	+ 7	- 21	+ 19
212	603	r	12.52	38	- 191	- 24	+ 12	- 13	+ 24
213	611	r	12.61	40	- 198	- 27	- 34	- 16	- 21
214	612	c	11.29	41	- 150	- 4	+ 38	+ 7	+ 51
215	613	r	12.97	42	+ 26	- 25	- 26	- 14	- 14
216	615	c	11.79	43	+ 118	+ 14	+ 2	+ 25	+ 13
217	620	r	12.43	46	- 263	- 25	+ 26	- 14	+ 39
218	624	i	13.00	47	+ 84	+ 6	- 71	+ 17	- 60
219	626	c	12.72	49	+ 76	+ 22	- 19	+ 33	- 7
220	627	c	11.81	49	+ 29	- 1	+ 21	+ 10	- 32
221	628	c	12.67	50	+ 13	- 2	- 5	+ 9	+ 7
222	630	c	11.25	51	- 133	+ 7	+ 29	+ 18	+ 41
223	635	c	11.91	53	- 17	+ 10	+ 55	+ 21	+ 67
224	636	c	12.51	53	- 190	+ 14	+ 34	+ 25	+ 47
225	641	r	12.64	55	- 30	+ 29	- 2	+ 39	+ 10
226	647	c	12.96	59	- 114	+ 13	- 28	+ 24	- 16
227	649	r	12.61	61	+ 19	- 20	- 13	- 8	- 2
228	654	c	12.21	62	+ 18	- 5	+ 9	+ 6	+ 21
229	658	r	11.71	65	- 190	- 19	+ 30	- 7	+ 42
230	659	r	12.55	66	+ 232	- 58	- 4	- 46	+ 6
231	665	c	12.37	71	- 110	- 6	+ 12	+ 6	+ 24
232	670	c	12.59	73	+ 116	- 60	+ 9	- 48	+ 20
233	675	r	13.03	75	- 307	+ 23	+ 19	+ 35	+ 32
234	671	c	12.02	75	- 42	+ 15	+ 1	+ 27	+ 13
235	680	r	11.71	78	+ 60	+ 1	- 15	+ 13	- 4
236	682	r	12.91	81	- 26	- 57	- 13	- 45	- 1
237	683	i	11.66	81	- 213	- 18	+ 68	- 6	+ 80
238	684	c	13.07	81	+ 158	- 55	- 54	- 43	- 43
239	685	c	12.61	82	+ 132	+ 11	- 1	+ 23	+ 9
240	686	c	12.78	+ 82	- 133	- 30	+ 17	- 18	+ 29

No.	Nord. No.	Group	Mag.	X	Y	ρ_a	ρ_b	ρ'_a	ρ'_b
241	690	r	12.58	+ 84	-314	- 17	- 5	- 5	+ 7
242	688	r	12.19	84	- 87	- 22	+ 3	- 10	+ 14
243	691	i	10.62	85	- 31	+ 52	+ 69	+ 64	+ 81
244	694	e	12.81	86	-147	- 9	+ 43	+ 3	+ 56
245	696	r	11.77	89	- 16	- 47	+ 12	- 35	+ 23
246	699	e	13.05	91	- 84	+ 31	- 31	+ 43	- 19
247	701	e	12.67	91	- 82	- 4	+ 13	+ 8	+ 25
248	705	e	12.11	93	+ 16	+ 33	+ 33	+ 45	+ 45
249	...	r	13.08	98	-340	0	- 65	+ 12	- 51
250	711	r	13.08	100	+208	- 60	+ 46	- 47	+ 58
251	713	e	12.81	102	- 40	+ 17	- 10	+ 30	+ 1
252	718	r	12.10	105	+157	+ 4	- 9	+ 17	+ 2
253	720	r	12.92	108	- 66	- 47	- 22	- 34	- 10
254	725	e	10.55	112	+ 11	- 2	+ 58	+ 11	+ 70
255	728	r	11.42	115	- 7	- 22	+ 8	- 9	+ 19
256	733	e	10.17	118	- 14	+ 27	+ 54	+ 40	+ 66
257	732	r	13.07	119	-324	+ 14	- 40	+ 27	- 27
258	736	r	12.91	122	-177	+ 10	- 5	+ 23	+ 7
259	740	e	11.90	127	+222	+ 30	+ 26	+ 43	+ 39
260	743	i	12.02	130	+ 69	- 63	- 5	- 50	+ 6
261	748	e	11.72	138	- 57	- 24	+ 26	- 11	+ 39
262	749	r	11.74	138	+ 90	- 36	- 3	- 23	+ 9
263	750	r	12.64	139	+105	- 34	- 21	- 21	- 10
264	753	r	12.87	140	+ 35	- 38	- 30	- 25	- 18
265	755	e	13.12	141	- 62	+ 19	- 7	+ 32	+ 6
266	766	e	12.16	154	-122	+ 16	+ 29	+ 29	+ 43
267	771	r	11.92	158	+ 77	- 16	- 19	- 3	- 5
268	777	i	10.32	163	+275	- 35	+ 61	- 23	+ 73
269	780	r	12.58	168	+168	+ 31	- 27	+ 44	- 16
270	781	i	12.98	169	-205	+ 57	- 97	+ 70	- 82
271	...	r	12.67	175	-303	+ 36	- 51	+ 50	- 37
272	786	r	11.91	177	+ 56	+ 14	- 10	+ 28	+ 4
273	792	r	12.77	183	- 55	- 40	- 16	- 26	- 3
274	...	r	12.71	188	+269	- 9	- 1	+ 5	+ 12
275	798	r	10.82	189	-202	- 47	+ 27	- 33	+ 41
276	804	e	13.15	197	-118	+ 43	- 14	+ 57	+ 1
277	808	r	11.42	202	+ 71	- 20	+ 45	- 6	+ 59
278	813	r	12.61	218	+154	- 19	- 23	- 6	- 10
279	...	r	12.77	222	-258	+ 10	+ 8	+ 24	+ 23
280	821	r	12.98	227	-205	- 35	+ 19	- 21	+ 34
281	822	r	12.68	231	+ 83	- 59	- 63	- 45	- 49
282	825	r	12.28	233	+193	+ 4	- 36	+ 19	- 22
283)	828	r	11.82	238	-150	- 39	+ 8	- 25	+ 24
284)		r	12.98	239	-149	- 27	- 35	- 13	- 20
285	830	r	13.31	245	- 29	+ 20	- 39	+ 35	- 24
286	831	r	12.78	247	+ 46	- 38	- 52	- 23	- 37
287	837	r	12.78	255	+113	- 39	- 46	- 24	- 32
288	838	r	11.00	260	-186	+ 52	- 64	+ 66	- 49
289	840	r	12.74	263	+ 56	- 35	- 9	- 20	+ 5
290	839	r	13.43	267	- 67	- 28	+ 3	- 13	+ 18
291	852	r	12.20	297	+ 5	- 33	- 30	- 18	- 15
292		r	12.03	+336	+ 13	- 5	+ 2	+ 11	+ 17

Yerkes Observatory, September, 1915.

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DEFINITIVE ORBIT OF COMET 1855 II,

BY GEORGES VAN BIESBROECK.

Comet 1855 II was discovered on the evening of June 3, 1855, by DONATI at Florence (*A. N.*, 41, 127, 1855) and the next day independently by DIEN at Paris, and by KLIN-KERFUES at Göttingen (*A. N.*, 41, 95 and 119, 1855; *C. R.*, 40, 1271-1272, 1855).

Not until June 5 were observations obtained that could be utilized. The series of observations extends through fifteen days only, to the 19th of June, when moonlight interfered. A last single observation was obtained on June 30 at Berlin. This observation is unfortunately in error and I have not been able to trace the origin of the error.

At the time of discovery, the comet had neither nucleus nor tail. Between June 3 and 5 DONATI estimated that its brightness was less than that of the cluster in *Hercules* (*A. N.*, 41, 127, 1855). BRUHNS (*A. N.*, 41, 111, 1855) describes the comet, on June 5, as a nebulous mass, of diameter slightly less than one minute of arc, without any tail and of brightness comparable with that of the cluster in *Aquarius*. For more precision on this point I estimated the brightness of these two clusters, using binoculars, and found the first to be 6^m.4 and the second 6^m.1 on the P. D. scale. The comet was therefore of about the 6^m when discovered. The computations indicate that the comet must have rapidly diminished in brightness. There is no precise data on this point, beyond the fact that on June 14 PAPE (*A. N.*, 41, 175, 1855) found the comet much fainter than on the preceding days.

The observational data taken as a whole are very unsatisfactory, both by reason of their small number and the short interval covered by them, as well as by the lack of precision in the settings on the comet, which had no well-defined nucleus. Besides, all the observations were obtained near the horizon, the highest altitude being only 19°, in the case of SECCHI's observation at Rome, on June 14.

Several sets of parabolic elements and two of elliptical elements have been published. A list of elements, as well as references to the observations, will be found in

CARL'S *Repertorium der Kometen-Astronomie*, München 1864, pp. 278-279. The following parabolic orbit by SCHULTZE (*A. N.*, 42, 200, 1855) represents the observations best:

$$\left. \begin{aligned} T &= 1855 \text{ May } 30.18468 \text{ Berlin M. T.} \\ \omega &= 22^\circ 36' 28''.1 \\ \oslash &= 260 \ 18 \ 53 \ .1 \\ i &= 156 \ 53 \ 10 \ .5 \end{aligned} \right\} \begin{array}{l} \text{Equinox of } 1855.0 \\ (a) \end{array}$$

$$\log q = 9.753396$$

It is based on twenty-nine observations up to June 19. These elements are a sufficiently good approximation, so that an ephemeris can be computed, normal positions formed and parallaxes and aberration-times deduced from them. In establishing this ephemeris I have computed the solar coördinates, from NEWCOMB'S tables: BAUSCHINGER'S *Tafeln zur Theoretischen Astronomie* were used for obtaining apparent places.

The determinations of position are 50 in number, both in right ascension and declination. They were obtained in the following observatories, listed according to longitude:

Vienna	Hamburg
Berlin	Leiden (the former observatory)
Rome (Collegio Romano)	Paris
Padua	Liverpool
Florence (the old observatory)	

Wherever possible I have resorted to the original data of observations, which has enabled me to account for some apparent discordances. In many cases, for lack of other data, I have had to be content with the published values, using of course only the differences between the coördinates of the comet and comparison stars. I consulted all available catalogues in the formation of the places of the comparison stars for 1855; and I applied to these catalogued positions the systematic corrections, given by AUWERS, as well as the weights.

The parallax factors were computed, adopting $8''.80$ for the solar parallax. For the aberration time for a distance unity, I used the value $498^s.5$

The differences "observed minus computed" were plotted, using the times as ordinates. A smooth curve through the points shows approximately the probable error of the single observations. The average probable error is

$$\text{in } \alpha, 0^s.98; \quad \text{in } \delta, 6''.9.$$

These large values indicate the great uncertainty of the settings on the comet. In deriving these means, I used all the values for right ascension, except that of June 30, but rejected two values of the declination (those of DONATI at Florence) which were so discordant that they could only vitiate the results. The weights were computed by the formulae

$$p_\alpha = \left(\frac{0.98}{\text{Deviation in } \alpha} \right)^2 \quad p_\delta = \left(\frac{6.9}{\text{Deviation in } \delta} \right)^2$$

for which I have substituted whole numbers, according to the following table:

WEIGHTS

Computed	Adopted
0.00 to 0.19	0
0.20 0.49	1
0.50 0.99	2
1.00 1.99	3
2.00 9.9	4
> 10.0	5

The observations were divided into ten groups, giving ten normal places for which the weights would not differ too greatly.

The time covered by the observations (27 days) appears to be short enough to allow of disregarding the perturbations due to the planets. I have, however, computed them, because it was found that the comet approached *Venus* as close as 0.22 (on June 14); at the same time it was plain that the corresponding deviation would affect more especially the declination of the comet. Now, as will be seen later, the observation of June 30 at Berlin, which would appear to be untrustworthy, differs from the computation merely in respect to declination. Therefore it was worth while to see in how far the perturbations could explain the deviation of the last observation, which was a very important one, as it doubled the time covered by the measures.

As the relative positions of the comet and *Venus* were changing rapidly about the 14th of June, I have computed the perturbations in two-day intervals, taking this date as the epoch of osculation. For the other planets, *Saturn*

being the farthest one taken into account, a ten-day interval was used.

PERTURBATIONS OF COMET 1855 II

1855	In α	In δ
June 3	$-0^s.009$	$-0''.17$
5	$- .007$	$- .11$
7	$- .004$	$- .06$
9	$- .002$	$- .03$
11	$- .001$	$- .01$
13	$.000$	$.00$
15	$.000$	$.00$
17	$.000$	$- .01$
19	$- .001$	$- .02$
30	$-0^s.002$	$-0''.10$

It was to be expected that these values would not be large, the motion of the comet being retrograde and its approach to *Venus* being only of very short duration. We see that these small values for the perturbations in no wise explain the lack of agreement of the isolated observation of June 30. Small as they are, I nevertheless took account of them in the formation of the normal places as given below:

NORMAL PLACES (1855.0)

No.	1855, June Berlin M.T.	α	Wt.	δ	Wt.
1	5.437243	$7^h 11^m 8^s.48$	26	$+36^\circ 15' 54''.1$	22
2	6.446742	$23 \ 25 \ .06$	26	$36 \ 04 \ 38 \ .7$	21
3	7.432310	$33 \ 53 \ .35$	23	$35 \ 50 \ 15 \ .7$	14
4	8.451520	$43 \ 19 \ .97$	12	$35 \ 33 \ 20 \ .7$	15
5	9.466766	$7 \ 51 \ 32 \ .09$	16	$35 \ 15 \ 27 \ .4$	12
6	11.417693	$8 \ 4 \ 31 \ .28$	26	$34 \ 40 \ 10 \ .4$	17
7	12.430838	$10 \ 5 \ .49$	18	$34 \ 22 \ 12 \ .7$	18
8	14.407880	$19 \ 9 \ .98$	10	$33 \ 48 \ 37 \ .2$	16
9	18.454501	$32 \ 24 \ .54$	23	$32 \ 47 \ 59 \ .1$	20
10	30.431404	$8 \ 51 \ 54 \ .87$	..	$(30 \ 39 \ 11 \ .0)$	

The last normal place (No. 10) is the isolated Berlin observation of June 30. Before computing definitive elements, this position was specially scrutinized. HORNSTEIN's method was used on this occasion. Representing precisely the normal places Nos. 1 and 9, which have the greatest weight, I have sought what values of the distance Δ_1 corresponding to the first normal place and of the reciprocal of the semi-major axis $\frac{1}{a}$ would best represent the intermediary normal places. I have successively assumed the following values which appeared, from former computations, to include the probable values of the unknowns:

Hypothesis	Δ_1	$\frac{1}{a}$
I	0.653000	0.040000
II	0.653500	0.040000
III	0.653000	0.050000

The normal places Nos. 1 and 9 being rigorously represented, the other normal positions, after being corrected for solar latitude, are as follows:

REPRESENTATION OF NORMAL PLACES ACCORDING TO THE THREE HYPOTHESES

No.	Normal Longitude	I	II	III	Normal Latitude	I	II	III
2	107°14'18".88	20".48	17".35	16".92	+13°51'36".07	34".31	34".08	34".37
3	109 25 60 .45	58 .18	52 .91	52 .65	13 57 3 .47	3 .17	2 .78	3 .41
4	111 25 47 .38	46 .57	39 .84	39 .94	14 00 4 .26	4 .79	4 .30	5 .21
5	113 10 45 .99	47 .36	39 .91	40 .47	14 01 4 .65	2 .10	1 .61	2 .73
6	115 59 14 .28	19 .99	12 .04	13 .95	13 58 46 .34	46 .57	46 .12	47 .44
7	117 12 29 .79	36 .34	29 .22	20 .74	13 56 7 .50	6 .70	6 .29	7 .63
8	119 13 26 .27	26 .09	20 .74	22 .71	13 49 1 .64	3 .03	2 .69	3 .87
10	126 54 74 .00	11 .84	32 .58	13 .03	12 32 24 .58	101 .00	103 .02	95 .06

It is clear that the above hypotheses nearly cover the possible range of the assumed unknowns, except for place No. 10, for which there are abnormal deviations in longitude, as well as in latitude. Solving the resulting equations of condition, excluding the 10th position, I find for the unknowns:

$$\Delta_1 = 0.653564 \pm 0.000296$$

$$\frac{1}{a} = 0.03108 \pm 0.00683$$

The mean error of a normal place of weight unity is 9".78. This solution represents as follows the normal places, the differences "observed minus computed" being given below:

Normal Place	Longitude	Latitude
2	- 1".2	+ 2".1
3	+ 3 .3	+ 0 .9
4	+ 2 .5	+ 0 .4
5	+ 0 .9	+ 3 .6
6	- 2 .3	+ 1 .1
7	- 3 .2	+ 2 .1
8	+ 2 .9	- 0 .3
10	(+39 .9)	(-81 .0)

The normal places Nos. 1 and 9 being represented rigorously, it is clear that this solution gives a good approximation, except for the place No. 10. Taking into account the mean errors of the unknowns found above, we find that the difference of +39".9 in longitude involves an uncertainty of $\pm 13".0$, while the difference in latitude has a mean error of only $\pm 4".3$. The observation of June 30 is therefore clearly in error. Yet if both ecliptic

coördinates are unreliable, it does not necessarily also follow that both equatorial coördinates are; in fact, it is improbable that the right ascension, which was obtained by difference of time of transit across the wires of the micrometer, would be much in error. In order to introduce this datum into the discussion, I have converted into equatorial coördinates the positions obtained for June 30 by the three hypotheses. The difference becomes (observed minus computed):

$$\text{In R. A., } +36".1 \pm 13".9$$

$$\text{In Decl., } -90".1 \pm 4".6$$

It is therefore the declination which is chiefly in error. In right ascension the difference is equally larger than its mean error, but in a smaller proportion. It should be pointed out that the mean error given here results from the assumption that the places Nos. 1 and 9, which the computation exactly represents, are rigorous. If we consider the respective uncertainty of these two normal places, the difference in right ascension will be almost within the limits assigned by its mean error, while in declination it will not. The preliminary computation therefore proves that the right ascension of June 30 is perhaps admissible, but that in declination the comet is about 1.5 minutes of arc farther north than the observed position.

On account of the importance of this Berlin observation—it doubles the duration of visibility—it was essential to verify the original observational data and reductions, to see if the abnormal difference could not be accounted for. At my request PROFESSOR HERMANN STRUVE was so kind as to have the original records at Berlin examined for me. MR. COURVOISIER writes that after a careful examination of all the records and of the

reductions, he finds that all seem correct: the comet was observed in right ascension by six transits across three wires, the telescope remaining fixed, and in declination by six bisections by a movable wire, the coincidence with the fixed wire being immediately determined. An error does not seem possible either in the time of the observation, within the narrow limits between twilight and setting of the comet, or in respect to the comparison star near which there was no other star. Likewise, the right ascension, obtained directly by difference of time of transit, could hardly be in error. As to the declination, the observer explicitly stated that the comet was south of the comparison star. The reduction shows that the difference in declination is 129".2, which corresponds to 130".9 after correcting for refraction (the comet being only 7° or 8° above the horizon). If, as the preliminary computations seem to prove, the comet was about 90" farther north than the observation states, that would not be a contradiction to the remark, "Comet south of the star"; but would be an indication of an error in reading the micrometer either in setting on the comet or in setting the fixed wire. An error of ten revolutions = 153" would be too great. Such would have been the difference, if there had been a confusion between the fixed wires, three in number, separated by 2'.5, which would be about ten revolutions for the Fraunhofer telescope used at Berlin. The difference of 90" corresponds to an error of:

$$5.9 \text{ revolutions} \approx 0.3.$$

To assume an error of five or rather six revolutions would be sufficient to account for the difference, but it seems to me too arbitrary to introduce either of these hypotheses into the computation. It therefore becomes necessary to abandon the declination observed on June 30 at Berlin, and to use merely the right ascension of that date in the computation of the definitive orbit.

The computation* was made using the elliptic elements corresponding to the preliminary solutions above. There is a slight difference between these elliptic elements and those which result from the preliminary solution, because the computation was made with a slightly different distribution of weights.

The preliminary elements, as well as their comparison with the ten normal places, follow:

$$\left. \begin{aligned} T &= 1855 \text{ May } 30.192112 \text{ Berlin M. T.} \\ \omega &= 22^\circ 32' 23''.92 \\ i &= 156 \ 52 \ 21 \ .37 \\ \oslash &= 260 \ 15 \ 4 \ .60 \\ \log q &= 9.7540028 \\ \log e &= 9.9939687 \left(\log \frac{1}{a} = 8.385603 \right) \end{aligned} \right\} (b)$$

Normal Place	Differences O - C	
	In α	In δ
No. 1	+ 0".03	+ 0".11
2	- 1 .45	+ 2 .74
3	+ 3 .44	+ 1 .04
4	+ 2 .59	+ 0 .77
5	+ 1 .66	+ 4 .27
6	- 2 .21	+ 2 .44
7	- 2 .35	+ 3 .58
8	+ 4 .49	- 0 .45
9	- 0 .24	+ 0 .01
10	+32 .67	(-93 .27)

The coefficients of the equations of correction were computed according to the form given by BAUSCHINGER, *Die Bahnbestimmung der Himmelskörper*, p. 459, the equator being adopted as a fundamental plane.

EQUATIONS OF CORRECTION (Logarithmic Coefficients)

Wt.	RIGHT ASCENSION						
26	0.0145 dq	+9.4899 dS	+8.8601 dP	+9.3018 _n dQ	+3.6938 _n dT	+8.4428 _n $d\frac{1}{a}$	= 8.3836
26	0.0170	9.3384	8.8445	9.2179 _n	3.6123 _n	8.4632 _n	0.0689 _n
23	0.0175	9.1445	8.8211	9.1355 _n	3.5281 _n	8.4769 _n	0.4454
12	0.0170	8.8328	8.7904	9.0499 _n	3.4359 _n	8.4871 _n	0.3237
16	0.0156	7.7583	8.7548	8.9643 _n	3.3374 _n	8.4946 _n	0.1321
26	0.0125	8.9693 _n	8.6763	8.7996 _n	3.1217 _n	8.5066 _n	0.2595 _n
18	0.0108	9.1321 _n	8.6311	8.7435 _n	2.9877 _n	8.5125 _n	0.2877 _n
10	0.0076	9.3104 _n	8.5358	8.5437 _n	2.6290 _n	8.5255 _n	0.5718
23	0.0021	9.4835 _n	8.3061	8.1753 _n	2.4629	8.5599 _n	9.3048 _n
(4)	9.9891	9.6534 _n	7.6291 _n	7.0972	3.0001	8.6925 _n	1.4485

* I wish to acknowledge here the kindness of Mr. H. VANDERLINDEN, Volunteer Assistant at the Royal Observatory, Uccle, who computed the corrections to the elements by the method of least squares in duplicate with me, giving a valuable check over the most laborious part of the work.

EQUATION OF CORRECTION (Continued)

Wt.	DECLINATION						
22	9.5188 dq	+8.3764 dS	+9.4697 _n dP	+9.9114 dQ	+2.8314 _n dT	+7.6769 _n $d\frac{1}{a}$	= 9.0414
21	9.4464	7.8340	9.5126 _n	9.8860	2.6736 _n	7.6454 _n	0.4378
14	9.3737	7.5072 _n	9.5461 _n	9.8605	2.5276 _n	7.6166 _n	0.0170
15	9.2966	7.9443 _n	9.5741 _n	9.8336	2.3946 _n	7.5918 _n	9.8865
12	9.2176	8.0459 _n	9.5967 _n	9.8062	2.2875 _n	7.5729 _n	0.6304
17	9.0571	8.0016 _n	9.6301 _n	9.7534	2.1773 _n	7.5558 _n	0.3874
18	8.9667	7.8943 _n	9.6434 _n	9.7258	2.1668 _n	7.5561 _n	0.5539
16	8.7693	7.3861 _n	9.6641 _n	9.6720	2.1921 _n	7.5682 _n	9.6532 _n
20	8.0722	7.9617	9.6919 _n	9.5611	2.2907 _n	7.6197 _n	8.0000
..	8.6660 _n	8.4684	9.7266 _n	9.1947	2.4137 _n	7.8893 _n	

At first, the observation of June 30 was entirely omitted, that is to say the last equation in right ascension. In taking account of the weights and in introducing the auxiliary unknowns x, y, z, t, u and w , defined as follows (logarithmic coefficients)

$$\begin{aligned} dq &= 0.4570x & dQ &= 0.5989t \\ dS &= 0.9841y & dT &= 6.7802_u \\ dP &= 0.8391z & d\left(\frac{1}{a}\right) &= 1.9407_w \end{aligned}$$

the homogeneous equations were computed, which gave the normal equations:

$$\begin{array}{rccccccc} 0.84600x + 9.73090y + 8.83091z + 8.56585_t + 0.53329u + 0.80562w & = & 9.96762 \\ 9.73090 & 0.47134 & 9.36116 & 9.60439_n & 0.25904 & 9.08009 & 9.40620 \\ 8.83091 & 9.36116 & 0.71749 & 0.64961_n & 9.31165 & 9.12450 & 0.42815_n \\ 8.56585_n & 9.60439_n & 0.64961_n & 0.65218 & 9.51101_n & 9.38970_n & 0.40257 \\ 0.53329 & 0.25904 & 9.31165 & 9.51101_n & 0.39709 & 0.46677 & 9.70196 \\ 0.80562 & 9.08009 & 9.12450 & 9.38970_n & 0.46677 & 0.77251 & 9.84624 \end{array}$$

The equations of elimination are:

$$\begin{array}{rccccccc} 0.84600x + 9.73090y + 8.83091z + 8.56585_t + 0.53329u + 0.80562w & = & 9.96762 \\ & 0.46523 & 9.35122 & 9.60130_n & 0.19140 & 9.56832_n & 9.26387 \\ & & 0.71600 & 0.64658_n & 8.72016 & 8.99957 & 0.43187_n \\ & & & 9.81783 & 8.69152_n & 9.24859_n & 9.40295 \\ & & & & 7.26735 & 7.14083 & 6.29449 \\ & & & & & 6.81981 & 7.06393_n \end{array}$$

The coefficients of the unknowns u and w in the last two equations being very small, the first four equations of elimination were used to express the unknowns in terms of u and w , for the sake of greatest precision compatible with the data. In substituting them in the homogeneous equations of condition, a set of equations in terms of u and w only was obtained; these in turn served to express u in terms of w , the last unknown, which was so directly expressed in terms of the second members of the equations of condition. In this way the last two equations of elimination were obtained. The solution gives the following values for the unknowns:

$$\begin{aligned} dT &= -0^d.000866 \pm 0^d.01329 \\ d\omega &= -11''.36 \pm 147''.3 \\ d\Omega &= -3.09 \pm 54.0 \\ di &= -3.12 \pm 26.6 \end{aligned}$$

$$dq = +0.0000153 \pm 0.0002036$$

$$d\left(\frac{1}{a}\right) = +0.0007420 \pm 0.009636$$

Mean error of an observation of weight unity = $8''.89$.

Considering the small corrections of the elements it is seen that solution (b) was already a very satisfactory one: every correction is much smaller than its mean error. The following definitive elements correspond to these corrections:

$$\left. \begin{array}{l} T = 1855 \text{ May } 30.19125 \pm 0.0133 \text{ Berlin M. T.} \\ \omega = 22^\circ 32' 12''.6 \pm 147''.3 \\ \Omega = 260 \ 15 \ 1.5 \pm 54.0 \\ i = 156 \ 52 \ 18.3 \pm 26.6 \end{array} \right\} \text{Equinox of 1855.0} \quad (c)$$

$$\begin{aligned} \log q &= 9.754015 \pm 0.000156 \\ \log e &= 9.99378 \pm 0.00241 \end{aligned}$$

Period = 252 years (between 155 and 523)
 The normal places are represented as follows:

DIFFERENCES (observed — computed)
 Elements *c*

Normal Pl. No.	$\Delta \alpha \cos \delta$		$\Delta \delta$	
	From Elements <i>c</i>	From Equations	From Elements <i>c</i>	From Equations
1	— 0 ^u .3	— 0 ^u .4	— 1 ^u .6	— 1 ^u .6
2	— 1 .4	— 1 .5	+ 1 .0	+ 1 .0
3	+ 2 .6	+ 2 .6	— 0 .6	— 0 .6
4	+ 2 .1	+ 2 .0	— 0 .8	— 0 .8
5	+ 1 .3	+ 1 .2	+ 2 .7	+ 2 .7
6	— 1 .9	— 1 .9	+ 1 .0	+ 0 .9
7	— 2 .1	— 2 .1	+ 2 .1	+ 2 .1
8	+ 3 .7	+ 3 .6	— 1 .8	— 1 .8
9	— 0 .2	— 0 .1	— 1 .2	— 1 .2
(10)	(+29 .6)	(+29 .6)	(—93 .9)	(—93 .8)

The check furnished by the agreement of the two sets of computations is sufficient (computed to seven decimals.)

A better agreement for the normal places Nos. 1 to 9 could not be hoped for. For the right ascension of

June 30, which was not used in the computation, solution (*c*) leaves a discrepancy of about 2^s. That is not abnormal, considering the uncertainty of the elements. If we compute by the equations of condition the uncertainty of the coördinates of June 30, we find

$$\begin{aligned} &= 123''.8 \text{ for } \alpha \cos \delta \\ &= 22''.3 \text{ for } \delta \end{aligned}$$

This confirms our previous result, *viz.*, that the observation of June 30 is admissible in right ascension, but not in declination.

In a second solution we have, besides the preceding equations of condition, used the supplementary equation corresponding to right ascension of June 30. We arbitrarily assigned to it a weight 4, which corresponds to an observation of fair quality. The Berlin observations are almost the best of the epoch, which would justify *a priori* a maximum weight of 5; but considering the conditions of visibility so difficult, we have thought the weight ought to be reduced at least to 4. Perhaps this value is too high, but fortunately it has no great influence on the result. Under these conditions the normal equations (logarithmic coefficients) become:

$$\begin{array}{ccccccc} 0.85429\,x + 9.51581\,y + 8.82151\,z + 8.56289_n t & + & 0.52956\,u & + & 0.81954\,w & = & 0.35953 \\ 9.51581 & & 0.51679 & & 9.36539 & & 9.60482_n & & 0.26977 & & 9.30782_n & & 0.26922_n \\ 8.82151 & & 9.36539 & & 0.71749 & & 0.64981_n & & 9.31228 & & 9.11727 & & 0.43048_n \\ 8.56289_n & & 9.60482_n & & 0.64961_n & & 0.65218 & & 9.51108_n & & 9.38899_n & & 0.40302 \\ 0.52956 & & 0.26977 & & 9.31228 & & 9.51108_n & & 0.39818 & & 0.46007 & & 9.32181 \\ 0.81954 & & 9.30782_n & & 9.11727 & & 9.38899_n & & 0.46007 & & 0.79537 & & 0.44634 \end{array}$$

and the equations of elimination:

$$\begin{array}{ccccccc} 0.85429\,x + 9.51581\,y + 8.82151\,z + 8.56289_n t & + & 0.52956\,u & + & 0.81954\,w & = & 0.35953 \\ & & 0.51479 & & 9.35965 & & 9.60298_n & & 0.23194 & & 9.70406_n & & 0.29308_n \\ & & & & 0.71610 & & 0.64684_n & & 8.73640 & & 9.02202 & & 0.41133_n \\ & & & & & & 9.87905 & & 8.71307_n & & 9.26352_n & & 9.00988 \\ & & & & & & & & 7.66924 & & 7.92466 & & 9.26871 \\ & & & & & & & & & & 7.60558 & & 9.09766 \end{array}$$

the two last equations were obtained indirectly, as in the preceding case. The results are:

$$\begin{aligned} dT &= +0^d009735 \pm 0^d011874 \\ d\omega &= +225''.44 \pm 114''.0 \\ d\Omega &= +107 .16 \pm 18 .5 \\ di &= + 23 .39 \pm 9 .4 \\ dq &= -0.0002969 \pm 0.0000589 \\ d\left(\frac{1}{n}\right) &= -0.013133 \pm 0.004111 \end{aligned}$$

$$\left. \begin{array}{l} T = 1855 \text{ May } 30.20185 \pm 0.01187 \text{ Berlin M.T.} \\ \omega = 22^\circ 36' 9''.4 \pm 114''.0 \\ \Omega = 260 \ 16 \ 51 .8 \pm 18 .5 \\ i = 156 \ 52 \ 44 .8 \pm 9 .4 \end{array} \right\} \begin{array}{l} \text{Mean Equinox} \\ 1855.0 \end{array} \right\} (d)$$

$$\begin{aligned} \log q &= 9.753776 \pm 0.000045 \\ \log c &= 9.99724 \pm 0.00102 \end{aligned}$$

Period = 847 years (between 529 and 1687)

Mean error of an observation of weight unity = 9^u.38.

The corrections are much larger than in the first case, but by reason of the increase in interval of time, the mean errors have become less. The normal places are now represented as follows:

DIFFERENCES (observed — computed)

Normal Place	$\Delta \alpha \cos \delta$		$\Delta \delta$	
	From Elements <i>d</i>	From Equations	From Elements <i>d</i>	From Equations
1	+0".6	+0".2	— 2".1	— 2".3
2	— 1 .5	— 1 .9	+ 0 .9	+ 0 .8
3	+2 .2	+1 .9	— 0 .2	— 0 .5
4	+1 .8	+1 .4	— 0 .3	— 0 .4
5	+1 .4	+1 .1	+ 3 .4	+ 3 .3
6	— 0 .9	— 1 .2	+ 1 .6	+ 1 .5
7	— 0 .6	— 0 .9	+ 2 .7	+ 2 .6
8	+5 .2	+4 .8	— 1 .6	— 1 .7
9	— 1 .4	— 1 .7	— 2 .0	— 2 .1
10	+2 .1	+1 .7	(—98 .5)	(—98 .9)

There is an almost constant difference between the values obtained by the elements and those which result from the equations of condition: this difference is from 0".3 to 0".4 in $\alpha \cos \delta$, and 0".1 to 0".2 in δ . This is not caused by an error in computation, but by the largeness of the corrections which certain elements have undergone, in particular the element $\frac{1}{a}$, which is reduced by more than half. For such variations there is a sensible change in the values of the coefficients of the equations of condition, which easily explains the discordance, which is however very small in comparison with the uncertainty of the observations.

The right ascension of June 30 is sufficiently well represented, but for the declination the discrepancy is still greater:

$$-98''.7 \pm 9''.6$$

Again it appears that the declination is not usable.

On the other hand, the differences have become greater in the two coördinates for the first nine normal places. The mean error of an observation of weight unity has increased from 8".89 to 9".38 in going from solution (c) to solution (d). It accordingly appears doubtful whether solution (d) is more advantageous than the solution (c). The uncertainty of the observation of June 30 is to be considered, the comet having no definite nucleus, being only 7° above the horizon, and having diminished theoretically by three magnitudes since its discovery, which would reduce its brightness to hardly 9th magnitude. Hence there would be little justification for wishing to better the result obtained by using all the observations which mutually agree, by taking into

account an additional isolated one of very doubtful character and certainly erroneous in one coördinate. WE THEREFORE ADOPT THE ELEMENTS (c) AS DEFINITIVE. The large uncertainty of the result is shown in the mean errors, by which the unknowns are affected. This was to be expected since we are dealing with an object observed only during some fifteen days under difficult conditions. From solution (d) we retain only the evidence that a supplementary observation, incomplete and unreliable, would tend to increase the eccentricity; that is to say to extend the period. In any event, the ellipticity of the orbit is too pronounced to admit of any possibility of representing the observations by a parabolic orbit.

As early as 1855, in comparing the provisional elements obtained at that time with those of previous comets, DONATI (*M. N.*, 16, 14, 1855) and ALEXANDER (*Astronomical Journal*, 4, 106, 1855) pointed out the possible identity of this comet with the first comet of 1362. BURCKHARDT (*Monatl. Corresp. Zach.*, 10, 166, 1804) computed a rough orbit of the latter, based chiefly on Chinese observations. These are not of sufficient number to permit the computation of an orbit, but BURCKHARDT filled them out by assuming two plausible hypotheses as to the latitude of the comet. He thus obtained the following elements, beside which for comparison are tabulated those obtained above for comet 1855 II (reduced to equinox of 1362):

	BURCKHARDT		1855 II
	1st Hypothesis	2d Hypothesis	
<i>T</i> (old style)	1362 March 11.2	1362 March 2.3	
ω	30°	20°	22°.7
$\oslash$	249	237	253 .5
<i>i</i>	159	148	156 .9
log <i>q</i>	9.659	9.672	9.754

BUT BURCKHARDT'S elements do not represent well the mean position, as HOLETSCHEK pointed out (*Untersuchungen über die Grösse und Helligkeit der Kometen*, 1, 74, 1896): the differences in longitude and in latitude for the first hypothesis are:

$$\Delta \lambda = -12^\circ \quad \Delta \beta = +14^\circ$$

and for the second:

$$\Delta \lambda = -29^\circ \quad \Delta \beta = -1^\circ$$

Rough as this approximation is, it shows, however, that there is in fact a certain similarity in respect to the position of the plane of the orbit. The analogy is less satisfactory as to perihelion distance. On the other hand,

it is not certain that the two orbits obtained by BURCKHARDT constitute the limiting values, which represent the data. Before attempting a new computation, I examined them carefully. The principal sources relating to the apparition of 1362 are found in the *Cometographie* by PINGRE. The vague information which European chroniclers have left us would have been altogether useless without the observations collected in China and set down in the *Chinese Annals*. Among the different interpretations which have been given to these, the most trustworthy seems to be that of J. WILLIAMS (Observations of Comets from B. C. 611 to A. D. 1640 extracted from the *Chinese Annals*, London, 1871, p. 72.) This author discusses the earlier sources (PINGRE, BIOT, etc.), and gives a better interpretation of some of the doubtful points.

Observed in China on March 5, 1362 (old style) before sunrise, the comet was described as being bluish-white in color. Its right ascension was 7.2 degrees (circle divided into $365\frac{1}{4}$ degrees) from the constellation *Wei*, which begins with α *Aquarii*, that is $\alpha = 323^\circ.2 + 7^\circ.1 = 330^\circ.3$ (equinox 1362). Neither the place nor the time of observation are stated; we assume that the observer was at Peking, and that he observed about 5 o'clock in the morning. Since the comet was observed a little before sunrise, this cannot be far from the truth. The European observers add the detail that the comet appeared near *Venus*, which led BURCKHARDT to assume the latitude zero for that date. The Chinese observation unfortunately is not precise as to the declination. On March 17th the comet passed near the western star of *Lekung*, formed by the three groups of two stars each: λ , μ ; η , σ , and ν , τ *Pegasi*. The most western star is σ *Pegasi*, the position of which in 1362 was

$$\alpha = 333^\circ.0, \quad \delta = +26^\circ.0$$

The star λ differs little in right ascension from σ , and for a northern observer, looking at the constellation rising in the east, λ would appear more to the right, and might have been taken as the farthest west. In that case, the position would be

$$\alpha = 334^\circ.1, \quad \delta = +20^\circ.3$$

But in view of the way the Chinese were accustomed to count right ascensions, we believe that the first interpretation is the most probable, and we have adopted it in the computation.

On the following days, the comet developed a long tail, but the linear unit used to express the length gives no indication as to the angular extent. On March 28, the nucleus was not observed, but the tail alone, which is described as a white cloud of curved form, crossing the sky, directed toward the west and passing *Arcturus*. In

a first approximation we can consider the tail opposite the *Sun* and conclude that the nucleus could not be far from the arc of a great circle passing through *Arcturus* and the *Sun* on that date.

On April 1st the comet was star-like, without a tail, in size compared to a cup of liquor, and passed before *Ta Yang*. According to WILLIAMS this refers to χ *Ursæ Majoris*. But this is contradicted by a more definite, additional statement: the comet was then 6 degrees from the *Pleiades*. This gives the right ascension,

$$\alpha = 47^\circ.5 + 5^\circ.9 = 53^\circ.4$$

while that of χ *Ursæ Majoris* is 168° .

We have submitted this contradiction to FATHER R. P. GELUY of the Mission de Scheut (near Brussels), who has in his possession a copy of the *Sheke*, or voluminous *Chinese Annals*. This sinologue has been kind enough to call our attention to the fact that the constellation reproduced on Plate 14 of the book by WILLIAMS is designated as *Ta Yang Shou*, while this word *Shou* (meaning star) is not found in the text. The Chinese designation therefore would not refer to a star, but to the vaguer notion of "uncertain or dawning luminosity." I have assumed that it perhaps referred to the twilight, and that the author meant to say that the comet passed its conjunction with the *Sun* at that time, as the given right ascension shows. The Chinese observer adds that that day the comet appeared like a star, without tail. We cannot therefore assume, as was the case on March 28, that it was not the nucleus but the tail which would pass before the star χ *Ursæ Majoris*. However that may be, we will use the above right ascension of April 1st, but we will assume that the observation was made about 7.00 P. M. local time. The *Chinese Annals* add that on April 7th the comet began to disappear and was not seen later.

From a geometrical point of view, we can only derive from these Chinese records the two data of sufficient accuracy:

$$\begin{aligned} t_1 &= \text{March } 4.39 \text{ G.M.T.} & \alpha_1 &= 330^\circ.3 \\ t_4 &= \text{March } 31.81 \text{ G.M.T.} & \alpha_4 &= 53^\circ.4 \end{aligned}$$

In addition we know that on March 17 the coördinates were nearly those of σ *Pegasi*:

$$t_2 = \text{March } 16.39 \text{ G.M.T.}, \quad \alpha_2 = 330^\circ.0, \quad \delta_2 = +26^\circ.0$$

and that at the instant $t_3 = \text{March } 27.39 \text{ G.M.T.}$, the head of the comet was not far from the arc joining the *Sun* and *Arcturus*. If we assume that the comet was exactly on the circle, we will have exhausted all the available geometrical data. This gives us five conditions to be fulfilled, that is, just what is necessary for the computation of a parabolic orbit, except that the data

are in an unusual form: three right ascensions, $\alpha_1, \alpha_2, \alpha_3$; one declination, δ_2 , and the condition as to one date, t_3 , that the object should be on a certain great circle. We have carried out the solution in the following manner: the geocentric coördinates of the comet at an epoch t may be expressed as follows:

$$(a) \quad \begin{cases} \cos \alpha \cos \delta = X + fx_2 + gx'_2 \\ \sin \alpha \cos \delta = Y + fy_2 + gy'_2 \\ \sin \delta = Z + fz_2 + gz'_2 \end{cases}$$

X, Y and Z being the coördinates of the *Sun* at the instant t , x_2, y_2 and z_2 the heliocentric coördinates of the comet at the instant t_2 considered as an origin, and x'_2, y'_2 and z'_2 the derivatives of these coördinates at the same moment; f and g are coefficients, functions of the dynamic elements only, which for epochs t , close to t_2 , have the approximate value:

$$f \neq 1 \quad \text{and} \quad g \neq (t - t_2)k$$

k being GAUSS'S constant.

The two first equations (a) give:

$$(b) \quad \tan \alpha = \frac{Y + fy_2 + gy'_2}{X + fx_2 + gx'_2}$$

The right ascensions corresponding to the epochs t_1 and t_2 give us two equations of condition of that form. If we assume the geocentric distance, ρ_2 , corresponding to the second date, we have at once x_2, y_2 and z_2 , since the geocentric position is known. These quantities substituted in (b) give two linear equations of condition in x'_2 and y'_2 , which permit the calculation of the values of these derivatives; a first calculation will give approximate values, since f and g are known only approximately. The condition that the orbit is parabolic

$$(c) \quad x'_2{}^2 + y'_2{}^2 + z'_2{}^2 = \frac{2}{r}$$

permits us to find z'_2 and consequently to compute the dynamic elements. We will obtain from these more precise values for f and g , which will give more approximate values for x'_2 and y'_2 , and so on, until the quantities change no longer. This process is very converging. Rarely has it been necessary to renew the calculation after the second approximation. With the final values obtained we find by equations (a) relative to the instant t_3 the position on that date.

The computation is gone through again with another value of the geocentric distance ρ_2 and that quantity is varied so that the third position should be 90° distant from the pole of the great circle passing through *Arcturus*

and the *Sun*. We have made this computation using the following three hypotheses with regard to ρ_2 :

Elements	Log $\rho_2 = 9.80$	Log $\rho_2 = 9.70$	Log $\rho_2 = 9.50$
Ω	316°	228°	195°
ω	22	304	297
i	145	140	114
log q	9.518	9.438	9.672
T 1362 March	$-3^d.43$	-5.14	-9.78

The corresponding geocentric positions are:

Epoch	Log $\rho_2 = 9.80$		Log $\rho_2 = 9.70$		Log $\rho_2 = 9.50$	
	α	δ	α	δ	α	δ
t_1	330°	$+ 2^\circ$	330°	$- 8^\circ$	330°	-24°
t_2	333	$+26$	333	$+26$	333	$+26$
t_3	4	$+66$	5	$+72$	0	$+78$
t_4	53	$+73$	53	$+78$	53	$+83$

Distance from comet to great circle *Sun-Arcturus* at epoch t_3 :

Log ρ_2	9.80	9.70	9.50
Distance	16°	19°	20°

We see that in each case the direct data are entirely represented (numbers in italics) but that none of the three hypotheses satisfies the condition that the comet at the epoch t_3 is on the great circle in question. Yet the distance is not considerable, and it is little different in the three hypotheses. We can consider that all three represent almost equally the data. The deviation is in such a direction that the comet's tail would be in retard on the radius vector joining the *Sun* to the comet. The Chinese observer described the tail as curved, which is one more reason that there should not be complete coincidence. The smallest distance (16°) produced by log $\rho_2 = 9.80$ seems to indicate that a more perfect coincidence would be obtained by increasing still more the geocentric distance. But we are limited on this side, if we restrict ourselves to a parabolic orbit, the values of z' found from equation (c) becoming imaginary for larger values of ρ_2 . On the other hand, if we consider the declination on the first date t_1 , we see that an increase in ρ_2 places the comet farther and farther north. But according to an indication, vague it is true, by European observers the comet was not far from *Venus* on that date. The position of that planet was at that moment:

$$\alpha = 315^\circ \quad \delta = -18^\circ$$

as derived from the tables of NEUGEBAUER (*Veröffentl. des Kgl. Rechen-Instituts*, Berlin, No. 25). We see that the smallest values of ρ_2 place the comet much nearer

Venus, which limits also the extent of the plausible values of that quantity.

All things considered, there exists a large uncertainty in the solution. This uncertainty is even greater than the preceding computations indicate: we have assumed in fact that on date t_2 the comet coincided with α *Pegasi*, while the observer merely signified a proximity. This

nearness is susceptible of many different interpretations, which introduces a still further cause of uncertainty. We see that it is indeed difficult to reach precise conclusions.

We have made still another trial. Taking as they are the elements of the comet 1855 II brought down to 1362, we have computed, making several hypotheses on the time of perihelion, the geocentric positions of the comet:

T 1362	February 23.0		March 5.0		March 10.0		March 20.0		Observed	
	α	δ	α	δ	α	δ	α	δ	α	δ
Mar. 4.39	319°.0	+ 2°.2	319°.6	- 10°.7	321°.7	- 14°.3	329°.3	- 18°.3	330°.3	
Mar. 16.39	5 .1	+46 .1	337 .5	+17 .5	332 .2	+ 5 .1	330 .5	- 9 .2	330 .0	+26°.0
Mar. 31.96	73 .0	+47 .5	58 .1	+49 .8	44 .1	+50 .3	7 .3	+33 .0	53 .4	

In order to represent the first place, it would be necessary to have $T = \text{March 21}$; to represent the second, $T = \text{March 3}$ would give the best result; the third would require that $T = \text{March 7}$. We find again that the representation of all the data at once is therefore almost impossible.

The objection could be made that in the interval of the two apparitions the orbit of the comet could have been sensibly affected by perturbations. But the uncertainty of the period is such (see elements c) that if indeed we have the same comet in 1362 as in 1855 it is not possible to affirm that between the two apparitions there may not have been one or even two intermediate returns. That is to say, that all attempt to compute perturbations over a long interval of time would be illusory.

To summarize, considering only the geocentric side of the question, the identity of the comets of 1362 and

1855 remains doubtful. The difference in physical characteristics between the two apparitions appears to be a more certain argument against the identity. At its maximum brightness the comet of 1855 was a small nebulosity, nearly round, without tail, of an apparent brightness of about 6 magnitude. It was not then visible, or only with difficulty, to the naked eye. The former apparition deals with a bright comet, visible to the naked eye in twilight, and developing at a certain time a tail more than 90° long, if it is true that on March 28th it extended to *Arcturus*. The difference in intrinsic brightness between the two apparitions is so great that it appears difficult to admit that it is one and the same object at successive returns.

The identity of the comet of 1362 with that of 1855 cannot, in my opinion, be considered as in any way proved.

Yerkes Observatory, January 28 1916.

NOTE ON THE SYSTEM OF THE BRIGHTER α STARS.

By H. C. PLUMMER.

As MR. C. A. MANEY remarks in his article on this subject (*A.J.*, Nos. 679-680) "the amount of profitable work... seems almost without limit." And the suggestions contained in his contribution may prove very valuable. At the same time it seems particularly desirable to bring independent views as far as possible to a common measure and the following remarks are offered to this end.

MR. MANEY has solved the equation

$$V_0 \cos d + K - V = 0$$

by least squares, according to different arrangements of the data. In this way several pairs of values of V_0 and K are obtained. It may be suggested at once that no physical significance need be attached to the divergences among the values of K . The measure of variability of

the quantity V is shown to be of the order of 12 km./sec. Hence without any elaborate analysis we should expect from n^2 equations an uncertainty in K of the order $12/\sqrt{n}$. For the six groups in Table IV $n = 7$ and we should expect an uncertainty of about 1.7. The total range among the six values of K is 3.54 and there is nothing surprising in this figure. For the two groups, galactic and extra-galactic, in Table III $n = 12$ and the uncertainty should be about 1. The actual difference is only 0.63. In Table I $n = 15$ on the average and the uncertainty may be about 0.8 while the actual range is 1.6. The case here, however, is different because the material of the three solutions is not independent. But if any point needs explanation it is not the Yerkes result, which practically reproduces the first Lick solution, but the difference between the two Lick results. The effect of

increasing the number of stars by only 20 per cent. is perhaps rather surprising. Before attributing the divergence between the Yerkes and the second Lick solution to the distribution of the stars it is necessary to be sure that there is no systematic discordance between the Lick and Yerkes velocities. MR. MANEY makes no allusion to this point.

The differences found among the values of V_0 in Table IV and particularly Table III are more important. MR. MANEY connects them with some criticism of the galactic hypothesis of stellar motion which I have employed in certain investigations. It must be confessed that it is equally difficult to criticize or to defend this hypothesis because it can be taken to be true only in the extreme sense of the pragmatist philosopher. All that is claimed is that it is sufficiently true of a sufficient number of stars to have a practical value.

However this may be, it is certain that MR. MANEY has misconceived the bearing of his results on the hypothesis. Indeed, he seems to have misconceived the meaning of the quantity V_0 . The values of V_0 represent the velocity of a group of stars as a whole relative to the *Sun*. So far as they are valid at all the peculiar velocities of the stars (*i. e.* their motions relative to the mean in each group) compensate one another and are eliminated in each solution. It follows that Table V is based on a complete misapprehension. The value of V_0 should be constant in all the groups of Table IV and this should be so whether we adopt the two-stream, the ellipsoidal or the galactic hypothesis. The differences in V_0 imply relative motion between groups of stars differently situated in space and this is equally subversive of any uniform hypothesis. After Table III, which indicates that the extra-galactic stars are moving with a velocity of six km./sec. relative to the galactic stars, MR. MANEY remarks that "the effect of star streaming is quite marked." The expression "star streaming" in this connection seems distinctly unfortunate because it has been consecrated to a quite different technical meaning. Elsewhere he speaks of "a large preferential motion relative to the *Sun*." But what we commonly understand by preferential motion is a property of the internal motions of the stars when the movement of the *Sun* has been eliminated as far as possible.

MR. MANEY is therefore not justified in thinking that on any hypothesis the different zones will yield values of $V_0 \cos \lambda$ instead of a constant V_0 . Having disproved this unreal expectation he suggests another formula for V_0 . As a matter of fact $25.3 \cos^3 \lambda$ will answer the purpose practically as well as his more complicated form. But what is the use of such formulæ when they apply only to four of the six zones and are obviously out of harmony with the other two results in Table IV? It

seems unprofitable to follow the deductions from them further.

But when all this has been said with regard to the interpretation to be placed on MR. MANEY's results, the results themselves, as expressed in Tables III and IV, remain. It is, however, difficult to form any judgment as to their value. The figures are incomplete because they need to be accompanied by the probable errors which result from the process of solution. MR. MANEY may be invited to supply this necessary information.

Provisionally it is possible to guess at the order of magnitude of the probable errors of V_0 . Were the distribution of the stars symmetrical in every case, the normal equations would be

$$K = \Sigma V/n$$

$$V_0 = \Sigma V \cos d / \Sigma \cos^2 d$$

or the corresponding probable errors being k , v_0 and v ,

$$k = v/\sqrt{n}$$

$$v_0 = v/(\Sigma \cos^2 d)^{1/2}$$

The distribution of the stars is indicated in the annexed Table I, for the several groups in galactic latitude λ and different distances d from the apex. It is easily deduced that on the average $\Sigma \cos^2 d$ is about 16 and therefore if $v = 12$ the probable error of V_0 in Table IV is of the order 3.

TABLE I

λ	0-7°	8-13°	14-24°	25-34°	35-49°	50-90°	
d							
0-39°	6	6	5	10	5	4	36
40-69	6	8	9	9	10	18	60
70-89	14	14	4	4	7	11	54
90-109	3	10	11	6	8	5	43
110-139	8	8	13	12	14	10	65
140-180	8	5	4	3	4	2	26
	45	51	46	44	48	50	284

Now the range in the value of V_0 for the four groups: I, III, IV, VI, is 3.8 and under the circumstances these results may be considered remarkably satisfactory. On the other hand the results in groups II and V each differ by about six from the mean. These are less satisfactory certainly, and yet it is permissible to regard the deviations as accidental.

This view may be confirmed in another way. If the six values of V_0 as they stand be regarded as independent determinations of one constant quantity, the "mean square" deviation from the mean may be calculated. It is 3.8. This again implies a probable error in a single determination of V_0 of the order 3 or rather less.

When the galactic and extra-galactic groups are taken together as in Table III, $\Sigma \cos^2 d$ is about 49 in each

division. The probable error to be expected is therefore rather less than 2. Hence the difference of 6 km./sec. actually found is certainly large. It is, however, very much the same difficulty under another form, for the greater part of the discrepancy is to be attributed to the abnormal results in groups II and V. It is not necessary to accept this figure as expressing exactly the relative velocities of the galactic and extra-galactic systems of *A* stars, but the suggestion that the two systems are moving relatively to one another need not be hastily dismissed.

On the whole, however, it seems abundantly clear that the differences among MR. MANEY'S values of V_0 must be very largely discounted in view of the inherent uncertainty in the determinations. And it is fortunate that this is so. For if it were necessary to take his figures at their face value and suppose that all the stars in one galactic zone are really moving as a whole relatively to the stars in another zone with the velocity 12 km./sec., we should be dealing with a chaos in which the search for law would be practically hopeless. Incidentally, such a conclusion would reduce MR. MANEY'S own investigation to an inconsistency. For if the velocities of a group are so widely variable in amount it cannot be supposed that they are identical in direction. Yet the whole is based on the assumption of a particular direction, $\alpha = 270^\circ$, $\delta = +30^\circ$.

Table II exhibits the distribution of the residual radial velocities which result from the general solution, according to the several groups. It will be noticed that there is an excess of large residuals in groups II and V. But there is

TABLE II

km./sec.	I	II	III	IV	V	VI	All
-40	2	5	0	1	2	0	10
-30	3	4	3	1	5	1	17
-20	7	7	9	9	4	3	39
-10	10	12	10	13	13	15	73
0	11	9	10	11	16	23	80
+10	8	7	7	7	5	6	40
+20	4	6	4	2	1	2	19
+30	0	1	3	0	2	0	6
+40							
							284
Means	12.1	14.1	13.1	10.0	11.4	7.5	(without sign)
	-1.1	-2.1	+1.7	-1.5	-1.2	+2.5	(with sign)

no sign of that marked dissymmetry which would be expected if the departure of the group solutions from the general solution were real. This point is confirmed by the mean residuals with regard to sign for the different groups, which are appended to the table. These are certainly not abnormally large. The residuals of this table have a bearing on the hypothesis of galactic motions. In the first five groups the effect of foreshortening is small and rather uncertain. In the last group, above galactic latitude 50° , it is well marked both in the increase of residuals falling within the limits ± 10 and by the mean without regard to sign. The effect can be variously explained, but the figures are fairly in accord with the galactic hypothesis.

1915, November 15.

SAN LUIS DECLINATIONS.

By ARTHUR J. ROY.

In an attempt to ascertain and eliminate the personal differences among the three fundamental observers at San Luis during 1909 and 1910, so much of interest has appeared that it seems advisable to publish an outline of the preliminary results in advance of the final discussion. At the very outset of the reductions, it was found that there were decided personalities which must be evaluated because they could not be directly eliminated, and the first approximation was made on broad lines with a view to ascertaining a system of fundamental positions which would found a homogeneous set of zenith distances. Naturally, all corrections for known errors have been applied and, as far as possible, outstanding uncertainties and unknown errors have been eliminated. The leading object was to avoid deforming the system rather than to attain the highest precision on individual stars.

There were pronounced systematic differences between the runs as simultaneously determined by two readers, but this would have no appreciable systematic effect and could scarcely amount to $\pm 0''.03$ for any star. The division correction was thoroughly determined at Albany, redetermined to the fifteen degree points both at San Luis and on the return to Albany. No discrepancies appeared that could call in question the original determination. The inclination of the fixed zenith distance wire was very precisely determined by occasionally making two pointings and readings on equatorial stars. The flexure of the circles has been repeatedly determined and the only notable discordance was in the sine flexure of circle B, for which the San Luis result differed $0''.2$ from the previous and subsequent determinations made in Albany. The various measures of the sine flexure of

TABLE I. SAN LUIS LATITUDES

Decl.	AE			AW			BE			BW		
	<i>R</i>	<i>V</i>	<i>T</i>	<i>R</i>	<i>V</i>	<i>T</i>	<i>R</i>	<i>V</i>	<i>T</i>	<i>R</i>	<i>V</i>	<i>T</i>
-113°	4.18	4.86	4.70	4.24	4.30	4.77	4.52	4.52	4.33	4.69	5.23	5.13
-109	4.72	4.26	4.90	3.92	4.15	4.34	4.03	4.91	3.97	4.34	4.85	4.40
-104	4.50	4.92	4.32	4.27	4.19	4.27	3.92	4.76	4.20	4.44	4.83	4.84
-100.6	4.40	5.12	4.32	4.28	4.48	4.00	5.07	4.61	4.53	5.11	5.41	4.74
-96.3	4.94	5.34	4.45	4.04	4.88	4.49	4.59	4.87	4.62	4.83	5.33	4.71
-92.1	5.13	5.02	4.71	4.72	4.53	4.69	4.75	5.23	4.96	5.33	4.84	4.94
-87.9	5.11	5.35	4.85	4.87	4.50	4.64	4.74	4.83	5.07	5.24	5.02	5.12
-83.7	5.10	5.05	5.03	4.57	4.64	4.86	5.03	4.77	5.13	5.19	5.07	5.19
-79.4	5.43	5.13	5.13	4.73	4.78	4.96	4.96	5.03	5.36	5.26	5.23	5.22
-76.4	5.57	5.28	5.49	4.73	4.61	5.16	5.17	5.41	5.34	5.54	5.44	5.30
-71.0	5.49	5.25	5.29	4.87	4.67	5.17	4.99	5.33	5.30	5.40	5.30	5.12
-66.5	5.59	5.53	5.49	5.03	5.01	5.34	5.26	5.13	5.55	5.42	5.22	5.41
-62.0	5.31	5.31	5.30	4.86	5.01	4.87	5.13	5.06	5.57	5.57	5.28	5.39
-56.3	6.00	5.82	5.92	5.36	5.36	5.74	5.68	5.63	6.06	5.83	5.57	5.81
-48.5	6.31	5.87	6.07	5.43	5.25	5.74	5.73	5.77	5.89	5.96	5.78	5.87
-41.5	5.95	5.72	6.03	5.22	5.17	5.53	5.67	5.58	6.12	5.72	5.68	5.90
-36.0	5.96	5.86	6.00	5.31	5.16	5.58	5.70	5.57	5.97	5.51	5.43	5.81
-30.1	5.85	5.98	5.97	5.32	5.29	5.55	5.76	5.85	6.18	5.53	5.62	5.83
-23.6	5.96	5.58	5.91	5.36	5.29	5.79	5.65	5.72	6.03	5.47	5.60	5.91
-15	5.77	5.56	5.91	5.26	5.18	5.64	5.51	5.56	5.89	5.51	5.61	5.74
-5	5.80	5.65	6.01	5.34	5.27	5.80	5.50	5.48	5.87	5.58	5.63	5.98
+5	5.97	5.75	6.09	5.49	5.39	5.76	5.79	5.77	6.03	5.66	5.77	6.09
+15	6.15	6.07	6.40	5.68	5.56	6.10	5.70	5.96	6.23	5.83	5.97	6.25
+25	6.32	6.12	6.68	5.72	5.82	6.26	5.87	6.02	6.42	5.91	6.21	6.60
+31.5	6.54	5.83	6.50	5.74	5.78	5.75	5.90	5.84	6.16	5.85	6.21	6.39
+34.5	6.68	6.61	6.69	6.56	5.82	6.58	6.27	6.44	6.58	6.39	6.80	7.16
+37.5	6.53	6.80	7.44	5.91	6.51	6.71	6.62	6.53	6.81	6.40	6.73	7.14
+40.5	7.56	7.20	7.86	6.39	6.41	6.90	6.53	6.84	7.12	7.03	6.82	7.14
+43.5	7.27	7.08	7.61	6.58	6.84	7.49	6.51	6.91	7.42	7.79	7.11	7.72
+46.5	7.98	7.26	8.44	7.14	6.28	7.56	7.75	9.06	7.96	7.81	8.76	8.99
+49	8.47	7.55	8.00	6.83	6.86	8.32	6.99	7.86	7.57	8.80	7.38	9.16

the telescope, by means of small opposing collimators, have been quite accordant, and though the apparent probable error was $\pm 0''.03$, it was believed that the real probable error might be more than $\pm 0''.06$.

Reflection observations for the determination of the cosine flexure of the telescope have been taken but definitive results have not yet been obtained.

The Pulkova refraction tables, with the necessary extension of the barometric factor strictly in proportion to the pressure, were used. The nadir correction was determined at irregular intervals, less frequently in long series or subseries than in short series. Where the evidence seemed to warrant it the correction was assumed to vary uniformly with the time, and the mean correction for any subseries was assumed to be independent of the adjacent subseries. The variation was much larger and more frequent on the west pier than on the east. Corrections were also applied for personalities dependent upon feet north or feet south (S-N) and for variation of

latitude as shown in ALBRECHT's papers in *A. N.*, Nos. 4414, 4504, and 4588.

The corrected zenith distances combined with the positions of the *P. G. C.* furnished determinations of the latitude which were tabulated for each star, for each observer in the four combinations of the two circles *A* and *B*, and in the two positions clamp east and clamp west. Means were formed in very small groups, and lest an erroneous declination should have an undue influence, weights were assigned as follows: 1 obs., weight 1; 2 or 3, weight 2; 4 to 8, weight 3; 9 or more, weight 4. These were later combined in larger groups as exhibited in Table I, from which $-33^{\circ}17'40''$ has been omitted. Assuming that each column might be represented by the equation $a + b \cdot \tan z$, least square solutions were made and the results are shown in Table II. Coarsely approximate declinations are given, but for the extreme groups the mean $\tan z$ was computed. In these solutions the weights were again reduced on a sliding scale, the

TABLE II

CONSTANT TERM OF LATITUDE						TANGENT TERM OF LATITUDE					
	AE	AW	BE	BW	Mean		AE	AW	BE	BW	Mean
R	45".82	45".26	45".50	45".57	45".54	R	-.351	-.337	-.322	-.229	-.310
V	45 .69	45 .19	45 .57	45 .58	45 .51	V	-.281	-.326	-.301	-.303	-.303
T	45 .86	45 .59	45 .87	45 .79	45 .78	T	-.503	-.433	-.382	-.435	-.439
Mean	45".79	45".35	45".65	45".65	45".61	Mean	-.378	-.365	-.335	-.322	-.350

divisor varying from about 6.5 in the weaker groups to 14 in the strongest. The weights of the extreme groups were still further modified by factors varying from 0.9 at 65° to 0.1 at 82° zenith distance.

Applying to Table I the corrections indicated in Table II, including the constants to reduce each to the mean latitude 45".61, the similar part of Table III results, and combining the twelve columns with equal weights, the mean is obtained. The next column gives the sum of the weights as used in the least square solution. The

other three columns are obtained by combining, with equal weights, the four positions for each observer and taking the differences.

The observed values of S-N, with their apparent probable errors, were as follows: $R + 0''.65 \pm 0''.08$, $V - 0''.12 \pm 0''.04$, $T + 0''.32 \pm 0''.03$. These values would only need to be changed to $R + 0''.57$, $V - 0''.07$, $T + 0''.35$ to remove between -90° and $+30^\circ$ the systematic personal differences here shown.

The unfavorable polar distance of the station rendered

TABLE III. CORRECTED LATITUDES

Decl.	AE			AW			BE			BW			Mean	Wt.	R-V	V-T	T-R
	R	V	T	R	V	T	R	V	T	R	V	T					
-113	5.93	6.41	7.52	6.55	6.55	7.30	6.57	6.41	6.29	6.06	7.05	7.51	6.68	12.7	+.32	+.56	-.88
-109	5.88	5.28	6.62	5.59	5.84	6.05	5.40	6.12	5.16	5.28	6.06	5.91	5.77	21.0	+.28	+.12	-.40
-104	5.29	5.65	5.50	5.59	5.55	5.52	4.96	5.66	5.05	5.15	5.73	5.89	5.46	28.5	+.40	-.16	-.24
-100.6	5.03	5.72	5.28	5.42	5.69	5.06	5.96	5.37	5.19	5.70	6.17	5.60	5.52	34.5	+.21	-.46	+.25
-96.3	5.43	5.82	5.21	5.07	5.96	5.38	5.35	5.51	5.13	5.33	5.97	5.40	5.46	51	+.52	-.54	+.02
-92.1	5.50	5.41	5.29	5.63	5.50	5.42	5.40	5.77	5.35	5.76	5.38	5.47	5.49	30	-.05	-.14	+.19
-87.9	5.39	5.66	5.31	5.69	5.38	5.27	5.31	5.29	5.35	5.60	5.48	5.54	5.44	32	-.05	-.08	+.13
-83.7	5.31	5.31	5.39	5.32	5.46	5.40	5.53	5.17	5.33	5.51	5.47	5.53	5.39	56	-.07	+.06	+.01
-79.4	5.59	5.35	5.41	5.44	5.55	5.43	5.42	5.39	5.52	5.55	5.59	5.49	5.48	48	-.03	-.01	+.04
-76.4	5.69	5.47	5.72	5.40	5.35	5.59	5.60	5.74	5.47	5.81	5.77	5.53	5.60	51	-.04	.00	+.04
-71.0	5.56	5.39	5.45	5.49	5.35	5.54	5.37	5.61	5.35	5.63	5.58	5.28	5.47	65	-.03	-.08	+.11
-66.5	5.61	5.63	5.58	5.60	5.64	5.64	5.59	5.36	5.52	5.60	5.45	5.51	5.56	90	-.08	+.04	+.04
-62.0	5.29	5.39	5.33	5.40	5.62	5.13	5.43	5.27	5.54	5.75	5.49	5.44	5.42	44	-.03	-.08	+.11
-56.3	5.93	5.85	5.88	5.85	5.91	5.93	5.92	5.79	5.95	5.96	5.73	5.80	5.88	80	-.10	+.07	+.03
-48.5	6.20	5.87	5.97	5.89	5.77	5.89	5.94	5.90	5.75	6.07	5.90	5.82	5.91	75	-.16	.00	+.16
-41.5	5.79	5.69	5.85	5.63	5.65	5.61	5.83	5.67	5.94	5.82	5.76	5.78	5.75	96	-.08	+.11	-.03
-36.0	5.77	5.80	5.79	5.69	5.61	5.63	5.44	5.63	5.72	5.56	5.48	5.66	5.68	116	-.08	+.07	+.01
-30.1	5.62	5.90	5.69	5.66	5.70	5.55	5.86	5.88	5.92	5.57	5.64	5.62	5.72	109	+.10	-.08	-.02
-23.6	5.68	5.45	5.56	5.65	5.65	5.72	5.70	5.70	5.70	5.46	5.57	5.64	5.62	113	-.03	+.07	-.04
-15	5.45	5.40	5.51	5.51	5.51	5.53	5.53	5.51	5.52	5.48	5.55	5.43	5.49	107	.00	+.01	-.01
-5	5.41	5.43	5.51	5.53	5.53	5.60	5.45	5.37	5.42	5.51	5.51	5.58	5.49	96	-.02	+.07	-.05
+5	5.48	5.45	5.14	5.58	5.55	5.43	5.62	5.57	5.47	5.52	5.56	5.56	5.52	96	-.03	-.05	+.08
+15	5.55	5.68	5.60	5.66	5.62	5.64	5.46	5.67	5.55	5.62	5.67	5.59	5.61	117	+.09	-.06	-.03
+25	5.55	5.59	5.63	5.54	5.72	5.59	5.47	5.58	5.57	5.59	5.76	5.72	5.61	97	+.12	-.03	-.09
+31.5	5.59	5.16	5.20	5.39	5.52	4.86	5.34	5.25	5.10	5.41	5.59	5.29	5.31	24.5	-.05	-.27	+.32
+34.5	5.59	5.84	5.22	6.09	5.44	5.54	5.59	5.74	5.37	5.86	6.09	5.91	5.69	21.0	.00	-.27	+.27
+37.5	5.30	5.91	5.74	5.29	5.99	5.47	5.80	5.70	5.44	5.78	5.88	5.70	5.67	26.5	+.33	-.28	-.05
+40.5	6.16	6.14	5.90	5.57	5.69	5.40	5.52	5.86	5.56	6.27	5.82	5.43	5.78	24.0	.00	-.31	+.31
+43.5	5.58	5.82	5.15	5.52	5.89	5.60	5.27	5.63	5.52	6.83	5.85	5.67	5.69	14.9	.00	-.32	+.32
+46.5	5.77	5.61	5.38	5.65	4.88	5.11	6.13	7.44	5.48	6.57	7.06	6.37	5.95	4.4	+.22	-.67	+.45
+49	5.84	5.47	4.08	4.80	4.87	5.05	4.69	5.64	4.56	7.20	5.17	5.80	5.26	1.8	-.34	-.42	+.76

it impracticable to independently determine the refraction constants, but it will be noted that this adjustment of the observations to the system of the *P. G. C.*, very closely fits the stars within seventeen degrees of the pole, both above and below; and another approximation to adjust to the pole is not necessary. Assuming that $-33^{\circ} 17' 45''.47$, the latitude indicated by these polar stars, is the true latitude of San Luis, and subtracting from it the mean values in Table III, we have the indicated correction to the *P. G. C.* The resulting smoothed curve is shown in Table IV.

TABLE IV
PRELIMINARY CORRECTIONS TO *P. G. C.*

-90°	0".00	-40°	+0".28	$+10^{\circ}$	+0".07
85	0 .00	-35	+0 .23	15	+0 .09
80	0 .00	-30	+0 .18	20	+0 .11
75	0 .00	-25	+0 .13	25	+0 .13
70	0 .00	-20	+0 .09	30	+0 .15
-65	0 .00	-15	+0 .05	$+35$	+0 .18
60	+0 .18	-10	+0 .01	40	+0 .20
55	+0 .38	-5	+0 .01	45	+0 .24
50	+0 .40	0	+0 .01	$+50$	+0 .27
45	+0 .37	$+5$	+0 .04		

Inasmuch as this is only a first approximation, an exhaustive discussion is inappropriate, but, in conclusion, attention may be directed to several points. First, the large constant and progressive differences that exist in Table I and are evaluated in Table II. In these are involved, with the uncertainties of the instrumental

corrections, the various and varying personalities, as well as the tangent term which must be largely due to a refraction factor. This factor in the mean would be about 0.9916. The differences of mean latitude among the observers may be entirely due to personalities in determining the nadir correction; or possibly the S-N should not be equally distributed each side of the zenith for all observers. While the tangent term well reconciles the observations to the far north, it deviates very rapidly at the far south where a decreasing correction to the refraction is needed, and particularly so for *T*. There are few other series of observations made from stations at so great an altitude (800 meters), but comparison can be made with TUCKER's results as given particularly on p. 153 of Vol. VI of the publications of the Lick Observatory, where he finds in general a much smaller correction to the refraction, though increasing rapidly at extreme zenith distances.

No explanation of the abrupt change in the indicated corrections to the *P. G. C.* near -60° has as yet been found. It seems certain that if it were personal or instrumental there would be a similar change north of the zenith, no vestige of which appears in the details. The same reasoning almost equally applies to the relation of the instrument to the building and other surroundings. An examination of the foundation curves of the *P. G. C.* revealed no minor corrections which had been ignored. While the weights of the standard positions vary from 1 to 31, all have been given equal weight in the discussion, and it was to be expected that the greatest divergences for individual stars would appear in this region.

Dudley Observatory, May 1, 1915.

ELEMENTS OF COMET 1916 *a* (NEUJMIN),

(FROM THREE POSITIONS, FEB. 29, MARCH 4 AND 8, WITH THE 12-INCH TELESCOPE OF THE YERKES OBSERVATORY),

By G. VAN BIESBROECK.

$T = 1916 \text{ March } 12.17135 \text{ G.M.T.}$
 $\omega = 194^{\circ} 30' 43''$
 $\Omega = 327 \ 31 \ 19$
 $i = 10 \ 34 \ 50$
 $\varphi = 34 \ 53 \ 43$
 $\mu = 641''.317$
 $\log a = 0.4953 \text{ (Per. } 5.53 \text{ years)}$

EPHEMERIS FOR GREENWICH M. MIDNIGHT:

Date	α_{vera}	δ_{vera}	$\log \rho$	Mag.
1916 March 14	9 ^h 5 ^m 44 ^s	$+6^{\circ} 25' .5$	9.6104	10.6
16	7 32	5 27 .2		
18	9 32	4 29 .9		
20	11 43	3 34 .2	9.6287	10.7
22	14 5	2 39 .9		

Date	α_{vera}	δ_{vera}	$\log \rho$	Mag.
1916 March 24	9 ^h 16 ^m 37 ^s	$+14^{\circ} 6' .9$		
26	19 18	0 55 .9	9.6503	10.8
28	22 8	+0 6 .1		
30	25 7	-0 42 .0		
April 1	28 15	1 28 .6	9.6744	10.9
3	31 30	2 13 .7		
5	34 53	2 57 .3		
7	38 23	3 39 .2	9.7009	11.1
9	41 58	4 19 .8		
11	45 40	4 58 .9		
13	49 32	5 34 .4	9.7298	11.3

Adopted magnitude 10.5 on March 4.

Yerkes Observatory, March 14, 1916.

OBSERVATIONS OF COMET 1915 c (TAYLOR),

MADE WITH THE FILAR MICROMETER OF THE 12 $\frac{1}{2}$ -INCH REFRACTOR OF DETROIT OBSERVATORY,
BY BERNHARD H. DAWSON.

1915-1916	Ann Arbor M. T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	α	δ	Log $p\Delta$ for α	for δ
Dec. 10	11 ^h 57 ^m 35 ^s	1	10, 10	- 6.63	+ 7 53.4	5 ^h 20 ^m 25.87	+ 0 49 25.7	8.150 _n	0.763
Jan. 7	9 51 7	2	8, 8	-27.93	- 6 0.1	5 6 49.73	+10 14 31.7	8.295 _n	0.666
7	10 35 5	2	8, 8	-28.38	- 5 12.4	5 6 49.28	+10 15 19.4	8.813	0.667
8	10 22 0	3	- , 5		-12 0.0		+10 40 34.1		0.661
8	10 37 12	3	7, -	+31.63		5 6 42.35		8.886	

Mean Places of Comparison Stars.

*	α 1915.0	Red. to App. Pl.	δ 1915.0	Red. to App. Pl.	Authority
1	5 ^h 20 ^m 27.48 1916.0	+5.02	+ 0 41 21.0 1916.0	+11.2	A. G. Nicolajew 1300.
2	5 7 15.52	+2.14	+10 20 25.7	+ 6.1	A. G. Leipzig I, 1547.
3	5 6 8.55	+2.17	+10 52 27.9	+ 6.2	B. D. +10°726, connected with *4.
4	5 5 20.14	+2.17	+10 47 12.2	+ 6.2	A. G. Leipzig I, 1535.

NOTES.

*2 was connected with A. G. Leipzig I, 1553. $\Delta\alpha = -40.97$, $\Delta\delta = +1' 58''.6$.

All observations and star connections were made by direct micrometer measurement of $\Delta\alpha$ and $\Delta\delta$, and have been corrected for differential refraction. The observation of January 8 was interrupted by clouds.

Ann Arbor, 1916, January 28.

OBSERVATIONS OF SOLAR ECLIPSE OF FEB. 3, 1916,

By J. G. PORTER.

The partial eclipse of the *Sun* of February 3 (civil date) was observed at the Cincinnati Observatory by Dr. YOWELL and myself, using respectively the eleven-inch and the four-inch equatorials. The sky was clear and definition sharp.

The Cincinnati mean times of the contacts are as follows:

		BEGINNING	ENDING
E. I. YOWELL	Feb. 2,	21 ^h 25 ^m 11 ^s	23 ^h 24 ^m 05 ^s
J. G. PORTER	Feb. 2,	21 25 11	23 24 06

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OBSERVATIONS OF SOLAR ECLIPSE OF FEB. 3, 1916, by J. G. PORTER AND E. I. YOWELL.

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AN ERROR IN HILL'S THEORY OF JUPITER AND SATURN,

By R. T. A. INNES.

HILL's theory of *Jupiter* and *Saturn* is originally printed in Vol. IV of the *Astronomical Papers of the American Ephemeris*, Washington, 1890, and has been reprinted, evidently from the original stereotyped plates, in Vol. III of his collected works, as in the latter known errors are not corrected in the text.

In deriving the various perturbing functions required in HANSEN's method of computing the perturbations which he uses, HILL has followed the CAUCHY-HANSEN method of development in preference to a purely algebraical one. He had therefore to perform numerous enormously long multiplications of trigonometrical series. HANSEN had expressed the opinion that it would be almost impossible to develop the disturbing functions by algebraical process in the case of the mutual action of these two planets. But the invention of the NEWCOMB operators, described by their author in Vol. V of the *Astronomical Papers*, and subsequent improvements, have greatly simplified an algebraical development and it therefore seemed worth while to attempt one. One of the first results of this work has been to find a serious error in HILL's formulæ which unfortunately vitiates the whole of his work on the perturbations arising from the third powers of the masses. The error is on page 341, lines 10 and 12, where $-\frac{1}{2}\left(\frac{a'}{\Delta}\right)^3$ should be $+\frac{3}{2}\left(\frac{a'}{\Delta}\right)^3$.

This error can be proved by performing the somewhat involved logarithmic differentiations of the two corresponding formulæ on p. 201. But the error was found through numerical work. If we adopt NEWCOMB's method of development, there is a wonderful simplification of the formulæ.

If following NEWCOMB, we put P for the development of the perturbing function we have in the HANSEN-HILL notation

$$a\Omega = \mu \left[\frac{a'}{\Delta} - a' \frac{r}{r'^2} H \right]$$

Hence

$$a r \frac{d\Omega}{dr} = \mu \left[-\frac{1}{2} \left(\frac{a'}{\Delta} \right)^3 \left\{ a^2 \left(\frac{r}{a} \right)^2 - \left(\frac{r'}{a'} \right)^2 \right\} - \frac{1}{2} \frac{a'}{\Delta} - a' \frac{r}{r'^2} H \right]$$

as written by HILL, p. 66.

In NEWCOMB's notation, we have much more simply

$$\frac{a}{\mu} \Omega = P$$

$$\frac{a}{\mu} r \frac{d\Omega}{dr} = DP$$

Turning now to the formulæ in which the error exists (p. 341), HILL writes

$$\begin{aligned} a_0 \left(r \frac{d}{dr} \right)^3 \Omega = \mu \left\{ -\frac{15}{8} \left(\frac{a'}{\Delta} \right)^7 \left[a^2 \left(\frac{r}{a} \right)^2 - \left(\frac{r'}{a'} \right)^2 \right]^3 \right. \\ + \left(\frac{a'}{\Delta} \right)^5 \left[\frac{9}{8} \left\{ a^2 \left(\frac{r}{a} \right)^2 - \left(\frac{r'}{a'} \right)^2 \right\}^2 \right. \\ + \frac{9}{2} \left(\frac{r'}{a'} \right)^2 \left(a^2 \frac{r^2}{a^2} - \frac{r'^2}{a'^2} \right) \left. - \frac{1}{2} \left(\frac{a'}{\Delta} \right)^3 \left(\frac{r'}{a'} \right)^2 \right. \\ \left. \left. - \frac{1}{8} \left(\frac{a'}{\Delta} \right)^3 \left[a^2 \left(\frac{r}{a} \right)^2 - \left(\frac{r'}{a'} \right)^2 \right] - \frac{1}{8} \frac{a'}{\Delta} - a' \frac{r}{r'^2} H \right\} \right\} \end{aligned}$$

and

$$\begin{aligned} a_1' \left(r' \frac{d}{dr'} \right)^3 \Omega = \mu' \left\{ \frac{15}{8} \left(\frac{a'}{\Delta} \right)^7 \left[a^2 \left(\frac{r}{a} \right)^2 - \left(\frac{r'}{a'} \right)^2 \right]^3 \right. \\ + \left(\frac{a'}{\Delta} \right)^5 \left[\frac{9}{8} \left\{ a^2 \left(\frac{r}{a} \right)^2 - \left(\frac{r'}{a'} \right)^2 \right\}^2 \right. \\ - \frac{9}{2} a^2 \left(\frac{r}{a} \right)^2 \left(a^2 \frac{r^2}{a^2} - \frac{r'^2}{a'^2} \right) \left. - \frac{1}{2} \left(\frac{a'}{\Delta} \right)^3 a^2 \frac{r^2}{a^2} \right. \\ \left. \left. + \frac{1}{8} \left(\frac{a'}{\Delta} \right)^3 \left[a^2 \left(\frac{r}{a} \right)^2 - \left(\frac{r'}{a'} \right)^2 \right] - \frac{1}{8} \frac{a'}{\Delta} - a' \frac{r'}{r'^2} H \right\} \right\}. \end{aligned}$$

It is to be noted that $-\frac{1}{2}\left(\frac{a'}{\Delta}\right)^3$ in these two expressions

is to be replaced by $+\frac{3}{2}\left(\frac{a'}{\Delta}\right)^3$.

With NEWCOMB, we put with equal precision

$$a_0 \left(r \frac{d}{dr} \right)^3 \Omega = \mu D_3 P$$

and

$$a_0' \left(r' \frac{d}{dr'} \right)^3 \Omega = \mu' (-1 - D)_3 P.$$

The very onerous labor involved — to use HILL's own words — in working out the numerical values of his formulæ is only too obvious. Only a glutton for figure-work would face it.

The error was discovered by comparing the constant terms in these formulæ with HILL's results. The comparison is not quite precise as my data depend upon HILL's revised theory, but the difference due to this must be small. HILL's calculations for each derivative are essentially different and, as shown, tend to become very complex. With the algebraical method, the calculations are precisely similar; I therefore present all the constant terms, adding HILL's values divided by μ or μ' when necessary so as to give an immediate comparison.

Jupiter and Saturn

Terms arising from	D_0	D_1	D_2	D_3
$\frac{1}{2}P_{0,0}^{(0)}$	1.091165	0.224358	0.66714	2.6125
$P_{-1,1}^{(-1)}$	-159	-726	-403	-277
$P_{-2,2}^{(-2)}$	-3	-20	-17	-17
$\dot{P}_{-2,0}^{(-1)}$	-1	-4	-3	-3
$\dot{P}_{-1,-1}^{(-1)}$	+1	+8	+6	+6
$\dot{P}_{0,-2}^{(-1)}$	0	+1	+1	+1
	1.091003	0.223617	0.66298	2.5835
HILL	1.090999	0.223609	0.66292	-1.820
	p. 53	p. 66	p. 205	p. 345

Then $D_n' = (-1 - D)^n$ gives

	D_1'	D_2'	D_3'
	-1.314620	2.20122	-6.3343
HILL	-1.314608	2.20113	-7.663
	p. 69	p. 208	p. 345

It will be seen that the disagreement in the cases of D_3 and D_3' is complete. *A priori*, we know that one of HILL's values is wrong because D_3 and D_3' must differ in sign. As proved above, HILL's formulæ require the corrections

$$+ 2 \left(\frac{a'}{\Delta} \right)^3 \left(\frac{r'}{a'} \right)^2 \quad \text{and} \quad + 2 \left(\frac{a'}{\Delta} \right)^3 a'^2 \left(\frac{r'}{a'} \right)^2$$

which can be obtained from page 60, namely:—

Correction	+4.406	+1.329
HILL, p. 345	-1.820	-7.663
HILL corrected	+2.586	-6.334
which compares with		
my D_3 and $D_3' P$ thus:—	+2.584	-6.334

It will be seen that for $D_0 P$ I have 1.091003, whilst HILL has 1.090999, the difference is very small and might well be due to the slightly different data adopted, but it is worth remarking that HILL's value depends immediately upon the constant term on p. 46; namely, 1.090769379, which depends upon the eccentric anomaly of *Jupiter*. The simple conversion of this to the mean anomaly by means of HILL's own figures gives not 1.09099923 but 1.091001223.

It should be remarked that the *Jupiter* constants k_3 and k_4 on p. 95 are interchanged.

From the above remarks, it will be apparent that I have given some time to the HANSEN HILL theory of *Jupiter* and *Saturn*. In later work, especially as regards the two integrations to find ndz , considerable modifications of the processes used by HANSEN and HILL are possible without any loss of precision or generality, and reduce the work by 50 to 60%. The impression I have at the moment is that with the NEWCOMB development of the perturbing function and the HANSEN method of applying the perturbations, the task of computation is simple and certain, but necessarily somewhat lengthy. The roughest of estimates would guess that LE VERRIER's methods were ten times as lengthy and lacked certainty.

Union Observatory, Johannesburg, 23d November, 1915.

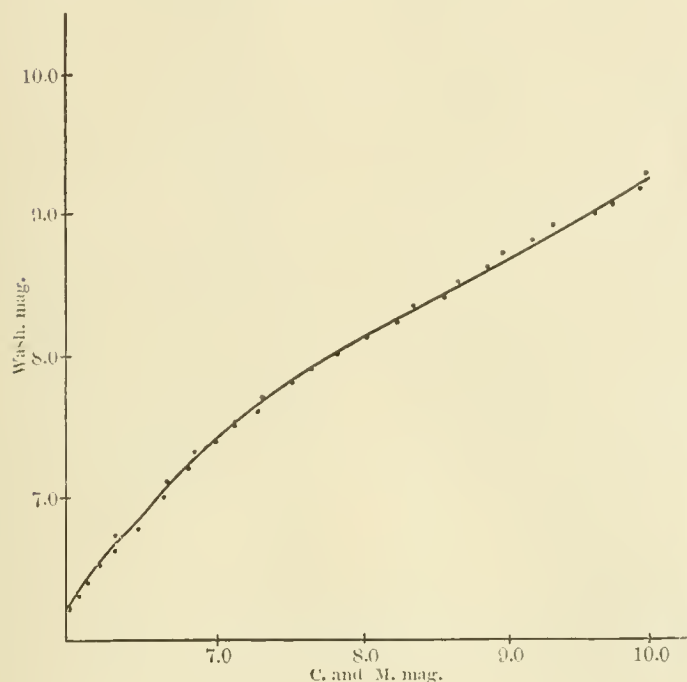
NOTE ON THE MAGNITUDE SCALE OF WASHINGTON A. G. CATALOGUE.

By R. J. POCOCK AND T. P. BHASKARAN

The reference stars for the *Hyderabad Astrographic Catalogue Zone -17°* are taken from the Washington A. G. C. During the progress of the astrographic work here, the magnitudes as derived from the photographic plates had to be compared with the visual magnitudes of the Washington Catalogue. This naturally led us to a detailed examination of the scale of the Washington mag-

nitudes. The simplest way of investigating the peculiarities of the scale is to make systematic counts of stars of different magnitudes. This was done in the case of stars whose declinations lie approximately between $-15^{\circ} 55'$ and $-18^{\circ} 5'$, these being the reference stars of the *Hyderabad Catalogue Zone -17°*. One feature immediately apparent is that the even magnitudes greatly

preponderate over the odd ones and particularly so in the case of magnitude 9.0. By means of these counts the Washington scale has been converted into a photographic scale comparable with that of CHAPMAN and MELOTTE using the table given by PROF. TURNER in *Monthly Notices R. A. S.*, 1915, Jan., p. 145. The results of the counts and comparison are given in the table.



In the above figure the C. and M. magnitudes are plotted against the corresponding Washington magnitudes. It will be seen that they lie on a smooth curve, for the later values the conversion from one scale to another may be effected by a straight line formula, *viz.*, C. and M. magnitudes = 1.62 (Wash. mag.) - 5.1.

Nizamiah Observatory, Hyderabad, 1915, December 18.

COMPARISON OF WASHINGTON MAGNITUDES WITH THOSE OF CHAPMAN AND MELOTTE

Wash. Mags.	Number	Cumulative Total	C. and M. Mags.
Brt. Stars	44	44	
6.2	6	50	6.02
6.3	2	52	6.06
6.4	4	56	6.11
6.5	8	64	6.22
6.6	6	70	6.29
6.7	0	70	6.29
6.8	16	86	6.45
6.9	8	94	6.52
7.0	13	107	6.63
7.1	6	113	6.67
7.2	22	135	6.81
7.3	6	141	6.85
7.4	28	169	7.00
7.5	31	200	7.14
7.6	37	237	7.28
7.7	16	253	7.33
7.8	73	326	7.55
7.9	44	370	7.65
8.0	139	509	7.93
8.1	65	574	8.04
8.2	153	727	8.24
8.3	84	811	8.34
8.4	204	1015	8.54
8.5	117	1132	8.64
8.6	282	1414	8.85
8.7	133	1547	8.93
8.8	462	2009	9.18
8.9	270	2279	9.30
9.0	822	3101	9.60
9.1	336	3437	9.70
9.2	699	4136	9.88
9.3	227	4363	9.93
9.4	293	4656	10.00
9.5	48	4704	10.01
9.6	41	4745	10.02

NOTE ON THE MAGNITUDE EQUATION OF WASHINGTON A. G. CATALOGUE,

By R. J. POCKOCK.

In the course of the Astrographic work at Hyderabad the places of the bright stars on the Hyderabad plates are compared with the places given in A. G. Washington. Since the Hyderabad plate constants are obtained by using the Washington places the differences obtained are referred to the mean of the Washington stars.

The differences in the sense Hyderabad — Washington

for the stars from 4^h to 10^h in Zone -17° have been arranged according to magnitude. Since the photographic results may be considered sensibly free of errors depending on the magnitudes of the stars, the mean differences give the magnitude equation of the Washington Catalogue. The results are given in the table — it will be noticed that the numbers of stars vary much from

MAGNITUDE EQUATION OF A. G. WASHINGTON
(Determined from Hyderabad Astrographic Plates)

Magnitude	No. of Stars	Mean Diff. in R. A.	Mean Diff. in Decl.
		Unit 0".003	
Brighter than } 7.1	31	- 65	-100
7.1 to 7.5	32	+ 15	-113
7.6 to 8.0	116	+ 39	- 35
8.1 and 8.2	63	- 16	- 23
8.3 and 8.4	109	+ 97	- 32
8.5 and 8.6	155	+ 24	+ 7
8.7	47	+ 23	- 39
8.8	156	+ 78	+ 7
8.9	96	+ 22	+ 2
9.0	255	+ 65	+ 9
9.1	93	- 14	- 7
9.2	200	- 22	+ 24
9.3	49	- 84	+ 22
9.4	61	-110	- 13
9.5 } 9.6 }	19	-268	+ 89

Nizamiah Observatory, Hyderabad, 1915, December 1.

group to group—a fact which is discussed in the previous paper.

The abnormal result in R. A. for the group 8.1 and 8.2 magnitudes seems real on an inspection of the individual values and not due to a few exceptional cases.

All differences amounting to 3".0 or over were omitted. In declination the errors are large for the bright stars, but beyond 8.4 with the exception of the small group of 8.7 magnitude they are quite small.

In R. A. the errors are larger than in declination and as usual are positive for bright stars and negative for faint stars (zero about 9.0) meaning that the bright stars are observed early relative to the fainter stars.

Excluding the few stars of 9.5 magnitude the maximum correction is only about 0".34, much smaller than has been obtained for the Cambridge Catalogue.*

It should be understood that the above are the *corrections* to the "Standard Co-ordinates," derived from the Washington places. If they are applied direct to the R. A. and declinations, the figures in the last column must be applied to the numerical values of the declinations (*i.e.* regardless of the - signs), this will correct the positions to the mean magnitude (about 9.0) as standard.

*Monthly Notices, R. A. S. 1912, Dec., p. 88 or Oxford Astro. Cat., Vol. VIII, part 1.

OCCULTATIONS OF STARS BY THE MOON.

OBSERVED WITH THE 26-INCH AND 12-INCH EQUATORIALS OF THE U. S. NAVAL OBSERVATORY
[Communicated by Captain J. A. HOOGEWERFF, U. S. Navy, Superintendent.]

Date	Object	Phen.	Wash. Sid. T.	Wash. M.T.	See'g	Instrum't	Pow'r	Obs.	Remarks
			h m s	h m s					
1914 July 8	17 Capricorni	DB	19 5 31.4	12 0 19.6	f	26-inch	250	B	
8	17 Capricorni	RD	20 29 17.8	13 23 52.3	f	26-inch	250	B	Good reappearance.
8	17 Capricorni	DB	19 5 30.6	12 0 18.9	f	12-inch	160	W	
8	17 Capricorni	RD	20 29 17.9	13 23 52.4	f	12-inch	160	W	
15	20 H ¹ Arietis	DB	22 49 51.7	15 16 31.8	f	26-inch	250	H	
15	20 H ¹ Arietis	DB	22 49 53.2	15 16 33.3	f	12-inch	160	W	
17	16 Tauri	DB	21 15 31.2	13 34 38.0	p	26-inch	183	H	Eye and ear.
17	16 Tauri	RD	22 10 9.3	14 29 4.1	f	26-inch	...	H	
17	16 Tauri	RD	22 10 9.2	14 29 4.0	f	12-inch	115	W	Dark limb visible.
17	17 Tauri	RD	21 44 16.2	14 3 15.3	...	12-inch	160	W	Reappearance gradual.
17	γ Tauri	DB	21 32 5.0	13 51 6.0	f	26-inch	183	H	
17	γ Tauri	DB	21 32 4.4	13 51 5.5	p	12-inch	160	W	Very uncertain.
17	γ Tauri	RD	22 28 45.7	14 47 37.5	f	12-inch	115	W	
17	20 Tauri	DB	21 43 34.4	14 2 33.6	f	26-inch	...	H	
17	20 Tauri	RD	22 38 57.0	14 57 47.1	f	26-inch	...	H	Eye and ear.
17	20 Tauri	DB	21 43 33.8	14 2 33.0	p	12-inch	160	W	Good disappearance.
17	20 Tauri	RD	22 38 57.1	14 57 47.2	f	12-inch	115	W	
17	21 Tauri	DB	21 53 16.7	14 12 14.3	f	26-inch	183	H	
17	21 Tauri	DB	21 53 17.3	14 12 14.9	p	12-inch	160	W	Very uncertain.
17	22 Tauri	DB	21 54 24.2	14 13 21.6	f	26-inch	183	H	

OCCULTATIONS OF STARS BY THE MOON — *Continued*

Date	Object	Phen.	Wash. Sid. T.	Wash. M. T.	See'g	Instrum't	Pow'r	Obs.	Remarks	
1914										
July 17	22 Tauri.	DB	^h 21 ^m 54 ^s 25.3	^h 14 ^m 13 ^s 22.7	p	12-inch	160	W	Very uncertain. Eye and ear. This is Star No. 4 of the Yale measurement. See <i>Transactions of the Astronomical Observatory of Yale University</i> , Vol. 1, p. 389.	
17	A.G. Berlin B. 1116	RD	22 27 54.2	14 46 46.1	f	26-inch	183	H		
Aug. 7	λ Aquarii	DB	20 0 41.7	10 57 23.6	p	26-inch	250	H		
7	λ Aquarii	RD	21 19 56.8	12 16 25.6	p	26-inch	183	H		
7	λ Aquarii	DB	20 0 37.4	10 57 19.3	f	12-inch	160	W		
7	λ Aquarii	RD	21 19 56.5	12 16 25.4	f	12-inch	115	W		
7	78 Aquarii	DB	21 38 10.4	12 34 36.4	p	26-inch	250	H		
7	78 Aquarii	DB	21 38 9.4	12 34 35.3	f	12-inch	160	W		
7	78 Aquarii	RD	22 59 39.8	13 55 52.4	f	12-inch	115	W		
30	τ Sagittarii	DD	19 37 30.9	9 3 50.6	f	26-inch	183	H		
30	τ Sagittarii	RB	20 54 46.2	10 20 53.3	f	26-inch	250	H		
30	τ Sagittarii	DD	19 37 31.0	9 3 50.8	p	12-inch	235	W		
30	τ Sagittarii	RB	20 54 52.2	10 20 59.3	p	12-inch	160	W		
Sept. 3	213 B. Aquarii	DD	2 27 50.3	15 37 19.2	p	26-inch	183	H	Time uncertain. Double star	
3	213 B. Aquarii	DD	2 27 55.1	15 37 24.0	p	12-inch	160	W	Uncertain.	
4	316 B. Aquarii	DB	20 28 53.4	9 35 25.2	f	12-inch	160	W		
4	316 B. Aquarii	RD	21 46 5.2	10 52 24.3	f	12-inch	160	W	A little late.	
12	406 B. Tauri	DB	1 20 8.2	13 54 25.0	p	26-inch	250	H		
12	406 B. Tauri	RD	2 30 1.9	15 4 7.3	f	26-inch	183	H		
12	406 B. Tauri	DB	1 20 9.8	13 54 26.6	p	12-inch	160	W		
12	406 B. Tauri	RD	2 30 1.8	15 4 7.2	p	12-inch	115	W	Dark limb visible.	
14	κ Geminorum	DB	1 41 59.5	14 8 20.9	p	26-inch	250	B	Brighter component.	
14	κ Geminorum	DB	1 41 59.3	14 8 20.7	p	12-inch	160	W	Brighter component.	
14	κ Geminorum	RD	2 13 37.3	14 39 53.6	f	12-inch	115	W	Brighter component.	
14	κ Geminorum, comp	RD	2 13 13.7	14 39 30.0	g	26-inch	183	B		
14	κ Geminorum, comp	RD	2 13 13.8	14 39 30.1	f	12-inch	115	W		
Nov. 5	354 B. Tauri	RD	8 5 41.2	17 6 32.6	f	12-inch	160	B	Probably a little late.	
6	415 B. Tauri	RD	23 33 58.6	8 32 17.8	g	26-inch	183	H		
6	415 B. Tauri	DB	22 51 26.2	7 49 52.4	f	12-inch	160	B	Disappearance gradual.	
6	415 B. Tauri	RD	23 33 58.6	8 32 17.8	g	12-inch	160	B		
7	39 Geminorum	RD	0 53 30.5	9 47 40.8	g	26-inch	183	H	Haze.	
7	39 Geminorum	RD	0 53 30.1	9 47 40.4	f	12-inch	115	B	Haze. Star faint.	
11	49 Leonis	DB	4 34 16.2	13 12 6.7	p	12-inch	160	B	Time uncertain. Clouds.	
11	49 Leonis	RD	5 1 36.7	13 39 22.7	f	12-inch	235	B	Haze.	
Dec. 3	136 Tauri	DB	3 30 9.3	10 41 40.3	f	26-inch	250	H	Clouds passing over.	
3	136 Tauri	RD	4 58 10.0	12 9 26.6	f	26-inch	183	H	Clouds passing over. Probably late. Re-appeared through clouds. Star very faint.	
3	136 Tauri	DB	3 30 9.6	10 41 40.6	f	12-inch	160	B	Time uncertain. Thin clouds	
23	21 Piscium	RB	1 8 43.0	7 1 58.9	f	26-inch	250	H	Haze.	
23	21 Piscium	DD	23 45 11.4	5 38 41.0	f	12-inch	235	B	Haze.	
1915	23	21 Piscium	RB	1 8 43.6	7 1 59.5	p	12-inch	160	B	Haze.
Jan. 1	A Geminorum	DB	2 39 49.2	7 57 27.0	f	26-inch	250	H		
1	A Geminorum	RD	3 48 29.2	9 5 55.7	g	26-inch	250	H		
1	A Geminorum	DB	2 39 49.4	7 57 27.2	f	12-inch	160	B	Disappearance gradual.	
1	A Geminorum	RD	3 48 29.8	9 5 56.3	g	12-inch	160	B		
2	η Cancri	DB	12 16 18.0	17 28 25.4	f	26-inch	250	H	Early 0 ^s .2.	
2	η Cancri	DB	12 16 16.8	17 28 24.2	p	12-inch	160	B	Disappearance gradual. Perhaps a little early.	
8	75 Virginis	DB	12 31 9.6	17 19 39.0	g	26-inch	250	H	Clouds and haze. Not a very good observation.	
8	75 Virginis	DB	12 31 9.5	17 19 38.9	f	12-inch	160	B	Disappearance gradual. Uncertain. Thin clouds and haze.	
28	39 Geminorum	RB	2 42 10.6	6 13 38.3	f	26-inch	250	H	Late 0 ^s .5.	

OCCULTATIONS OF STARS BY THE MOON — *Continued*

Date	Object	Phen.	Wash. Sid. T.	Wash. M. T.	Sec'g	Instrum't	Pow'r	Obs.	Remarks
1915			h m s	h m s					
Jan. 28	40 Geminorum	RB	2 40 53.9	6 12 21.8	f	26-inch	250	H	Late 0.5.
29	9 Cancri	DD	8 54 33.6	12 21 4.4	f	26-inch	388	B	Thin clouds.
Feb. 9	58 G. Sagittarii . . .	RD	15 27 8.1	18 9 19.6	f	26-inch	183	H	
9	58 G. Sagittarii . . .	RD	15 27 8.1	18 9 19.6	f	12-inch	235	B	
25	181 B. Geminorum . . .	RB	9 8 3.2	10 48 22.2	p	12-inch	160	B	Thin clouds. Obs. late.
26	η Cancri	DD	7 32 51.8	9 9 30.5	f	26-inch	250	H	Eye and ear.
26	η Cancri	RB	8 30 51.9	10 7 21.1	f	26-inch	250	H	Eye and ear.
26	η Cancri	DD	7 32 51.9	9 9 30.6	f	12-inch	235	B	
26	η Cancri	RB	8 30 57.6	10 7 26.8	p	12-inch	160	B	Unsteady.
26	102 B. Cancri	DD	12 10 55.2	13 46 48.4	p	12-inch	235	B	
26	102 B. Cancri	RB	13 13 5.1	14 48 48.1	p	12-inch	160	B	
26	ϵ Cancri	DD	12 20 25.8	13 56 17.4	p	12-inch	235	B	
26	ϵ Cancri	RB	13 22 41.0	14 58 22.4	p	12-inch	160	B	
Mar. 31	75 Virginis	DB	15 26 9.3	14 51 45.5	f	26-inch	250	H	
31	75 Virginis	RD	16 33 49.8	15 59 14.9	f	26-inch	250	H	Reappearance late 0.2.
31	75 Virginis	DB	15 26 8.3	14 51 44.5	f	12-inch	160	B	Disappearance gradual.
31	75 Virginis	RD	16 33 49.8	15 59 14.9	p	12-inch	160	B	Windy.
Apr. 21	B.D. +23°.1810 . . .	DD	9 33 33.4	7 37 33.2	f	26-inch	250	H	
21	82 Geminorum	DD	9 37 44.2	7 41 43.3	f	26-inch	250	H	
21	82 Geminorum	RB	10 37 12.1	8 41 1.5	p	26-inch	250	H	Late.
21	82 Geminorum	DD	9 37 44.0	7 41 43.2	f	12-inch	235	B	A trifle lazy.
21	82 Geminorum	RB	10 37 6.5	8 40 55.9	f	12-inch	160	B	A little late.
May 5	ι Capricorni	DB	17 21 42.7	14 29 23.2	vp	26-inch	250	H	Gradual.
5	ι Capricorni	RD	18 33 48.8	15 41 17.5	f	26-inch	183	H	Driving clock failed.
5	ι Capricorni	DB	17 21 48.6	14 29 29.1	p	12-inch	160	B	Gradual.
5	ι Capricorni	RD	18 33 48.9	15 41 17.6	f	12-inch	235	B	
21	A Leonis	DD	14 30 23.8	10 35 37.8	p	26-inch	183	H	
21	A Leonis	DD	14 30 23.8	10 35 37.8	f	12-inch	235	B	Thin clouds.
25	75 Virginis	DD	16 42 2.8	12 31 11.5	f	26-inch	250	H	Clouds and haze.
25	75 Virginis	DD	16 42 2.8	12 31 11.5	p	12-inch	235	B	Clouds and haze.
June 4	13 Piscium	DB	20 24 35.7	15 33 48.9	p	26-inch	250	H	Gradual.
4	13 Piscium	RD	21 20 42.4	16 29 46.4	f	26-inch	183	H	Daylight, late 0.3.
4	13 Piscium	RD	21 20 41.8	16 29 45.8	f	12-inch	115	B	Daylight.
19	359 B. Leonis	RB	13 52 39.2	8 3 57.9	p	26-inch	250	H	Late.
19	359 B. Leonis	RB	13 52 42.6	8 4 1.3	p	12-inch	160	B	Somewhat late.
25	43 Ophiuchi	DD	20 16 3.1	14 2 43.5	p	26-inch	250	H	
27	51 Sagittarii	DB	19 32 52.4	13 11 48.0	p	26-inch	250	H	Eye and ear.
27	51 Sagittarii	RD	20 35 22.0	14 14 7.1	p	26-inch	250	H	Eye and ear.
27	51 Sagittarii	DB	19 32 53.2	13 11 48.8	p	12-inch	160	B	Gradual, a little haze.
27	51 Sagittarii	RD	20 35 22.1	14 14 7.8	f	12-inch	160	B	
27	h Sagittarii	DB	19 37 8.6	13 16 3.5	p	26-inch	250	H	Eye and ear.
27	h Sagittarii	RD	20 53 3.9	11 31 46.4	p	26-inch	250	H	Eye and ear.
27	h Sagittarii	DB	19 37 8.8	13 16 3.7	p	12-inch	160	B	Gradual, a little haze.
27	h Sagittarii	RD	20 53 4.3	14 31 46.8	f	12-inch	160	B	
July 23	10 G. Sagittarii	DD	19 9 50.6	11 6 36.1	p	26-inch	388	B	
23	10 G. Sagittarii	RB	19 43 24.7	11 10 1.9	p	26-inch	250	B	
26	Uranus	DB	23 12 30.6	15 26 13.9	p	26-inch	250	B	Uncertain, clouds.
Aug. 21	h Sagittarii	RB	18 27 2.9	8 29 51.2	p	26-inch	250	H	Late 0.5.
21	h Sagittarii	RB	18 27 2.9	8 29 51.2	p	12-inch	160	B	
21	51 Sagittarii	DD	17 11 21.8	7 14 25.5	p	26-inch	183	H	Unsteady.

OCCULTATIONS OF STARS BY THE MOON — *Continued*

Date	Object	Phen.	Wash. Sid. T.	Wash. M. T.	See'g	Instrum't	Pow'r	Obs.	Remarks
1915			h m s	h m s					
Aug. 21	51 <i>Sagittarii</i>	RB	17 52 3.5	7 55 0.6	p	26-inch	250	H	Late 0.5.
21	51 <i>Sagittarii</i>	DD	17 11 21.8	7 14 25.5	p	12-inch	235	B	
21	51 <i>Sagittarii</i>	RB	17 51 59.9	7 54 57.0	p	12-inch	160	B	
23	42 <i>Capricorni</i>	DD	0 0 39.8	13 54 44.7	f	26-inch	250	H	
23	42 <i>Capricorni</i>	RB	0 57 40.3	14 51 35.8	p	26-inch	250	H	Late.
23	42 <i>Capricorni</i>	DD	0 0 39.9	13 54 44.8	g	12-inch	160	B	
23	42 <i>Capricorni</i>	RB	0 57 41.4	14 51 36.9	p	12-inch	160	B	
Sept. 21	197 <i>G. Aquarii</i>	DD	21 38 4.0	9 38 30.9	p	26-inch	250	H	Eye and ear.
21	197 <i>G. Aquarii</i>	RB	22 45 18.1	10 45 34.0	p	26-inch	250	H	Late.
21	197 <i>G. Aquarii</i>	DD	21 38 4.6	9 38 31.5	f	12-inch	115	ECB	A little late.
27	16 <i>Tauri</i>	RD	21 49 26.3	9 26 15.9	f	12-inch	115	H	
27	17 <i>Tauri</i>	DB	20 54 28.3	8 31 26.9	vp	12-inch	160	H	Gradual.
27	<i>q Tauri</i>	DB	21 16 18.8	8 53 13.8	f	12-inch	160	H	Gradual.
27	<i>q Tauri</i>	RD	22 0 8.5	9 36 56.3	f	12-inch	115	H	
27	20 <i>Tauri</i>	DB	21 20 57.6	8 57 51.8	f	12-inch	160	H	Gradual.
27	20 <i>Tauri</i>	RD	22 18 55.0	9 55 39.7	f	12-inch	115	H	
27	22 <i>Tauri</i>	DB	21 38 20.9	9 15 12.3	f	12-inch	115	H	Gradual.
27	22 <i>Tauri</i>	RD	22 25 29.6	10 2 13.3	f	12-inch	115	H	
27	21 <i>Tauri</i>	DB	21 39 45.4	9 16 36.6	f	12-inch	115	H	Gradual.
29	112 <i>B. Aurigæ</i>	DB	0 9 36.8	11 38 11.6	vp	26-inch	250	H	Clouds.
29	112 <i>B. Aurigæ</i>	RD	1 12 48.6	12 41 13.1	f	26-inch	183	H	Clouds.
Oct. 2	<i>B.D. +21° 1812</i>	DE	3 19 21.1	14 35 37.1	f	26-inch	250	H	Gradual.
2	49 <i>B. Cancri</i>	DB	3 28 53.9	14 45 8.3	f	26-inch	250	H	Gradual.
2	49 <i>B. Cancri</i>	RD	4 37 11.1	15 53 14.3	f	26-inch	388	H	
2	49 <i>B. Cancri</i>	DB	3 28 54.2	14 45 8.6	g	12-inch	160	B	Gradual.
2	49 <i>B. Cancri</i>	RD	4 37 11.0	15 53 14.2	g	12-inch	235	B	
13	10 <i>G. Sagittarii</i>	DD	18 54 20.1	5 28 43.8	f	26-inch	183	H	Twilight.
13	10 <i>G. Sagittarii</i>	RB	20 10 43.9	6 44 55.1	p	26-inch	250	H	
13	10 <i>G. Sagittarii</i>	RB	20 10 44.3	6 44 55.5	p	12-inch	160	B	Haze.
15	336 <i>B. Sagittarii</i>	DD	21 5 22.2	7 31 32.6	g	26-inch	183	B	
25	χ <i>Tauri</i>	DB	23 40 10.6	9 26 36.6	p	26-inch	250	H	
25	χ <i>Tauri</i>	DB	23 40 9.2	9 26 35.2	f	12-inch	160	B	Gradual, double star.
25	χ <i>Tauri</i>	RD	0 35 58.3	10 22 15.2	f	12-inch	160	B	Thin clouds.
25	<i>A.G. Cam. (Eng.) 2060</i>	RD	0 36 57.5	10 23 14.2	p	26-inch	250	H	
25	<i>A.G. Cam. (Eng.) 2060</i>	RD	0 36 57.8	10 23 14.5	f	12-inch	160	B	A little late.
30	χ <i>Cancri</i>	DB	8 21 34.8	17 46 55.8	p	26-inch	250	H	
Nov. 24	ϵ <i>Geminorum</i>	DB	0 39 27.3	8 27 46.3	f	26-inch	250	H	
24	ϵ <i>Geminorum</i>	RD	1 11 52.1	9 0 5.8	f	26-inch	250	H	Late.
24	ϵ <i>Geminorum</i>	DB	0 39 27.4	8 27 46.4	g	12-inch	160	B	Good disappearance.
24	ϵ <i>Geminorum</i>	RD	1 11 51.7	9 0 5.4	g	12-inch	235	B	Good reappearance.
25	187 <i>B. Geminorum</i>	DB	3 35 11.6	11 19 5.8	p	26-inch	250	H	
25	187 <i>B. Geminorum</i>	RD	4 27 35.6	12 11 21.3	g	26-inch	183	H	
27	π <i>Cancri</i>	RD	3 1 38.0	10 37 45.9	p	12-inch	235	B	
Dec. 8	ψ <i>Sagittarii</i>	RB	22 24 11.2	5 17 49.6	p	26-inch	183	H	Late
8	ψ <i>Sagittarii</i>	RB	22 24 7.0	5 17 45.4	p	12-inch	160	B	
18	16 <i>Tauri</i>	RB	23 39 13.1	5 53 20.1	p	26-inch	250	H	
18	<i>q Tauri</i>	DD	23 2 25.4	5 16 38.4	p	26-inch	183	H	
18	<i>q Tauri</i>	RB	23 51 25.2	6 5 30.2	p	26-inch	250	H	
18	<i>q Tauri</i>	DD	23 2 25.3	5 16 38.3	p	12-inch	160	B	Windy.
18	<i>q Tauri</i>	RB	23 51 29.6	6 5 34.5	p	12-inch	160	B	Windy, late.

OCCULTATIONS OF THE STARS BY THE MOON—Continued

Date	Object	Phen.	Wash. Sid. T.	Wash. M. T.	See'g	Instrum't	Pow'r	Obs.	Remarks
1915			h m s	h m s					
Dec. 18	20 <i>Tauri</i>	DD	23 5 44.0	5 19 56.4	p	26-inch	183	H	
18	20 <i>Tauri</i>	DD	23 5 44.0	5 19 56.4	p	12-inch	160	B	Windy.
18	21 <i>Tauri</i>	DD	23 29 53.8	5 44 2.3	p	26-inch	183	H	
18	21 <i>Tauri</i>	RB	0 14 41.1	6 28 42.2	p	26-inch	250	H	Late.
18	21 <i>Tauri</i>	DD	23 29 53.8	5 44 2.3	p	12-inch	160	B	Windy.
18	21 <i>Tauri</i>	RB	0 14 39.4	6 28 40.6	p	12-inch	160	B	Windy, late.
18	22 <i>Tauri</i>	DD	23 27 29.2	5 41 38.1	p	26-inch	183	H	
18	22 <i>Tauri</i>	DD	23 27 29.2	5 41 38.1	p	12-inch	160	B	Windy.
18	22 <i>Tauri</i>	RB	0 22 57.0	6 36 56.8	p	12-inch	160	B	Windy, late.
21	ϵ <i>Geminorum</i>	DB	11 8 58.9	17 9 25.1	f	26-inch	250	H	Haze.
21	ϵ <i>Geminorum</i>	RD	12 11 57.8	18 12 13.7	f	26-inch	250	H	Haze.
21	ϵ <i>Geminorum</i>	DB	11 8 58.0	17 9 24.2	f	12-inch	160	B	Gradual, haze.
21	ϵ <i>Geminorum</i>	RD	12 11 57.6	18 12 13.5	f	12-inch	160	B	Haze.
22	<i>B.D. +23° 1744</i>	DB	9 41 35.2	15 38 19.9	p	26-inch	250	H	
22	<i>B.D. +23° 1744</i>	RD	10 55 14.8	16 51 47.4	p	26-inch	250	H	
22	<i>B.D. +23° 1744</i>	DB	9 41 36.3	15 38 21.0	f	12-inch	160	B	Gradual.
22	<i>B.D. +23° 1744</i>	RD	10 55 14.6	16 51 47.2	f	12-inch	160	B	
1916	22 <i>A.G. Berlin, B.3001</i>	RD	10 55 20.0	16 51 52.6	f	12-inch	160	B	
Jan. 14	<i>q Tauri</i>	DD	9 38 52.6	14 5 11.7	p	26-inch	183	H	
14	<i>q Tauri</i>	DD	9 38 52.6	14 5 11.7	f	12-inch	115	B	
14	<i>q Tauri</i>	RB	10 36 48.7	15 2 58.3	p	12-inch	160	B	
14	16 <i>Tauri</i>	DD	9 40 19.7	14 6 38.5	f	12-inch	115	B	
14	16 <i>Tauri</i>	RB	10 12 37.9	14 38 51.4	p	12-inch	160	B	
14	21 <i>Tauri</i>	DD	9 58 52.5	14 25 8.3	p	26-inch	183	H	
14	21 <i>Tauri</i>	DD	9 58 52.4	14 25 8.2	f	12-inch	115	B	
14	20 <i>Tauri</i>	DD	10 0 23.4	14 26 38.9	f	12-inch	115	B	
14	20 <i>Tauri</i>	RB	10 47 48.5	15 13 56.3	vp	12-inch	160	B	
14	22 <i>Tauri</i>	DD	10 2 6.7	14 28 21.9	p	26-inch	183	H	
14	22 <i>Tauri</i>	DD	10 2 6.8	14 28 22.0	f	12-inch	115	B	
18	48 <i>Geminorum</i>	DD	7 24 11.0	11 35 8.5	g	26-inch	250	H	
18	48 <i>Geminorum</i>	DD	7 24 11.0	11 35 8.5	f	12-inch	160	B	Thin clouds.
18	48 <i>Geminorum</i>	RB	8 50 33.1	13 1 16.5	g	12-inch	160	B	
Feb. 14	ϵ <i>Geminorum</i>	DD	1 46 48.4	4 12 31.6	p	26-inch	183	H	Light clouds, Gradual.
14	ϵ <i>Geminorum</i>	RB	2 16 35.8	4 42 14.1	f	26-inch	250	H	A trifle late.
14	ϵ <i>Geminorum</i>	DD	1 46 48.1	4 12 31.3	p	12-inch	235	B	Gradual, daylight.
14	ϵ <i>Geminorum</i>	RB	2 16 35.2	4 12 13.5	f	12-inch	160	B	Daylight.
14	ω <i>Geminorum</i>	DD	12 57 9.3	15 21 2.7	f	26-inch	250	H	Eye and ear.
14	ω <i>Geminorum</i>	DD	12 57 8.9	15 21 2.3	p	12-inch	235	B	

NOTES

DD signifies star disappeared at dark limb of Moon; DB, disappeared at bright limb; RD, reappeared at dark limb; RB, reappeared at bright limb.

Under "Seeing," g = good; f = fair; p = poor; vp = very poor.

Observers: H = A. Hall; B = H. E. Burton; W = C. B. Watts; ECB = E. C. Bower.

All observations were recorded on chronograph, except as noted under "Remarks."

Occluding strips are attached to the eye-pieces of power 160 and power 250, as described in *The Astronomical Journal*, Nos. 673-674, page 13.

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NO. 17

MEASURES OF THE FIFTH SATELLITE OF *JUPITER* MADE AT THE LOWELL OBSERVATORY IN SEPTEMBER, 1915.

At the request of Professor E. E. BARNARD the following measures of his fifth satellite of *Jupiter* have been sent to the *Astronomical Journal* for immediate publication. They are part of an investigation on the internal constitution of *Jupiter* and will eventually appear in a Bulletin and a Memoir on the subject in the publications of the Lowell Observatory.

Their value especially lies in that they were taken purposely at opposition in order to get rid of the differential effect of irradiation, a vitiating factor in such

observations generally, being much greater than is commonly supposed.

V was also tied up to I and II of both of which extensive measures were made during the same examination. The measures follow.

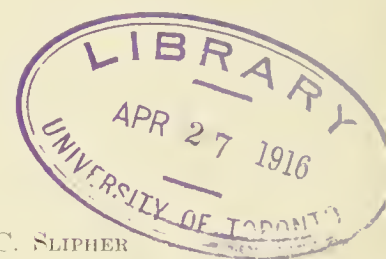
M. S. T. refers to Mountain Standard Time, which is seven hours slow of Greenwich Mean Time.

Ep. = eye-piece. X = coördinate *p*.

PERCIVAL LOWELL, *Director*.

DISTANCE OF SATELLITE V FROM *Jupiter* MEASURED ALONG X

M. S. T., 1915	At True Dist.	At Mean Dist.	Remarks	Observer
Sept. 9 ^d 14 ^h 9 ^m	37'' .73	28'' .77	To near edge. Aper. 24 in. Ep. 25 ^{mm} .	E. C. SLIPHER
	35 .65	27 .19		
12 ^d 2	89 .01	67 .88	To far edge.	
	88 .82	67 .73		
	88 .88	67 .78		
	89 .19	68 .01		
19	87 .90	67 .03		
20	36 .74	28 .02	To near edge.	
	36 .36	27 .73		
26 ^d 2	36 .55	27 .87		
10 13 56	37 .87	28 .86	To near edge. Aper. 24 in. Ep. 25 ^{mm} .	E. C. SLIPHER
	37 .24	28 .39		
	38 .34	29 .23		
	37 .34	28 .46		
	36 .88	28 .12		
	88 .92	67 .78	To far edge.	
	87 .89	67 .00		
	88 .41	67 .39		
	88 .66	67 .59		
14 15	88 .20	67 .24		
12 13 38	37 .62	28 .66	To near edge. Seeing 4. Aper. 24 in. Ep. 25 ^{mm} .	E. C. SLIPHER
	37 .22	28 .36		
42	37 .53	28 .59		



DISTANCE OF SATELLITE V FROM *Jupiter* MEASURED ALONG *X* — *Continued*

M. S. T., 1915	At True Dist.	At Mean Dist.	Remarks	Observer
Sept. 12 ^d 13 ^h 47 ^m	88 ^{''} .74	67 ^{''} .59	To far edge.	
	88 .91	67 .73		
	89 .19	67 .94		
	89 .31	68 .03		
	88 .48	67 .40		
54 ¹ / ₂	88 .95	67 .76		
55 ¹ / ₂	37 .59	28 .63	To near edge.	
	37 .13	28 .28		
	36 .54	27 .83		
	38 .21	29 .10		
	38 .49	29 .32		
	37 .12	28 .28		
14 2	37 .30	28 .41		
	88 .77	67 .62	To far edge.	
	89 .16	67 .92		
8	88 .94	67 .75		
16 8 14	33 .20	25 .27	To near edge.	Aper. 24 in.
	32 .18	24 .50		Ep. 25 ^{mm} .
	32 .20	24 .52		
	31 .93	24 .31		
21 ¹ / ₂	82 .09	62 .50	To far edge.	
	80 .90	61 .59		
	82 .12	62 .52		
	81 .29	61 .89		
26 ¹ / ₂	80 .31	61 .14		
	80 .34	61 .16		
	79 .93	60 .85		
	78 .81	60 .00		
32 ³ / ₄	28 .53	21 .72	To near edge.	
	26 .83	20 .12		
	28 .03	21 .34		
	26 .35	20 .06		
	26 .29	20 .01		
37 ¹ / ₄	76 .16	57 .98	To far edge.	
	78 .01	59 .41		
	77 .23	58 .80		
13 37	38 .29	29 .45	To near edge.	Seeing 7.
	37 .91	28 .87		Aper. 24 in.
	37 .98	28 .92		Ep. 25 ^{mm} .
42 ¹ / ₂	88 .71	67 .54	To far edge.	
	88 .76	67 .58		
	88 .97	67 .71		
	88 .93	67 .70		
47 ¹ / ₄	37 .10	28 .25	To near edge.	
	38 .18	29 .30		
	37 .09	28 .21		

DISTANCE OF SATELLITE V FROM *Jupiter* MEASURED ALONG X — *Continued*

M. S. T., 1915	At True Dist.	At Mean Dist.	Remarks	Observer
Sept. 16 ^d 13 ^h 53 ^m $1\frac{1}{4}$	37''.16	28''.29		
	87 .32	66 .48	To far edge.	
	88 .16	67 .12		
	87 .02	66 .25		
	86 .53	65 .88		
58	87 .57	66 .68		
	35 .36	26 .92	To near edge.	
	34 .25	26 .08		
	35 .28	26 .86		
14 4 ³ ₄	85 .97	65 .45	To far edge.	
	85 .67	65 .23		
18 8 9	31 .61	24 .07	To near edge. Seeing 6.	E. C. SLIPHER
	32 .04	24 .40	Aper. 24 in.	
11 ¹ ₂	81 .39	61 .98	To far edge. Ep. 25 ^{mm} .	
	80 .58	61 .36		
	80 .74	61 .48		

POSITION ANGLE BETWEEN SATELLITE V AND SATELLITE II AND DISTANCE MEASURED ALONG X

M. S. T., 1915	At True Dist.	At Mean Dist.	P. A.	Remarks	Observer
Sept. 18 ^d 8 ^h 21 ^m			62°.05	Seeing 6.	LOWELL
42	122''.44	93''.24		Aper. 24 in.	
	121 .94	92 .85		Ep. 25 ^{mm} .	
57	120 .77	91 .97			
58 ¹ ₂	120 .31	91 .61			
	120 .47	91 .74			
9 7	118 .48	90 .22			
22 ³ ₄			243 .6		E. C. SLIPHER
13 5 ¹ ₄			240 .2	Seeing 8.	LOWELL
6 ³ ₄			241 .4	Aper. 24 in.	
13	72 .84	55 .47		Ep. 25 ^{mm} .	
14	73 .39	55 .88			
14 ³ ₄	73 .07	55 .64			
20			241 .2		
21 ³ ₄			242 .4		

POSITION ANGLE BETWEEN SATELLITE I AND SATELLITE V AND DISTANCE MEASURED ALONG X

M. S. T., 1915	At True Dist.	At Mean Dist.	P. A.	Remarks	Observer
Sept. 18 ^d 13 ^h 31 ^m $1\frac{1}{4}$			247°.7	Seeing 7.	E. C. SLIPHER
32 ¹ ₄			248 .5	Aper. 24 in.	
38 ¹ ₂	61''.49	46''.83		Ep. 25 ^{mm} .	
39 ³ ₄	61 .40	46 .76			

POSITION ANGLE BETWEEN SATELLITE I AND SATELLITE V AND DISTANCE MEASURED ALONG X

(Continued)

M. S. T., 1915	At True Dist.	At Mean Dist.	P. A.	Remarks	Observer
Sept. 18 ^d 13 ^h 41 ^m ₂	61'' .05	46'' .49			
42 ^m _{3/4}	60 .98	46 .44			
43 ^m _{3/4}	60 .95	46 .41			
46 ^m _{1/2}			247° .5		
47 ^m _{3/4}			247 .9		
53 ^m _{1/4}	60 .89	46 .37			
55	60 .20	45 .85			
56 ^m _{1/2}	59 .50	45 .31			
57 ^m _{1/4}	60 .15	45 .80			
58 ^m _{1/4}	60 .41	46 .01			
59	60 .12	45 .78			
14 0	60 .54	46 .10			
1 ^m _{1/4}	60 .18	45 .83			
8 ^m _{1/2}			247 .1		
11 ^m _{1/2}	60 .78	46 .28			
14 ^m _{1/4}	61 .07	46 .51			
15 ^m _{1/4}	61 .55	46 .87			
16 ^m _{3/4}	60 .51	46 .08			
17 ^m _{1/2}	61 .15	46 .57			
18 ^m _{1/4}	61 .19	46 .60			
19	60 .69	46 .21			
20	61 .40	46 .76			
22 ^m _{1/2}			247 .6		
25			246 .9		

POSITION ANGLE BETWEEN SATELLITE V AND SATELLITE II AND DISTANCE MEASURED ALONG X

M. S. T.	At True Dist.	At Mean Dist.	P. A.	Remarks	Observer
Sept. 19 ^d 13 ^h 24 ^m	81'' .41	62'' .01		Seeing 7.	E. C. SLIPPER
26 ^m _{1/2}	80 .65	61 .43		Aper. 24 in.	
27 ^m _{1/2}	80 .04	60 .97		Ep. 25 mm	
28	80 .19	61 .08			
29	80 .54	61 .35			
31			248° .7		
32 ^m _{1/2}			249 .7		
37	78 .16	59 .53			
38 ^m _{1/2}	78 .98	60 .16		Better; well	
39 ^m _{1/2}	78 .62	59 .88		seen on wire.	
41	79 .03	60 .20			
42 ^m _{1/2}	78 .05	59 .15			
46	79 .22	60 .31			LOWELL
48	78 .59	59 .86		Satellite at	
50 ^m _{1/4}	77 .95	59 .38		limit of visi-	
55 ^m _{1/4}	78 .51	59 .80		bility.	
58	77 .39	58 .94			
11 41.			248 .3		

POSITION ANGLE BETWEEN SATELLITE I AND SATELLITE V AND DISTANCE MEASURED ALONG X

M. S. T., 1915	At True Dist.	At Mean Dist.	P. A.	Remarks	Observer
Sept. 27 ^d 12 ^h 14 ^m .6			76°.0	Seeing 6-7. Aper. 24 in. Ep. 25 ^{mm} .	E. C. SLIPHER
18 .6	22''.32	17''.08			
19 .8	22 .06	16 .88			
21 .4	21 .43	16 .39			
22 .7	21 .15	16 .18			
23 .9	20 .59	15 .75			
25 .8	20 .05	15 .34			
28 .0	19 .00	14 .54			
29 .4	19 .52	14 .93			
32 .6			78 .0		
36 .3			79 .2		
39 .8	16 .89	12 .92			
41 .1	16 .40	12 .55			
42 .8	15 .53	11 .89			
44 .1	15 .66	11 .98			
45 .3	14 .95	11 .44			
46 .7	14 .96	11 .44			
48 .0	14 .94	11 .43			
50 .0	14 .77	11 .30			
50 .7	14 .62	11 .18			
53 .2	14 .91	11 .41			
57 .1			83 .0		
59 .0			83 .2		

NOTE. — On August 30th picked up the satellite as a strange object while exploring the region about *Jupiter*, detecting its motion during the interval. It was seen without a bar in the eye-piece and with the planet in the field. On September 16th it was seen with an aperture of fifteen inches. — E. C. S.

Lowell Observatory, March, 1916.

A NEW VARIABLE, *B.D.* + 69.184,

By RAYMOND S. DUGAN.

I have recently completed a series of photometric observations of the eclipsing binary *RZ Cassiopeie*. During the early part of the series I used *B.D.* +69.184 = *a* as a comparison star, but later substituted *B.D.* +69.181 = *b*, as the measures with the former were apparently somewhat discordant. The observations with *b* are sufficiently numerous to define the light curve of *RZ Cassiopeie*. With the aid of this light curve it was then possible to find the variations in the light of *a* on the nights when it was observed. The number of nights when the star was of average brightness is greater than the number of nights when it was either brighter or fainter than the average. On two

nights, 36.0 days apart, it was observed descending from a brightness some three or four tenths of a magnitude greater than the minimum. Periods approximating six, three, or two days are indicated by a considerable portion of the data, but no period has been found which satisfies all the data. The average brightness of *a* is about 8.8 magnitude. In the following table I give, for nights when the light was nearly constant or when the observations were too few to surely show a change, the average of the Greenwich geocentric times of observation, the average magnitude-difference between *a* and *b*, and the number of observations. The observations made on the two nights referred to above are given in full.

J. D.	$b - a$ ^m	No.	J. D.	$b - a$ ^m	J. D.	$b - a$ ^m	No.
9126.6	+0.33	10	0240.525	+0.76	0276.553	+0.79	
9151.6	.43	5	.531	.74	.560	.79	
9153.6	.45	12	.537	.75	.569	.72	
9322.7	.43	12	.543	.76	.575	.72	
9323.7	.37	5	.549	.81	.611	.57	
9336.7	.48	13	.587	.57	.618	.56	
9359.7	.34	12	.599	.59	.627	.58	
9447.6	.45	14	.608	.53	.637	.36	
9475.7	.41	2	.614	.68	.647	.39	
9680.7	.40	19	.628	.61	.655	.30	
9715.5	.35	4	.634	.75	.664	.32	
9716.6	.35	14	.658	.68	0276.684	.36	
9721.6	.47	11	.665	.69	0443.6	.48	2
0227.55	.53	3	.680	.60	0447.5	.33	1
0233.65	+0.50	10	.688	.67	0451.6	.57	3
			.694	.55	0453.5	.41	4
			0240.701	+0.48	0457.5	.47	2
					0503.6	.37	6
					0624.7	+0.45	2

Princeton University Observatory, March 15, 1916.

OBSERVATIONS OF THE DOUBLE COMETS 1915 e (TAYLOR) AND 1915 a (MELLISH),

By E. E. BARNARD.

Observing TAYLOR's comet on 1916, February 9, with the large telescope, it was found that the comet was distinctly double with a difference of about one magnitude between the components. The smaller one was north and slightly following. Though their nebulosities mingled, each was a distinct comet in the center of which was a small condensation—in the larger one this was almost stellar. The magnitudes were estimated 14 and 15, but these may be too faint on account of moonlight. They were later estimated 13th magnitude and 14th magnitude.

Following are the measures of these objects:

C. S. T.	P. A.	C. S. T.	Dist.
7 ^h 33 ^m 39 ^s	21° 38' (4)	7 ^h 36 ^m 46 ^s	8'' .76 (4)
7 54 12	20 .95 (3)	7 57 44	9 .23 (4)
9 48 54	21 .46 (6)	9 54 39	9 .52 (4)

Apparently there was an increase in the distance, but the last measure was made under difficulties on account of increasing haze, which prevented any later observations.

THE COMPANIONS OF COMET 1915 a (Mellish)

The following additional measures of the components of this comet (see *A. J.* Vol. XXIX, p. 40, 1915) were obtained with the large telescope, on 1915, May 23.

A C			
C. S. T.	P. A.	C. S. T.	Dist.
14 ^h 30 ^m	291° .6 (5)	14 ^h 43 ^m	58'' .9 (6)
15 0	290 .9 (3)	15 5	58 .5 (4)
A B			
C. S. T.	P. A.	C. S. T.	Dist.
14 ^h 57 ^m	294° .5 (2)	14 ^h 53 ^m	35'' .5 (4)

A was about five times brighter than C, and was slightly condensed, but with no nucleus—it was more definite on the preceding side. Sometimes B appeared to have a nucleus. They were all involved in the common nebulosity of A.

The comet was very low near the southern horizon, among clouds, and the seeing was very bad.

Yerkes Observatory, Williams Bay, Wisconsin,
1916, February 10.

OBSERVATIONS OF COMETS,

MADE WITH THE FILAR MICROMETER OF THE 11-INCH EQUATORIAL AT SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.,

By HARRIET W. BIGELOW.

Greenwich Mean Time	*	Comp.	$\begin{smallmatrix} \nearrow & \nwarrow \\ \Delta\alpha & \Delta\delta \end{smallmatrix}$		$\begin{smallmatrix} \nearrow & \nwarrow \\ \alpha & \delta \end{smallmatrix}$'s Apparent		Log $p\Delta$	
			$\Delta\alpha$	$\Delta\delta$	α	δ	α	δ
Comet 1913 a (SCHAUMASSE)								
1913 h m s			m s	' "	h m s	° ' "		
May 10 18 50 35	1	15 7	+2 58.80	+2 7.2	20 39 46.05	+13 41 44.0	9.544	0.678
14 19 37 25	2	30 10	+0 3.72	-6 29.5	20 20 32.43	+18 5 50.9	9.371	0.587
June 5 15 11 13	3	22 6	+0 49.74	+4 0.2	16 15 27.81	+41 36 30.8	9.170	9.350
Comet 1915 a (MELLISH)								
1915								
Feb. 17 20 37 53	4	18 9	+0 20.27	-5 34.0	17 12 7.78	+ 2 37 0.6	9.546	0.757
18 21 32 39	6	20 9	-1 54.54	-6 7.9	17 13 26.91	+ 2 32 23.1	9.443	0.753
19 21 45 35	7	18 8	-3 20.70	+2 54.0	17 14 43.54	+ 2 27 50.1	9.404	0.753
20 21 46 22	8	19 8	-2 4.52	-1 42.7	17 15 59.75	+ 2 23 13.3	9.395	0.753
21 21 23 54	9	20 8	-0 49.73	-6 20.7	17 17 14.57	+ 2 18 35.3	9.444	0.755
Mar. 8 20 23 54	10	19 8	-1 18.42	-5 0.2	17 36 6.19	+ 1 5 49.6	9.486	0.764
9 20 39 56	11	20 8	-1 47.57	-5 47.8	17 37 22.33	+ 1 0 25.7	9.446	0.763
11 20 36 28	12	19 10	-1 6.16	-1 41.0	17 39 52.36	+ 0 49 27.3	9.442	0.765
23 19 0 25	13	20 9	+0 29.74	+1 36.8	17 54 45.37	- 0 26 43.1	9.554	0.772
24 19 26 33	14	20 9	+1 42.58	+0 21.4	17 56 1.10	- 0 34 24.1	9.511	0.773
26 20 7 58	15	15 7	-5 5.36	-2 10.2	17 58 32.35	- 0 50 21.4	9.413	0.776
Apr. 12 18 36 27	16	24 9	+2 37.56	-3 4.8	18 19 47.69	- 3 58 12.4	9.511	0.791
13 18 20 2	17	20 8	+1 13.79	-5 9.6	18 21 3.76	- 4 13 24.8	9.533	0.791
19 18 11 36	18	23 9	+0 2.20	-2 27.5	18 28 54.03	- 6 1 8.9	9.523	0.800
20 18 42 41	19	20 10	-1 54.35	+7 15.5	18 30 16.26	- 6 22 27.5	9.460	0.806
May 6 18 14 11	20	20 9	+2 28.96	-7 57.2	18 54 26.92	-15 6 11.2	9.449	0.846
9 18 31 14	21	20 8	+1 47.96	+0 56.6	19 0 6.25	-17 44 57.2	9.394	0.862
10 18 30 8	22	19 9	-0 5.75	+8 22.9	19 2 6.82	-18 43 51.2	9.394	0.866

Mean Places of Comparison Stars for the Beginning of the Year

*	α	Red. to App. Pl.	δ	Red. to App. Pl.	Authority
1	20 ^h 36 ^m 46 ^s .10	+1 ^s .15	+13° 39' 49".2	-12 ^s .4	A. G. Leipzig 1 8076
2	20 20 27 .39	+1 .32	+18 12 34 .2	-13 .8	A. G. Berlin A 8158.
3	16 14 35 .23	+2 .84	+41 32 40 .7	-10 .1	A. G. Bonn 10450.
4	17 11 46 .95	+0 .56	+ 2 42 51 .6	-17 .0	Comp. with 5 $\Delta\alpha$ +10 ^s .15 $\Delta\delta$ -3' 38".2
5	17 11 36 .80	+0 .56	+ 2 46 29 .8	-17 .0	A. G. Albany 5706.
6	17 15 20 .88	+0 .57	+ 2 38 48 .0	-17 .0	A. G. Albany 5730.
7	17 18 3 .65	+0 .59	+ 2 25 13 .0	-16 .9	A. G. Albany 5752.
8	17 18 3 .65	+0 .62	+ 2 25 13 .0	-17 .0	A. G. Albany 5752.
9	17 18 3 .65	+0 .65	+ 2 25 13 .0	-17 .0	A. G. Albany 5752.
10	17 37 23 .60	+1 .01	+ 1 11 6 .8	-17 .0	A. G. Albany 5881.
11	17 39 8 .87	+1 .03	+ 1 6 30 .5	-17 .0	A. G. Albany 5890.
12	17 40 57 .44	+1 .08	+ 0 51 25 .3	-17 0	A. G. Nicolajew 4400.
13	17 54 14 .25	+1 .38	- 0 28 3 .8	-16 .1	A. G. Nicolajew 4461.
14	17 54 17 .10	+1 .42	- 0 34 29 .4	-16 .1	A. G. Nicolajew 4462.
15	18 3 36 .28	+1 .43	- 0 47 55 .5	-15 .7	A. G. Nicolajew 4497.

Mean Places of Comparison Stars for the Beginning of the Year — Continued

*	α	Red. to App. Pl.	δ	Red. to App. Pl.	Authority
16	18 17 8.23	+1.90	- 3 54 54.1	-13.5	A. G. Strassburg 6141.
17	18 19 48.05	+1.92	- 4 8 2.0	-13.2	A. G. Strassburg 6161.
18	18 28 49.76	+2.07	- 5 58 29.8	-11.6	A. G. Wien-Ottakring 6218.
19	18 32 8.51	+2.10	- 6 29 31.9	-11.1	A. G. Wien-Ottakring 6233.
20	18 51 55.35	+2.61	-14 58 7.9	- 6.1	A. G. Washington 7005.
21	18 58 15.56	+2.73	-17 45 19.2	- 4.6	A. G. Washington 7050.
22	19 2 9.81	+2.76	-18 52 10.1	- 4.0	Argentine <i>Gen. Cat.</i> 26165.

THE APEX OF SOLAR MOTION DERIVED FROM THE
ALBANY ZONE CATALOGUE,

BY BENJAMIN BOSS.

The as yet unpublished zone observations taken at the Dudley Observatory from 1896 to 1900, principally of stars included in the zone -23° to -37° , have furnished the means of deriving a number of large proper-motions. While there is no pretence of a refined degree of accuracy, the proper-motions, on account of their size, will not deviate greatly from the true proper-motions. More than 700 proper-motions of $10''$ or greater were derived.

The zone was divided into twenty-four trapezia, each trapezium containing all the stars in an hour of right-ascension. For each group conditional equations were formed for the separate stars by the equations

$$X \sin \alpha - Y \cos \alpha = \mu \cos \delta$$

$$\text{and } X \cos \alpha \sin \delta + Y \sin \alpha \sin \delta - Z \cos \delta = \mu'.$$

A mean of the equations of condition was formed for each group. There are an average of twenty-eight stars in a group. Equal weight was given to each conditional equation when solving for the normal equation.

The following normal equations resulted:

$$\begin{aligned} +11.95 X & - 0.04 Y & = & +0.21 \\ - 0.04 X & +11.92 Y & = & -1.91 \end{aligned}$$

$$\begin{aligned} + 2.92 X & - 0.02 Y & - 0.14 Z & = +0.06 \\ - 0.02 X & + 2.94 Y & + 0.00 Z & = -0.37 \\ - 0.14 X & + 0.00 Y & +17.95 Z & = +1.97 \end{aligned}$$

or combining the two to secure greater accuracy

$$\begin{aligned} +14.87 X & - 0.06 Y & - 0.14 Z & = +0.27 \\ - 0.06 X & +14.86 Y & + 0.00 Z & = -2.28 \\ - 0.14 X & + 0.00 Y & +17.95 Z & = +1.97 \end{aligned}$$

Solving the latter normal equation

$$X = +.019, \quad Y = -.153, \quad Z = +.110$$

Then from the relations

$$\begin{aligned} X &= M \cos D \cos A \\ Y &= M \cos D \sin A \\ Z &= M \sin D \end{aligned}$$

A , D and M are computed, where M represents the mean parallactic motion, and A and D the coördinates of the solar apex

$$A = 277^{\circ}.0 \quad D = +35^{\circ}.4 \quad M = 19''.0$$

The result is surprisingly good in view of the fact that all the stars lie in a comparatively narrow zone.

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NO. 18

GRAPHICAL SOLUTION OF THE POSITION OF A BODY IN AN ELLIPTIC ORBIT,

By THORNTON C. FRY.

1. INTRODUCTION. The three equations,

$$(1) \quad M = E - e \sin E,$$

$$(2) \quad \frac{r}{a} = 1 - e \cos E,$$

$$(3) \quad \cos E - e = (1 - e \cos E) \cos v,$$

define the position of a body moving in an elliptic orbit, when M , the mean anomaly, and e , the eccentricity, are known. Equation (1) is known as KEPLER's equation, and its solution presents practical difficulties. The purpose of this paper is to explain a simple graphical method of finding the quantities $\frac{r}{a}$ and v , when too great accuracy is not required. The method will be immediately recognized as applicable in double star computations, for example, in which the accuracy furnished by the graphical solution is satisfactory.

The foundation for the work is the theory of Nomography, for which reference may best be made to D'OCAGNE, *Traité de Nomographie*. This work presents as an illustration a nomographic solution of KEPLER's equation. The solution is, however, of little practical value, at least in the form in which it is presented in the figure which accompanies the illustration.

Another nomographic solution of KEPLER's equation is that of V. LÁSKA.* For this chart the author claims an accuracy of about a fifth of a minute of arc, but the solution requires several auxiliary computations, which make its use somewhat tedious. The value of e is limited to be less than 0.25.

Since the construction of the present chart requires an extension on the common theory of Nomography which the author has been unable to find in the literature, it

will be necessary to carry out the solution without further reference to texts on the subject.

A determinant which will be met in the work is

$$\begin{vmatrix} x_1 & y_1 & 0 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The expansion of (4) is

$$y_3 - y_2 = \frac{y_1}{x_1} (x_3 - x_2).$$

That is, (4) states that the slope of the line joining (x_2, y_2) and (x_3, y_3) is $\frac{y_1}{x_1}$.

2. THEORY OF THE CHARTS. The equations (1), (2) and (3) may be written, respectively, in the forms:

$$\begin{vmatrix} 1 & e & 1 \\ 0 & M & 1 \\ \sin E & E & 1 + \sin E \end{vmatrix} = 0, \quad (5)$$

$$\begin{vmatrix} 1 & e & 1 \\ 0 & \frac{r}{a} & 1 \\ \cos E & 1 & 1 + \cos E \end{vmatrix} = 0, \quad (6)$$

$$\begin{vmatrix} 1 & e & 1 \\ \cos E & 1 & 1 \\ -1 & 1 & -\cos v \end{vmatrix} = 0. \quad (7)$$

This may be verified by actual expansion.

If the first column of each of these determinants be subtracted from the last, the following set of determinants results:

*A.N., Nr. 4770: "Ein Nomogramme zur Auflösung der Kepler'schen Gleichung."

$$(8) \quad \begin{vmatrix} 1 & e & 0 \\ 0 & M & 1 \\ \sin E & E & 1 \end{vmatrix} = 0,$$

$$(9) \quad \begin{vmatrix} 1 & e & 0 \\ 0 & \frac{r}{a} & 1 \\ \cos E & 1 & 1 \end{vmatrix} = 0,$$

$$(10) \quad \begin{vmatrix} 1 & e & 0 \\ \cos E & 1 & 1 - \cos E \\ -1 & 1 & 1 - \cos v \end{vmatrix} = 0.$$

If the third row of (7) be subtracted from the second, and the resulting determinant rearranged by elementary operations, another determinant

$$(11) \quad \begin{vmatrix} 1 & e & 0 \\ -\cos v & 1 & 1 + \cos v \\ -1 & 1 & 1 + \cos E \end{vmatrix} = 0,$$

results, which is, of course, equivalent to (10).

If E and v are not zero, (10) may be written in the form

$$(12) \quad \begin{vmatrix} 1 & e & 0 \\ \frac{\cos E}{1 - \cos E} & \frac{1}{1 - \cos E} & 1 \\ \frac{-1}{1 - \cos v} & \frac{1}{1 - \cos v} & 1 \end{vmatrix} = 0.$$

If E and v are not π , (11) may be written in the form

$$(13) \quad \begin{vmatrix} 1 & e & 0 \\ \frac{-\cos v}{1 + \cos v} & \frac{1}{1 + \cos v} & 1 \\ \frac{-1}{1 + \cos E} & \frac{1}{1 + \cos E} & 1 \end{vmatrix} = 0.$$

Define eight curves by the equations:

$$(14) \quad \begin{matrix} x_1 = 0 \\ y_1 = M \end{matrix} \quad \left. \vphantom{\begin{matrix} x_1 = 0 \\ y_1 = M \end{matrix}} \right\}$$

$$(15) \quad \begin{matrix} x_2 = \sin E \\ y_2 = E \end{matrix} \quad \left. \vphantom{\begin{matrix} x_2 = \sin E \\ y_2 = E \end{matrix}} \right\}$$

$$(16) \quad \begin{matrix} x_3 = 0 \\ y_3 = \frac{r}{a} \end{matrix} \quad \left. \vphantom{\begin{matrix} x_3 = 0 \\ y_3 = \frac{r}{a} \end{matrix}} \right\}$$

$$(17) \quad \begin{matrix} x_4 = \cos E \\ y_4 = 1 \end{matrix} \quad \left. \vphantom{\begin{matrix} x_4 = \cos E \\ y_4 = 1 \end{matrix}} \right\}$$

$$\left. \begin{matrix} x_5 = \frac{\cos E}{1 - \cos E} \\ y_5 = \frac{1}{1 - \cos E} \end{matrix} \right\} \quad (18)$$

$$\left. \begin{matrix} x_6 = \frac{-1}{1 - \cos v} \\ y_6 = \frac{1}{1 - \cos v} \end{matrix} \right\} \quad (19)$$

$$\left. \begin{matrix} x_7 = \frac{-\cos v}{1 + \cos v} \\ y_7 = \frac{1}{1 + \cos v} \end{matrix} \right\} \quad (20)$$

$$\left. \begin{matrix} x_8 = \frac{-1}{1 + \cos E} \\ y_8 = \frac{1}{1 + \cos E} \end{matrix} \right\} \quad (21)$$

These eight curves will be found sufficient for the solution of our problem, and the remainder of the paper will be devoted mainly to discussing them. For the present we shall confine our attention to (14) and (15).

If E be given a value $\mathfrak{E}$, (15) defines a pair of values x_2 and y_2 , and therefore a point (x_2, y_2) which we shall denote by $\mathfrak{P}_E$. In general a different point $\mathfrak{P}_E$ is determined for each value of E , and the aggregate of such points is a curve, $\mathfrak{C}$. Suppose that, when the point $\mathfrak{P}_E$ is marked, the value $\mathfrak{E}$ is written beside it, and that this is done for a sequence of values of E differing by a convenient "tabular difference." The result is a *scale* of the quantity E , arranged along the curve $\mathfrak{C}$.

That is, the equations (15) may be thought of as the equations of a *scale*. We shall suppose in what follows that this scale has been constructed.

Similarly, if M be given a value $\mathfrak{M}$, (14) defines a point $\mathfrak{P}_M(x_1, y_1)$, and a different point will be determined for each value of M , although, in this special case, the x -coordinate does not change from point to point. Hence (14) defines a scale of values of M . Let this scale also be constructed.

Let $\mathfrak{r}$, $\mathfrak{M}$ and $\mathfrak{E}$ be any set of values satisfying (8). Then (8) becomes, in virtue of (14) and (15),

$$\begin{vmatrix} 1 & \mathfrak{r} & 0 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0. \quad (22)$$

an equation which states that the slope of the line join-

ing $\mathfrak{P}_E$ and $\mathfrak{P}_M$ is $\mathfrak{r}$. Suppose values $M = \mathfrak{M}$, $e = \mathfrak{e}$ are given and the value $E = \mathfrak{E}$ is desired which, together with $\mathfrak{M}$ and $\mathfrak{e}$, satisfies equation (1). Through the point $\mathfrak{P}_M$ on the scale (14) draw a line of slope $\mathfrak{e}$. Equation (22) tells us that it will intersect scale (15) in the point $\mathfrak{P}_E$ to which the desired number $\mathfrak{E}$ corresponds. This number may be read off. This affords, therefore, a method of solution of the equation (1).

A similar chart may be constructed for (2) by using (16) and (17); for (3) by using (18) and (19), or by using (20) and (21). Since the first rows of (8), (9), (12) and (13) are identical, the direction corresponding to a value $e = \mathfrak{e}$ is the same on each chart. This will be found to be a very fortunate situation.

3. THE SCALES. Having shown that it is possible to construct a chart solving each of the equations (1), (2) and (3), we turn to a discussion of the scales which must be constructed in each of these charts.

In (14) $x_1 = 0$ is evidently the equation of the Y -axis. The scale is therefore a straight line. From

$$\begin{aligned} y_1 &= M \\ dy_1 &= dM. \end{aligned}$$

we obtain

If a set of values of M with a constant tabular difference dM be chosen, so that $dM = C$, we have

$$dy_1 = C.$$

That is, the divisions of the scale are equal in length. Such a scale is called *uniform*.

Elimination of E from equations (15) gives

$$x_2 = \sin y_2,$$

which shows that the scale is a sinusoid running along the Y -axis. Since

$$dy_2 = dE,$$

and is constant if the tabular difference of E is constant, it follows that the points of division of this scale are at the same *vertical* distance above one another. If the scale were distributed along a straight line, it would be uniform.

If in equations (14) and (15) $M = E$, then $y_1 = y_2$. That is, the numbers on the E -scale stand horizontally opposite the same numbers on the M -scale.

(16) is a set of equations exactly like (14), and represents a uniform scale on the Y -axis.

In (17) $y_4 = 1$ is the equation of a line parallel to the X -axis and one unit above it. The point on this scale numbered E is at the distance $\cos E$ from the Y -axis. Since $|\cos E| \leq 1$, this scale, for $0 \leq E \leq \pi$, is two units long. The scale is not uniform. The distance between

successive divisions is proportional to $d(\cos E) = -\sin E dE$, and is variable. Such a scale is technically called a *sinusoidal scale*.

We pass (18) and (19) for a time and consider (20) and (21) instead. From (20)

$$y_7 = x_7 + 1. \quad (23)$$

Also,

$$\begin{aligned} ds_7 &= \sqrt{(dx_7)^2 + (dy_7)^2} \\ &= \sqrt{2} \frac{\sin v}{(1 + \cos v)^2} dv \\ &= \frac{\sqrt{2}}{2} \tan \frac{v}{2} \sec^2 \frac{v}{2} dv \\ &= \frac{\sqrt{2}}{2} d\left(\tan^2 \frac{v}{2}\right). \end{aligned}$$

Hence, if s_7 be measured from the point numbered $v = 0$,

$$s_7 = \frac{\sqrt{2}}{2} \tan^2 \frac{v}{2}. \quad (24)$$

This is a linear scale, as shown by (23). It is not uniform. Equation (24) gives the distance from the point numbered $v = 0$ to that to which is affixed any other value of v , and therefore affords an easy method of constructing the scale.

Equation (21) gives

$$y_8 = -x_8. \quad (25)$$

$$s_8 = \frac{\sqrt{2}}{2} \tan^2 \frac{E}{2}. \quad (26)$$

This is also a linear scale. (26) shows that it is identical with (20) in appearance, although different in position.

From (20) and (21) it follows that, if $v = E$, $y_7 = y_8$. That is, any number on the v -scale stands horizontally opposite the same number on the E -scale.

In the solution of our problem it is necessary to use values of v and E between 0° and 180° . Our scales of v and E should be of service for this range of values. But substituting 180° for v or E in (24) or (26) gives $s = \infty$. Hence it is impossible to use these scales throughout the desired range of values. This difficulty may be overcome by the following device.

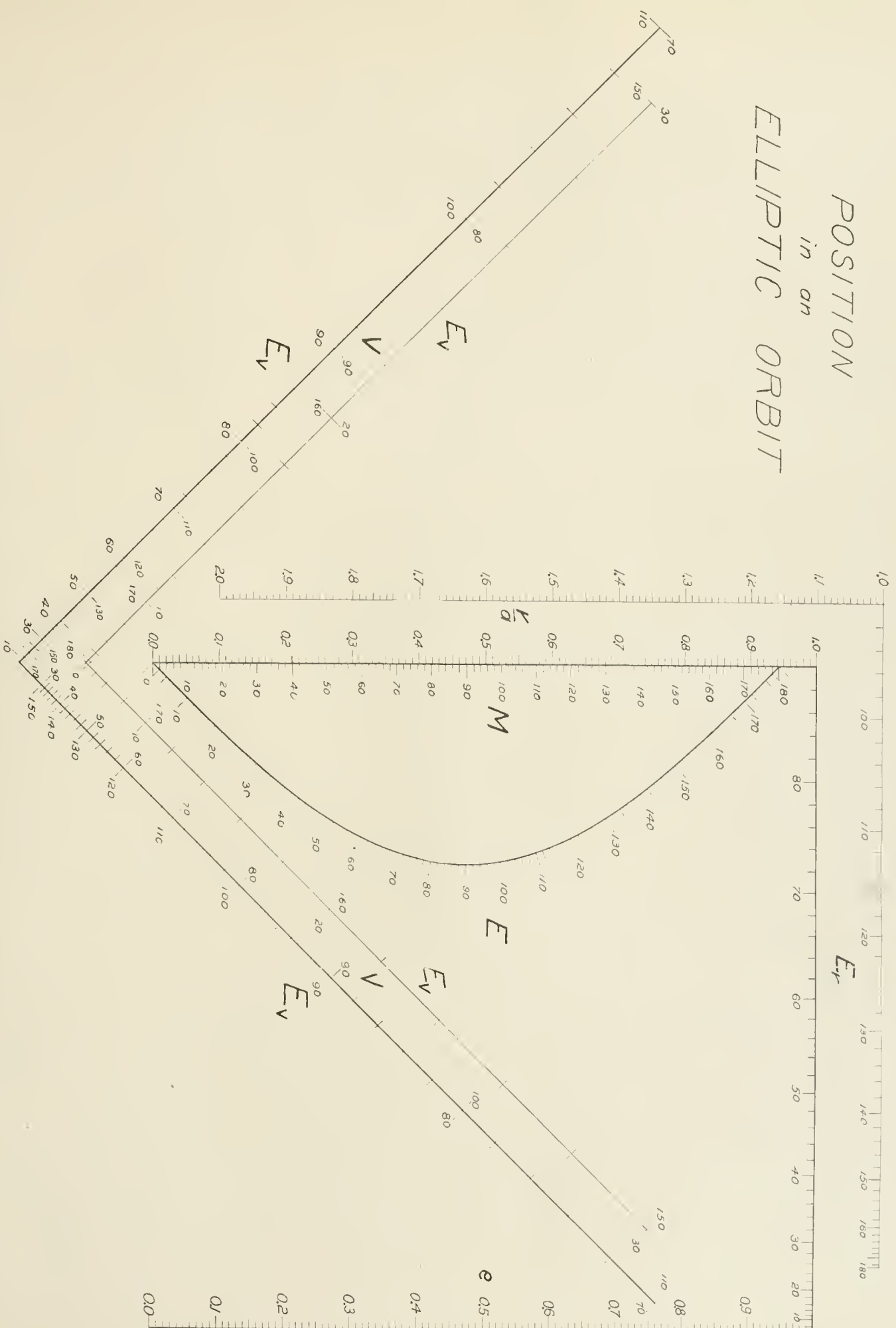
Let v_1 and E_1 satisfy (11) for a given value of e , and set

$$\begin{aligned} v_2 &= \pi - E_1 \\ E_2 &= \pi - v_1. \end{aligned}$$

(11) becomes

$$\begin{vmatrix} 1 & e & 0 \\ -\cos v_1 & 1 & 1 + \cos v_1 \\ -1 & 1 & 1 + \cos E_1 \end{vmatrix} \quad (27)$$

POSITION in an ELLIPTIC ORBIT



measure used in equations (14) and (15) must be the same; but a glance at the figure shows that it need not be the same as that used in equations (29). It is only necessary that Oa and af be two sides of a square.

Fig. 2 represents the chart of equation (2). Since

$$0 < r < 2a,$$

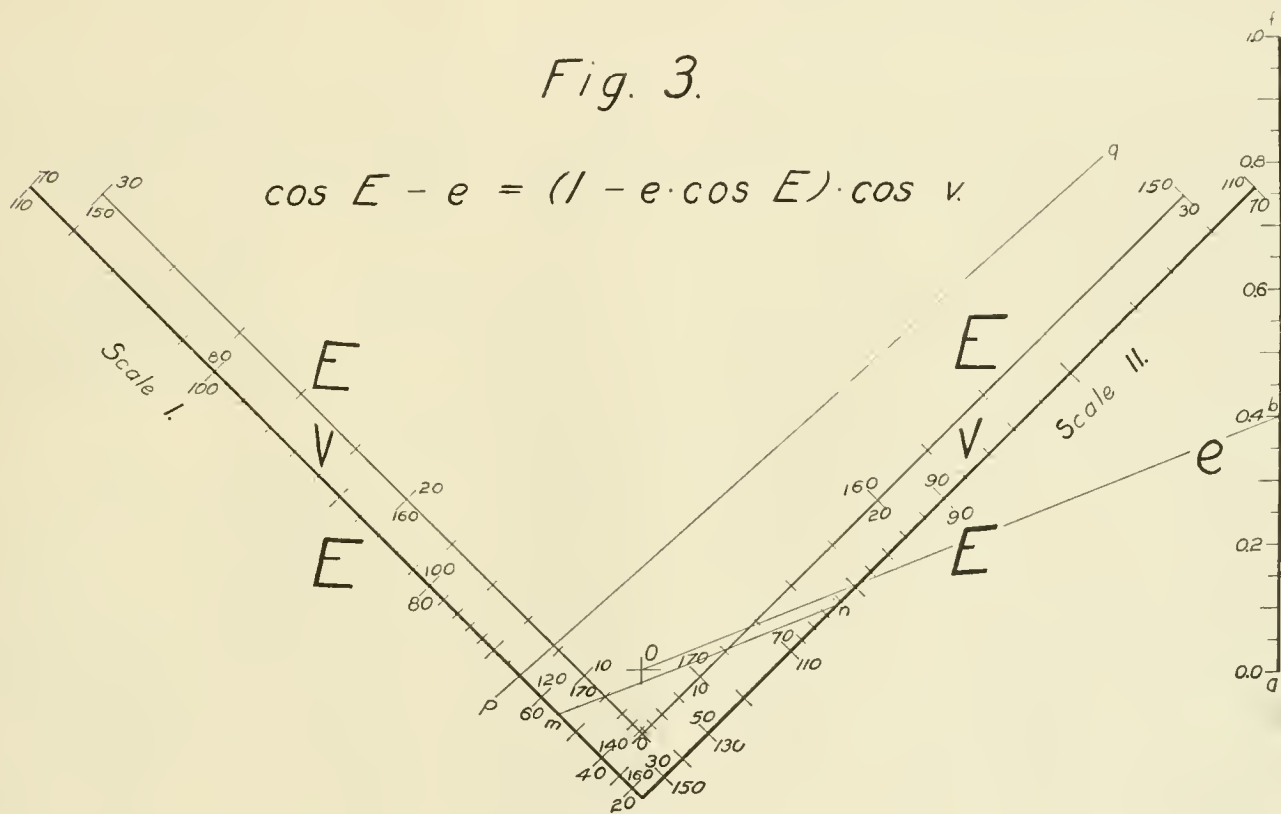
$$0 < \frac{r}{a} < 2,$$

and the $\frac{r}{a}$ -scale need be only two units long. As we have

already seen, the E -scale corresponding is but two units long. The lines Ob and gh represent the computation of $\frac{r}{a}$ for $e = 0.4$ and $E = 102^\circ.4$. The result is $\frac{r}{a} = 1.085$.

Since e is always positive, a value of E on the portion AD always corresponds to a value of $\frac{r}{a}$ on the portion OA , and a value of E on AC corresponds to a value of $\frac{r}{a}$ on AB . Hence the portion CAB of the diagram may be rotated through 180° into the position $C'A'B'$, since

Fig. 3.



this rotation carries any line into a parallel line — e.g., gh into $g'h'$.

Fig. 3 represents the chart of equation (3). O is the origin of the drawing. The chart is not altered by translating the E - and v -scales so that their intersection lies at any other point than that shown. They are constructed to read angles from 0° to 110° . The scales of supplementary angles are also shown. Because of the congestion of the scales in the neighborhood of the point of intersection, an auxiliary scale with a very much larger unit of measure has been constructed to read the first 30° . This is shown in Fig. 3 by means of a lighter line.

For convenience, numbers representing values of v have been arranged *between* the two lines, those representing values of E outside them.

The figure shows the computation of v for $E = 102^\circ.4$, $e = 0.4$. The result is $v = 124^\circ.5$.

Now it is evident that, if e be very large, there are values of v for which these rearranged scales will not suffice. For instance, if $e = 0.9$ and $E = 65^\circ$, no construction can be carried out, the line pq , Fig. 3, not intersecting the scale of v anywhere. The limiting value of e for which all values of E and v may be considered is that corresponding to the line joining $v = 110^\circ$, the end

of scale II, to the same number on scale I. This value is about 0.625. If the scale II of v and E were extended to 120° , the limiting value of e would be 0.8, which is large enough for most practical purposes. This would add to the scale II about one-half its present length. Such an increased length would not be inconvenient.* It was not included in the present drawing because it was deemed unnecessary for the purposes of the chart.

5. THE USE OF THE CHART. The three drawings of Figs. 1, 2 and 3 have been superposed in the chart shown in the accompanying plate. The preceding discussion of each separate part sufficiently explains the character of the chart itself. It is suggested to those who would use the method, however, that the scales of E , $\frac{r}{a}$ and v for which lighter lines have been used, be followed with a ruling pen and red ink. It will be found that this will in a very marked degree simplify the appearance of the chart, and eliminate liability to the error of reading from wrong scales.

The origin corresponding to the e -scale is at the common zero point of the M -, $\frac{r}{a}$ -, and E -scales. The first step in the use of the drawing is to so orient it upon the drawing board that the edge of a T-square passes through this origin and the proper value of e . In this position, any desired parallel may be constructed by simply moving the T-square along the board. No lines need be drawn on the chart, the readings being made while the

T-square is in position. For this purpose it is desirable to have a T-square with a celluloid blade, so that interpolation may be accurately made. Lacking this, an opaque square may be made to serve the purpose by using with it a pair of triangles placed as in drawing parallels. I have found the latter arrangement to be satisfactory.

After the drawing is oriented, the order in which the scales are used is as follows: The T-square being placed at the proper value of M , the indicated number on the scale E is read off. This need not be recorded, but may be held in mind. We shall call it $\mathcal{C}$. The T-square is now moved to the value $\mathcal{C}$ on the scales E_r and E_v in turn, and the values $\frac{r}{a}$ and v read off.

Care must be exercised in two respects. It must be borne in mind that the numbers *between* the E_v - and v -scales are values of v ; those *outside*, values of E_v . It must be remembered that when the heavily lined scale is used for E_r or E_v , the heavily lined scale for $\frac{r}{a}$ or v must be read, and *not* the light lined scaled. The first of these injunctions is no more troublesome than the mechanical distinction between tangent and cotangent columns in a table of trigonometric functions. The second is unnecessary if the lighter scales are inked in in red as advised above.

Where the number $\mathcal{C}$ occurs at two points of E_v , as for instance $\mathcal{C} = 90^\circ$, either point may be used.

The accuracy of the drawing is shown by the following set of values, which were the first read from the original of the accompanying plate. The table shows the complete list of results taken at this time, none having been omitted or altered. As the values of e differ from entry to entry, orientation of the drawing was not advisable, and the construction of parallels was carried out by the shifting of a straightedge and a single small triangle. This process is much more uncertain than the shifting of a T-square, so that the table may be thought of as constructed under relatively unfavorable circumstances.

e	M	E	ΔE	$\frac{r}{a}$	$\Delta \frac{r}{a}$	v	Δv
0.403	$110^\circ 30'$	119.3	$+0^\circ .4$	1.095	$+0.003$	$128^\circ .5$	$+0^\circ .5$
0.371	19 22	29 .9	$+0 .1$	0.678	0.000	12 .9	$+0 .3$
0.371	21 30	33 .1	0 .0	0.685	$+0.004$	47 .7	$-0 .3$
0.371	23 38	36 .0	$+0 .2$	0.695	$+0.005$	51 .1	$+0 .4$
0.371	25 46	39 .1	$+0 .1$	0.707	$+0.005$	55 .2	$+0 .3$
0.62	194 .1					184 .4	$-0 .2$
0.62	59 .1					131 .5	$+0 .2$

PHOTOGRAPHIC POSITIONS OF COMET *f* 1913 (DELAN),

By WILLIAM O. BEAL.

The plates from which these positions have been obtained were taken with the 10½-inch equatorial of the University of Minnesota. The exposure times varied from five to twelve minutes. The scale of the plates is 1^{mm} = 1'. The measurements were made with the Repsold measuring machine, and the star places were

taken from the A. G. Zones. In the table the following designations are used:

B = WILLIAM O. BEAL,
F = G. LEE FLEMING,
K = BESSIE L. KIRK,
W = HUGH B. WILCOX.

	Minneapolis M.T. 1914	α 1914.0	δ 1914.0	log pJ		No. of Exposures	No. of Comp. Stars	Computer	Photo. by
				α	δ				
1	Feb. 27 ^d 7 ^h 56 ^m 25 ^s	2 41 55.89	3 25 17.6	9.537	0.776	2	4	B	B
2	Mar. 3 7 48 49	2 43 17.85	4 9 7.4	9.545	0.773	2	4	B	B
3	Mar. 8 7 25 24	2 45 18.87	5 4 15.9	9.539	0.768	2	3	B	B
4	Mar. 11 7 28 46	2 46 41.69	5 37 42.3	9.556	0.767	4	5	B	B
5	Mar. 12 7 32 58	2 47 10.53	5 48 54.7	9.564	0.767	4	6	B	B
6	Mar. 15 7 34 19	2 48 42.50	6 22 33.0	9.576	0.767	3	6	B	B
7	Mar. 19 7 47 40	2 50 55.76	7 7 46.3	9.598	0.770	4	6	B	B
8	Mar. 23 7 47 40	2 53 20.28	7 53 11.8	9.606	0.771	2	5	K	B and F
9	Sept. 24 7 41 11	11 11 24.02	47 42 9.9	9.672	0.855	1	4	B	B and W

Minneapolis, Minn., January 22, 1916.

OBSERVATIONS OF ASTEROIDS,

MADE AT THE U. S. NAVAL OBSERVATORY WITH THE PHOTOGRAPHIC TELESCOPE,

By GEORGE H. PETERS.

[Communicated by Captain J. A. HOOGWERFF, U. S. Navy, Superintendent.]

Name	Mag.	Date 1915	G. M. T.	α 1915.0	δ 1915.0
(230) <i>Athamantis</i>	10.1	July 14	18 ^h 23 ^m .2	22 ^h 1 ^m 27 ^s .0	+3° 26' 19''
(230) <i>Athamantis</i>	10.1	July 18	18 39 .0	21 59 49 .7	+3 40 48
(420) <i>Bertholda</i>	12.4	Sept. 8	17 32 .0	23 11 25 .5	+4 35 00
(69) <i>Hesperia</i>	11.0	Sept. 10	16 07 .1	23 6 33 .8	-1 15 33
New	12.5	Sept. 10	17 47 .0	23 42 03 .0	+7 34 47
(403) <i>Cyane</i>	12.5	Sept. 12	14 55 .0	22 00 15 .6	+2 00 00
(420) <i>Bertholda</i>	12.4	Sept. 12	16 22 .0	23 8 46 .8	+4 17 08
New	12.5	Sept. 12	17 38 .0	23 40 15 .1	+7 34 35
(69) <i>Hesperia</i>	11.0	Sept. 13	15 14 .0	23 4 26 .0	-1 35 35
(420) <i>Bertholda</i>	12.4	Sept. 13	16 34 .5	23 8 06 .3	+4 12 24
(469) <i>Argentina</i>	13.5	Sept. 15	16 40 .2	23 44 28 .2	+4 49 43
(124) <i>Alkeste</i>	10.5	Sept. 15	18 15 .0	0 53 03 .7	+5 50 03
(239) <i>Adrastea</i>	13.0	Sept. 15	18 15 .0	0 58 46 .6	+5 27 32
(167) <i>Urda</i>	13.0	Sept. 15	18 15 .0	0 59 29 .2	+5 3 34
(787) [1914 U Q]	12.4	Sept. 15	18 15 .0	1 6 10 .6	+5 37 02
(469) <i>Argentina</i>	13.5	Sept. 16	16 45 .0	23 43 42 .5	+4 47 05
(124) <i>Alkeste</i>	10.5	Sept. 16	18 47 .0	0 52 21 .1	+5 44 26
(239) <i>Adrastea</i>	13.0	Sept. 16	18 47 .0	0 58 18 .4	+5 20 40
(167) <i>Urda</i>	13.0	Sept. 16	18 47 .0	0 58 50 .4	+4 58 38

OBSERVATIONS OF ASTEROIDS (Continued)

Name	Mag.	Date 1915	G. M. T.	α 1915.0	δ 1915.0
(787) [1914 U Q]	12.4	Sept. 16	18 ^h 47 ^m .0	1 ^h 5 ^m 37 ^s .0	+5° 25' 09"
(377) <i>Campania</i>	11.1	Oct. 9	15 54 .0	0 29 48 .1	+6 59 59
(57) <i>Mnemosyne</i>	10.1	Oct. 9	15 54 .0	0 30 26 .6	+6 56 38
(518) <i>Halaire</i>	12.7	Oct. 9	16 45 .2	0 50 42 .3	+7 33 18
(377) <i>Campania</i>	11.1	Oct. 10	15 05 .0	0 29 04 .4	+6 52 06
(57) <i>Mnemosyne</i>	10.1	Oct. 10	15 05 .0	0 29 49 .6	+6 47 05
(377) <i>Campania</i>	11.1	Oct. 11	13 15 .3	0 28 23 .0	+6 44 36
(57) <i>Mnemosyne</i>	10.1	Oct. 11	13 15 .3	0 29 14 .5	+6 37 59
(518) <i>Halaire</i>	12.7	Oct. 12	14 21 .8	0 48 29 .6	+7 4 52
(184) <i>Dejopeja</i>	12.7	Oct. 12	16 24 .3	1 07 23 .2	+8 25 05
(377) <i>Campania</i>	11.1	Oct. 25	13 09 .1	0 19 17 .7	+4 56 48
(184) <i>Dejopeja</i>	12.7	Oct. 27	15 30 .0	0 56 49 .1	+7 21 33

In the above list have been incorporated the corrections kindly furnished by PROFESSOR COHN, Director of the Royal Astronomical Computing Bureau of Berlin. Several designations were changed by him, and those that were in doubt were identified. One asteroid is new.

Asteroid observations are taken at the Naval Observatory in the following zones:

March, April, September, October + 4° to + 8°;
 May to August, inclusive, + 0° to + 4°;
 November to February, inclusive, +12° to +16°.

A NEW VARIABLE STAR,

1855.0 α 17^h 0^m 50^s.7 δ + 17° 22'.0
 1900.0 17 2 50.4 + 17 18.2

By E. E. BARNARD.

I have found a new variable star on my photographs with the Bruce telescope of this observatory. Its position with reference to *BD* + 17° 3168 (9.3) on 1916 March 18 was

Position Angle 26°.9
 Distance 248".8

from which the above position was determined. Visually

the star is yellow. It was bright (about 9½ magnitude) on 1916 March 6, and faint on 1911 May 24.

PROF. E. C. PICKERING kindly informs me that the variable is new. It is shown on a number of H. C. O. plates where it ranges from 10.1 magnitude to perhaps less than 15th magnitude. This data will be very valuable for the determination of its period and light curve.

Yerkes Observatory, Williams Bay, Wisconsin, 1916 March 27.

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NO. 19

SYSTEMATIC MOTION OF THE *B*-TYPE STARS,

By BENJAMIN BOSS.

There has been such a variety of opinion on the subject of the systematic motions of type *B* stars, that it is eminently desirable to bring to bear every method of attack.

As all the data necessary for a definitive solution of the problem are not available recourse must be had to statistical methods in which one or more assumptions are made.

In the present treatment of the subject it is assumed that the mean of the proper-motions in any limited sky area will furnish a better center of motion from which to determine systematic tendencies of motion, than a center of motion based for instance on an attempt to free the results from the effect of solar motion. To illustrate the point, let us assume the true projected velocity figure to be represented by an ellipse. If the limited data in a restricted area represented motions on one side of the major axis of the ellipse only, then manifestly the adoption of the true center, in this case the center of the ellipse, would yield results considerably at variance with the facts, whereas a center of motion determined solely by the actual proper-motions would closely reproduce the true preferences of motion. This method would also take account of the varying mean parallax for the stars in different regions.

The objection might be raised that the removal of the mean motion from the individual motions in a given area would at the same time remove the group motion sought. Quantitatively that is true, but as the aim in the present case is merely to determine the direction of preferences of motion, the removal of the mean group motion still leaves the question of direction determinate, as the stellar motions will tend to elongate along the line of the preferential directions. This to be sure is only true when the stars under discussion are at varying distances from the solar system, and when their individual motions vary.

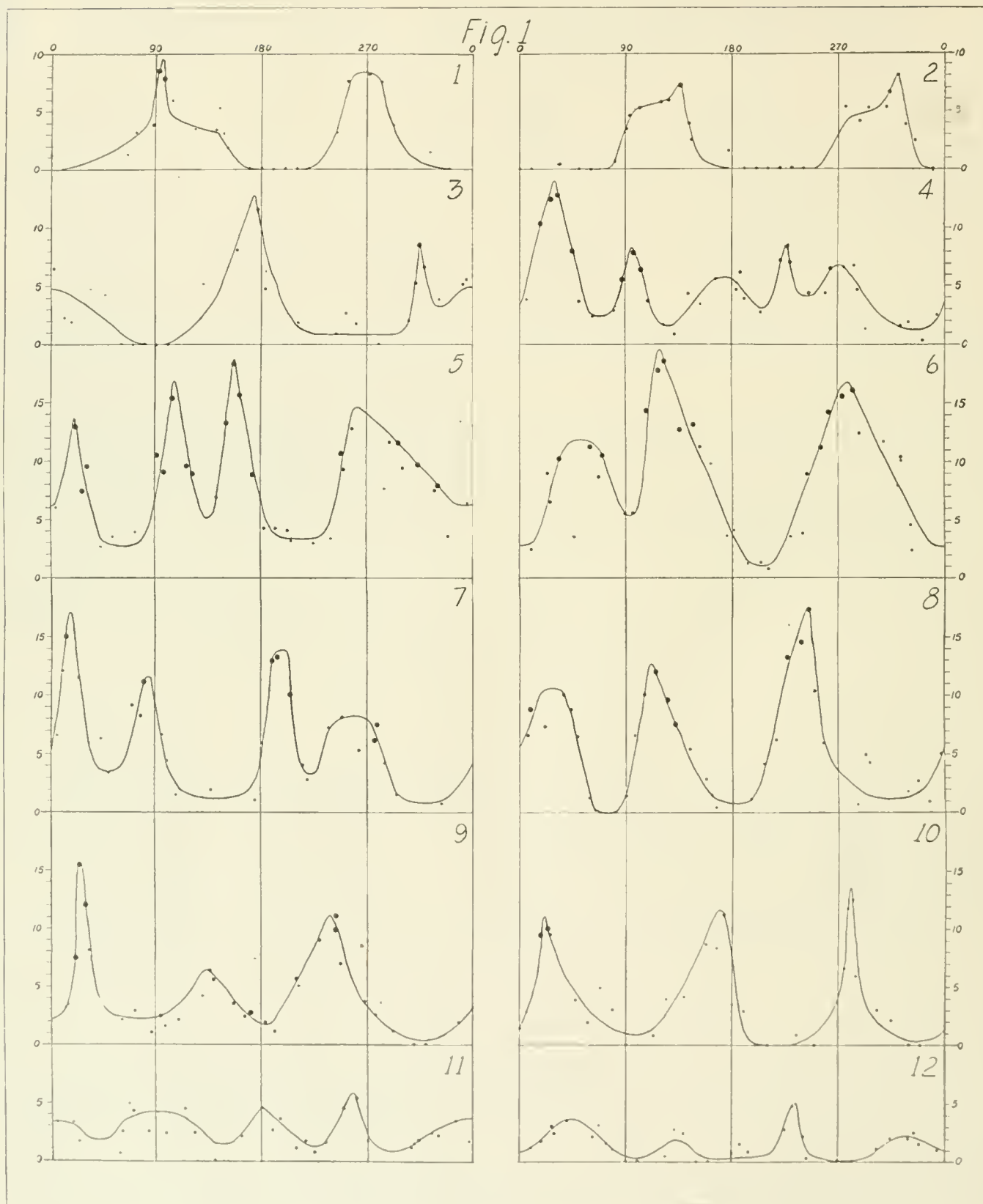
The method would not apply in the case of such a group as the *Taurus* group where there is a strict com-

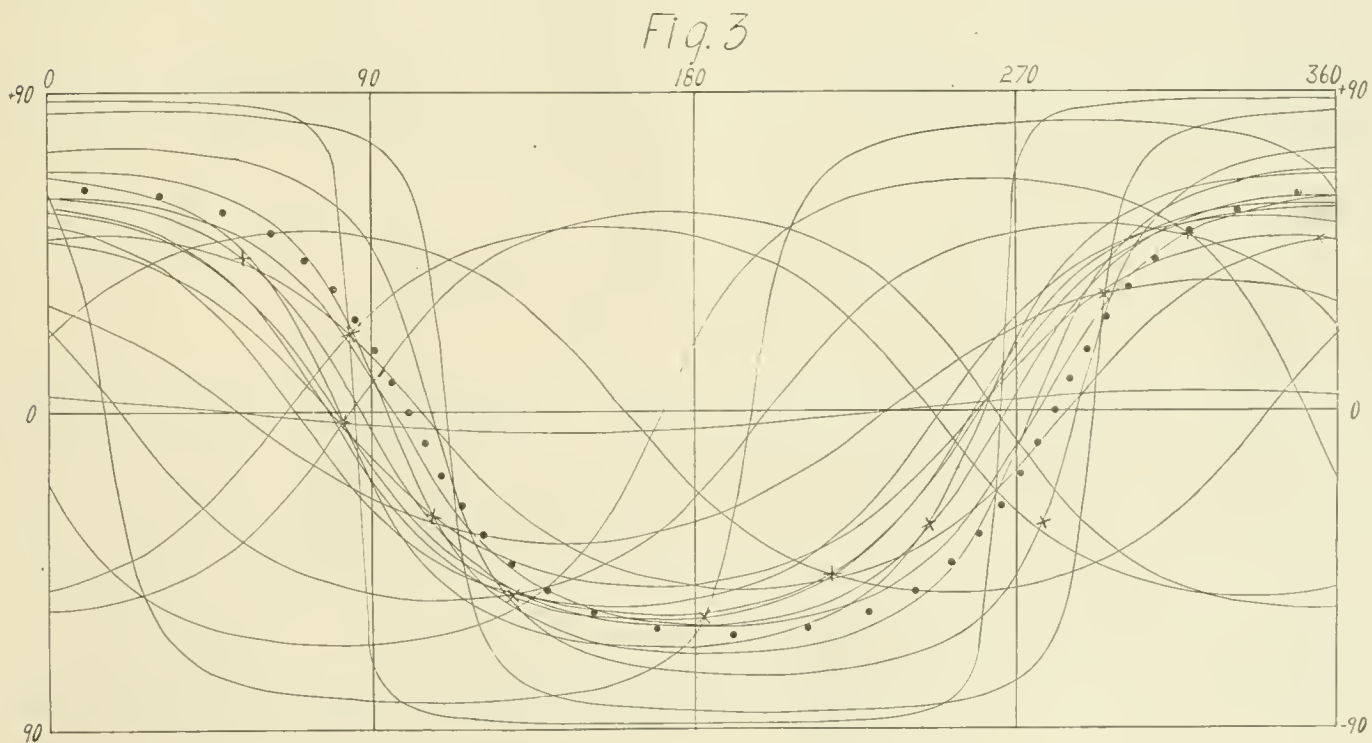
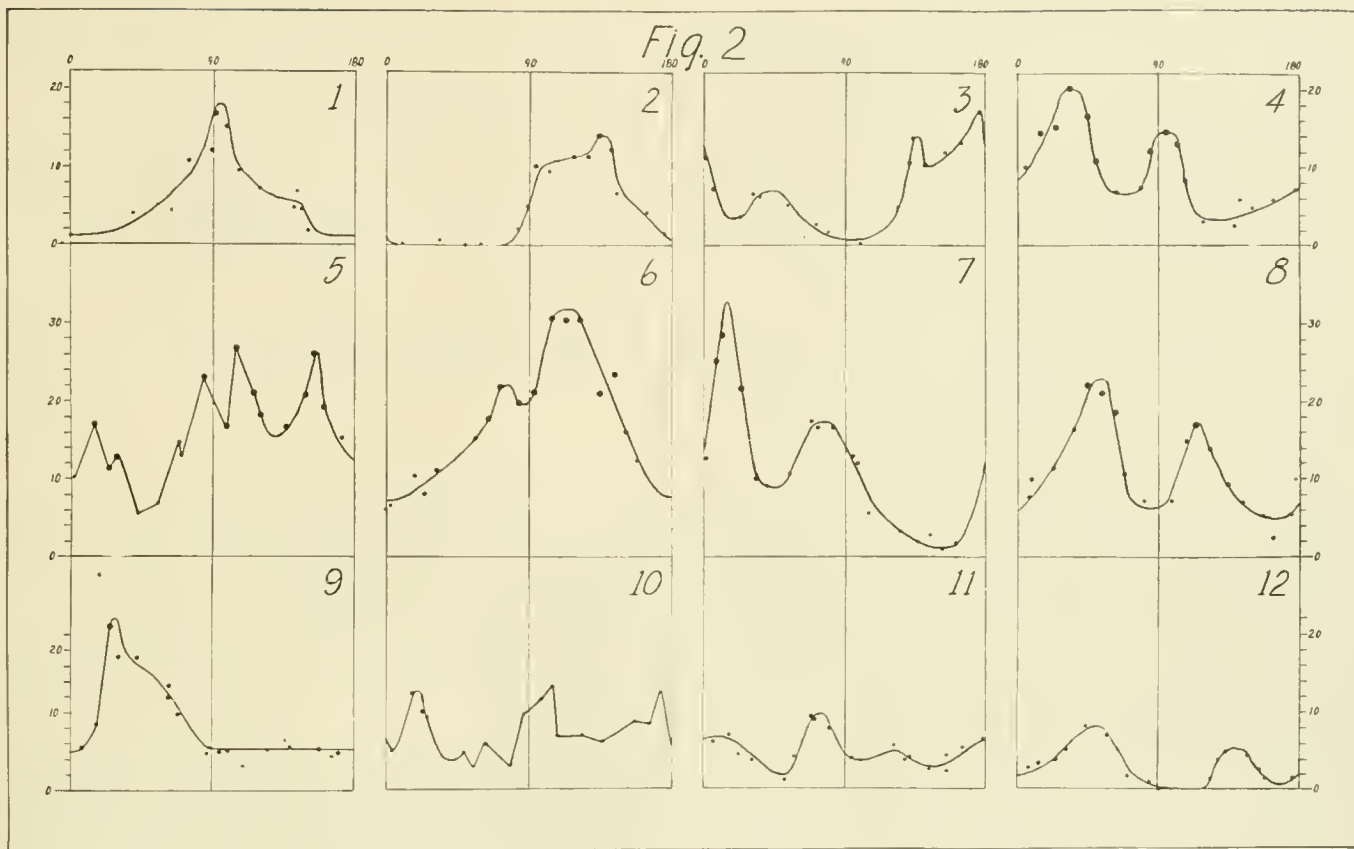
munity of motion. But there is little likelihood that such groups are predominant among the *B*-type stars in many regions. The close agreement of the position of the apex of solar motion as derived from a treatment of the proper-motions of type *B* stars, with the position derived from a general solution including all types, renders the existence of a large systematic drift among the *B*-type stars improbable, unless the drift is shared in common with stars of all types.

The proper-motions utilized in the present investigation were taken from the *Preliminary General Catalogue*. In order to discuss these proper-motions twelve irregular areas were formed, mainly based upon a natural grouping of the stars of types *B* to *B₅* inclusive. Roughly speaking the areas are bounded as follows.

Area	Limits in R.A.	Limits in Decl.	No. Stars
1	22 ^h 10 ^m to 1 ^h 0 ^m	+35° to +65°	22
2	2 50 to 4 30	+30 to +55	21
3	4 50 to 6 30	+ 5 to +35	24
4	4 20 to 6 30	-15 to +10	48
5	5 30 to 8 0	-15 to -40	62
6	7 40 to 10 0	-40 to -70	54
7	10 20 to 13 20	-45 to -75	38
8	13 30 to 15 30	-30 to -65	42
9	15 30 to 17 30	-15 to -50	40
10	17 30 to 20 0	-10 to -50	21
11	18 0 to 20 30	+15 to +55	25
12	20 30 to 22 10	+35 to +70	22

The proper-motions in right ascension and declination of the stars in these groups were assembled and straight means taken. These means were then subtracted from the individual proper-motions. The resulting motions were converted into polar coördinates and arranged in order of position-angle from the north pole. Preference of motion should now become manifest by concentrations of proper-motions, or by increase in the size of the motions at certain position-angles. The more stars in the





area the greater the determination of the preferential motion, but as we have to deal with a restricted number of stars per area it is necessary to take overlapping means in order to secure a smooth representation of the velocity figure. Thus the sums of the proper-motions between the position-angles 0° – 30° , 10° – 40° , etc., were formed, and the mean position-angle for each group taken. With the sums as ordinates and the position-angles as abscissae smooth curves were drawn to represent the observations, as shown in Fig. 1. The heavier the dot the greater the weight of the plotted point. The curves were drawn entirely mechanically; that is to say they were made to represent the plotted points as nearly as possible, without foreknowledge of the results they would lead to, and have not been revised. While all the curves are made up of more than one maximum, the maxima are not all independent, for a maximum located 180° from another merely indicates that the preferential motion extends in two opposite directions. Thus in curves 1, 2, 4, 6, 7, 11, and 12 there are one or more dependent maxima. In other cases a comparatively slight shift in the position of the adopted center of motion would convert seemingly independent maxima into dependent maxima.

Therefore in order to secure a greater degree of accuracy in locating the maxima, and as a means of smoothing out slight discrepancies in their positions, the position-angles in diametrically opposite directions were combined, resulting in Fig. 2 which is constructed in the same manner as Fig. 1. The maxima of curves 2 and 9 are evidently composites of two independent maxima, but no attempt has been made to separate them. The material from which curves 5 and 10 are constructed is so unevenly distributed that a change in the method of grouping according to position-angle would entirely change the

nature of the curve with the exception of the last maximum of curve 5, and the first maximum of curve 10, which are real. Considered as a whole the observed data are very well represented by the curves.

Fig. 3 represents the results graphically. The sky is laid out on the flat. Each cross represents the position of the center of an area, and from it great circles are drawn in the directions derived from the maxima of the curves of Fig. 2. The dots outline the path of the galactic equator. An inspection of the diagram yields the following results.

1. There is a strong tendency on the part of the *B*-type stars to preserve the plane of their present distribution.

2. The data are not sufficiently accurate to determine the character of the motion in this plane.

3. There are also strong tendencies of motion among the *B*-type stars directed outside of the plane of their distribution, but there is apparently nothing of a systematic character in these tendencies of motion.

4. In some regions the group motion directed outside the plane of the *B*-type stars is stronger than the motion in the plane.

Thus it would seem that the tendency toward random motion, which becomes increasingly manifest with progression in type, is present in the *B*-type stars. But as the strongest tendency of motion among the *B*-type stars is directed in the plane of their distribution, the plane must in large measure be preserved in the following stage in evolution, a marked feature in the distribution of the *A*-type stars.

The same general method will subsequently be applied to other types, though considerable modifications will be necessitated because of the undue influence exerted by large proper-motions.

THE SUN'S MEAN LONGITUDE,

By FRANK E. ROSS.

In *Astronomical Constants*, pp. 30-32, NEWCOMB gives the correction $\delta l''$ to the *Sun's* mean longitude from 1753 to 1892, compared with LEVERRIER's *Tables of the Sun*. No direct comparison with NEWCOMB's tables for this period has been made, so far as the writer is aware.

Examination of the observed phase times of the annular eclipse of the *Sun* of April 16-17, 1912, taken in connection with the correction to the *Moon's* mean longitude indicated by occultations, convinced the writer that the *Sun's* mean longitude given by NEWCOMB's tables must be sensibly in error at this epoch. Reference to

the Greenwich observations of the *Sun* during the years 1901 to 1910 strengthened this conviction, a tolerable agreement between the direct observations and results from the eclipse being found. A correction to the *Sun's* mean longitude was accordingly incorporated in the prediction for the phase times for the total eclipse of the *Sun* of August 14, 1911 (*Astronomical Journal*, No. 667).

It is highly desirable to compare with NEWCOMB's tables all the observations of the *Sun* from BRADLEY's to the present time. Fortunately the revision of the *Tables of Mars* just completed by the writer has made such comparison not a difficult undertaking. In that

revision it was necessary to compute from NEWCOMB'S tables the *Sun's* longitude for 394 scattering dates in the interval 1751 to 1899. A manuscript ephemeris of the *Sun* for each day from 1751 to 1865, computed from LEVERRIER'S tables, is in the archives of the *American Ephemeris*. Comparison of these two sets of positions gives the difference NEWCOMB minus LEVERRIER, or $N-L$, for a sufficient number of dates to eliminate the short period differences, due principally to differences in

eccentricity and perigee, and to furnish good values of the actual differences between NEWCOMB and LEVERRIER in mean longitude.

With the differences found in this way, the mean differences $N-L$ for the epochs adopted by NEWCOMB, given in the table which follows, are obtained. The periods, and correction to LEVERRIER'S, c , are taken from *Astronomical Constants*, pp. 22-24.

CORRECTION TO THE *Sun's* MEAN LONGITUDE FROM NEWCOMB'S TABLES 1750 — 1892

Period	c	$N-L$	$O-N$	Period	c	$N-L$	$O-N$	Period	c	$N-L$	$O-N$
Greenwich: —				Paris: —				Konigsburg: —			
1750—62	+0.33	+ 0.96	-0.63	1801—07	-1.78	+(0.7)	-2.5	1816—23	+0.30	+0.67	-0.37
65—71	+0.37	+ 0.65	-0.28	08—15	-0.65	+ 0.52	-1.17	24—30	+0.02	+0.45	-0.43
72—78	+0.74	+ 1.09	-0.35	16—22	+0.18	+ 0.67	-0.49	31—38	+0.23	+0.13	+0.10
79—85	+2.89	+(1.0)	+1.94	23—29	+0.01	+ 0.45	-0.44	39—45	+0.77	+0.42	+0.35
86—92	+1.51	+ 0.82	+0.69	37—44	+0.33	+ 0.46	-0.13	Oxford: —			
93—97	+1.87	+ 0.74	+1.13	45—52	+0.10	- 0.07	+0.17	1840—49	+2.49	+0.10	+2.39
98—02	+1.02	+(0.7)	+0.3	53—59	+0.66	+ 0.22	+0.44	60—68	+1.96	-0.20	+2.16
1803—06	+0.27	+(0.7)	-0.4	60—65	+0.38	- 0.21	+0.59	69—76	+0.92	-0.03	+0.95
07—10	-0.34	+(0.7)	-1.0	66—70	+2.29	- 0.13	+2.42*	80—87	-0.31	-0.16	-0.15
11—14	-3.33	+ 0.65	-3.98*	71—79	-0.26	- 0.20	-0.06	Pulkowa: —			
15—18	-1.99	+ 0.20	-2.19	80—89	+0.44	- 0.16	+0.60	1842—50	+1.20	+0.05	+1.15
19—22	-0.51	+ 0.87	-1.38	Palermo: —				61—70	-0.40	-0.20	-0.20
23—26	-1.08	+ 0.83	-1.91	1791—96	-0.07	+ 0.79	-0.86*	Dorpat: —			
27—30	-0.42	+ 0.35	-0.77	97—01	-2.33	+(0.7)	-3.0*	1823—30	+0.36	+0.45	-0.09
31—34	+0.76	+ 0.12	+0.64	1802—05	-3.11	+(0.7)	-3.8*	31—38	+0.45	+0.13	+0.32
35—38	+1.16	+ 0.14	+1.02	06—12	+5.92	+(0.7)	+5.2*	Cape of Good Hope: —			
39—42	+0.84	+ 0.57	+0.27	Cambridge: —				1884—90	-0.36	-0.10	-0.26
43—46	+0.15	+ 0.31	-0.16	1828—34	-0.13	+ 0.21	-0.34	Strassburg: —			
47—50	-0.10	- 0.20	+0.10	35—40	-0.18	+ 0.22	-0.40	1883—88	-1.65	-0.07	-1.58
51—54	+0.40	+ 0.11	+0.29	42—47	-0.21	+ 0.07	-0.14				
55—58	+0.36	+ 0.28	+0.08	50—58	-0.11	+ 0.25	-0.36				
59—62	-0.02	- 0.19	+0.17	Washington: —							
63—66	+0.31	- 0.17	+0.48	1846—52	-0.85	- 0.11	-0.74				
67—70	+0.35	- 0.10	+0.45	61—65	-0.53	- 0.26	-0.27				
71—74	+0.12	+ 0.06	+0.06	66—73	-0.22	- 0.05	-0.17				
75—78	-0.12	- 0.21	+0.09	74—81	-0.45	- 0.38	-0.07				
79—82	-0.05	- 0.40	+0.35	82—91	-0.79	- 0.06	-0.73				
83—88	-0.20	- 0.07	-0.13								
89—92	-0.44	- 0.29	-0.15								

*Rejected in subsequent computations.

Values of $N-L$ in parentheses are interpolated.

Greenwich Observations, 1893-1912. These have been taken from the annual volumes. The mean yearly values of the error of right ascension, $O-L$, have been formed, and collected in the following table. The third

column contains the corrections for equinox; the fourth, the reduction from LEVERRIER'S to NEWCOMB'S *Tables of the Sun*.

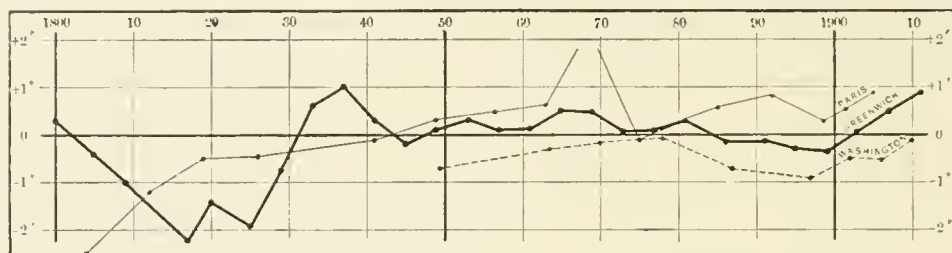
DIAGRAM 1

CORRECTIONS $O - C$ TO NEWCOMB'S LONGITUDE OF THE *Sun* FROM OBSERVATIONS
AT GREENWICH, PARIS AND WASHINGTON

Heavy line — Greenwich observations.

Light line — Paris observations.

Broken line — Washington observations.



Year	$O - L$ Mean	Syst. Cor.	$L - N$	$O - N$	Year	$O - L$ Mean	Syst. Cor.	$L - N$	$O - N$
1893	-.092	+.054	+.032	-.006	1893	-.011	+.038	+.032	+.059
94	-.116	+.054	+.034	-.028	1897	-.063	+.038	+.033	+.008
95	-.107	+.054	+.034	-.019	98	-.040	+.038	+.033	+.031
96	-.105	+.054	+.033	-.018	99	-.023		+.035	+.012
97	-.111	+.054	+.033	-.024	1900	+.006		+.037	+.043
98	-.111	+.054	+.033	-.024	01	-.003		+.027	+.024
99	-.114	+.054	+.033	-.027	02	+.021		+.027	+.048
1900	-.100	+.054	+.033	-.013	03	+.018		+.037	+.055
01	-.055	+.054		-.001	04	-.017		+.032	+.015
02	-.053	+.054		+.001	05	+.026		+.033	+.059
03	-.011			-.011	06	+.058		+.035	+.093
04	+.021			+.021	10				+.09*
05	+.015			+.015					
06	+.020			+.020					
07	+.028			+.028					
08	+.068			+.068					
09	+.046			+.046					
10	+.054			+.054					
11	+.064			+.064					
12	+.070			+.070					

*Extrapolated, to be used in combination curve.

Washington Observations. For 1894 to 1899, the ephemeris of comparison C , is based on HANSEN'S *Tables of the Sun*.

Paris Observations, 1890-1906. These have been treated in a similar manner, with results which are shown in the following table. The systematic correction given has been obtained by comparing 186 equatorial clock stars with NEWCOMB'S places for the epoch 1898.0.

Year	$O - L$ Mean	Syst. Cor.	$L - N$	$O - N$
1890	+.005	+.038	+.012	+.055
91	-.014	+.038	+.020	+.044
92	-.009	+.038	+.028	+.057

Year	$O - C$	Syst. Cor.	$C - N$	$O - N$
1894.5	-.098		-.027	-.125
1896	-.014		-.029	-.043
97	-.016		-.039	-.055
98	-.012		-.022	-.031
99	-.030		-.034	-.064
1900	-.052			-.052
01	-.017			-.017
02	-.036			-.036
03	-.032			-.032
04	-.004	-.028		-.032
05	-.032	-.028		-.060

Year	$O - C$	Syst. Cor.	$C - N$	$O - N$
1906	+ ^s .009	- ^s .028		- ^s .019
07	- .011	- .028		- .039
08	+ .012	- .028		- .016
09	+ .031	- .028		+ .003
10	+ .015	- .028		- .013
11	+ .006	- .028		- .022

Diagram 1 shows the correction to the mean longitude of the *Sun* from NEWCOMB's tables, given by the Greenwich, Paris, and Washington observations separately.

These three observatories were chosen for this comparison, on account of the long series of observations obtained by them. The systematic differences in the results are well marked in the diagram. It will be noticed that the Washington curve lies entirely below the Greenwich and Paris curves. We conclude that the Washington observations of the *Sun* for 60 years have given a result $0^s.05$ less in right ascension than Greenwich, and $0^s.08$ less than Paris for the same period. On account of the great number of observers which must have taken part in the observations over such an extended period, it is inconceivable that this is a result of

DIAGRAM II

FLUCTUATIONS IN THE MEAN LONGITUDE OF THE *Earth* AND THE *Moon*

Heavy line — *Sun's* longitude from all observations. Circles — *Moon's* minor residuals, multiplied by one-fifth for purpose of comparison.

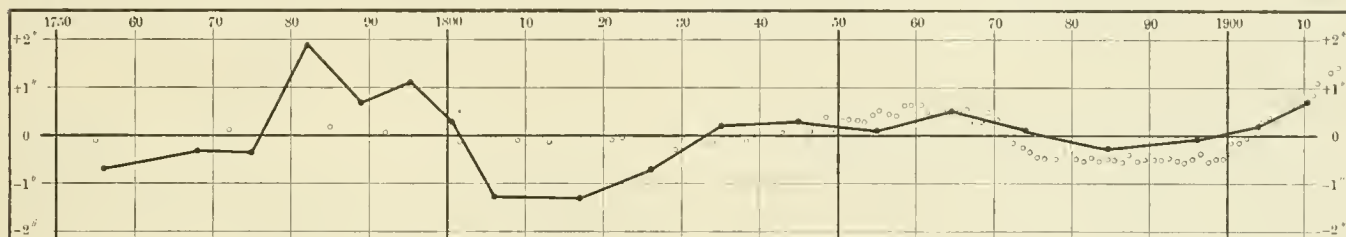


DIAGRAM III

FLUCTUATIONS IN THE MEAN LONGITUDE OF THE *Earth* AND *Mars*

Heavy line — *Sun's* longitude from new elements of *Mars*.

Light line — *Mars's* longitude from new elements of *Mars*.

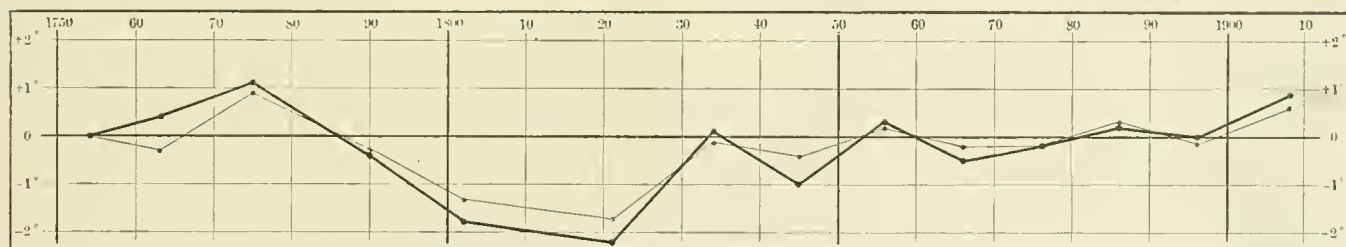
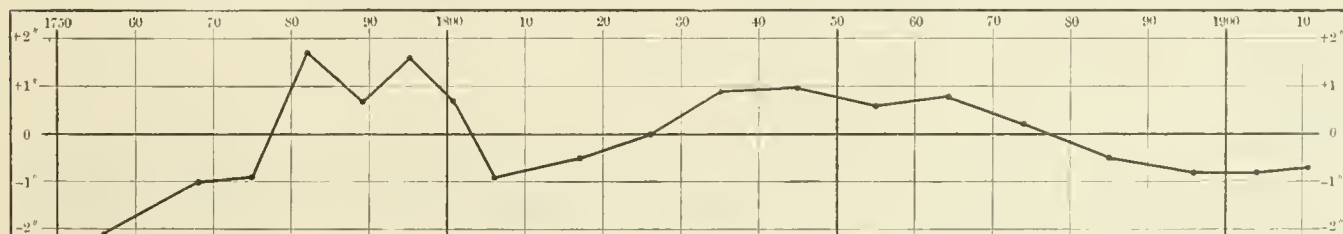


DIAGRAM IV

RESIDUALS IN THE *Sun's* LONGITUDE AFTER APPLYING A SECULAR ACCELERATION OF $3''.5$ 

personal equation. It must accordingly be considered as of instrumental or housing origin, or as a local systematic refraction.

The acceleration of the *Sun* in longitude, which began about 1988 is well shown by all three series. It is undoubtedly real. As to the fluctuations shown earlier, the evidence is not so conclusive.

In Diagram 2, results from all the observatories, collected in the preceding tables, have been combined into one curve, shown as a heavy continuous line. The minor residuals in the *Moon's* longitude, reduced to one-fifth, shown as a series of circles, are also given for comparison. The correlation between the deviations of *Sun* and *Moon* is well marked since 1830. From 1750 to 1830, no connection whatever is found. NEWCOMB's reduction of the occultations of the *Moon* show only small and unimportant fluctuations in the *Moon's* longitude during this period, as the diagram indicates, while the apparent fluctuations in the *Sun's* longitude are large. That they are due entirely to changes in the *Earth's* rotation is thus open to question.

To obtain additional data on the subject of fluctuations, the corrections to NEWCOMB's longitude of the *Sun*, δL , and the longitude of *Mars*, δl , given in *Investigations on the Orbit of Mars*, A.J., No. 692, page 161, are plotted in diagram No. 3 which is appended. The major fluctuation of the *Sun* from about 1760 to 1830, shown in Diagram 2 is corroborated. In view of the entirely independent character of the determinations lying at the base of the curves in the two diagrams, it can hardly be doubted that this is a real fluctuation, and not a result of errors of observation. That the *Sun* is now accelerating its motion is also well established from the two curves.

If it should be established that minor residuals in the *Moon's* longitude have a counterpart in the *Sun's* longitude, the question arises, is there also a counterpart in the *Sun's* longitude for the long period empirical term in the *Moon's* longitude? Positive affirmation or denial of this important question is not now possible. It is unfortunately entangled with the minor fluctuations just considered, and the secular acceleration of the *Sun's* mean longitude.

Secular acceleration of the Sun's mean longitude. COWELL (*M. N.*, Vol. 66) and FOTHERINGHAM (*M. N.*, Vol. 75) have independently from ancient eclipses and occultations respectively concluded that there is a secular acceleration in the *Sun's* longitude. Their values are in close agreement, being 4''.1 and 3''.2 respectively.

In order to determine if an acceleration of this order of magnitude is consistent with modern observations of the *Sun*, the residuals shown in Diagram 2 were corrected for the mean of COWELL's and FOTHERINGHAM's acceleration, $-3''.5 T^2$. A solution was then made by least squares for correction to epoch and mean motion with result —

Correction to NEWCOMB's longitude of the *Sun*,

$$\delta l = -0''.62 + 1''.20 T;$$

(T in centuries from 1850.0)

the total correction applied to NEWCOMB being, therefore,

$$\delta L = -0''.62 + 1''.20 T + 3''.50 T^2.$$

Applying δL to the residuals in Diagram 2, new residuals were formed and plotted, as shown in Diagram 4. The representation is seen not to be satisfactory. While not conclusive, it appears quite unlikely that a secular acceleration of the *Sun* now exists.

The residuals in Diagram 2 were solved on the assumptions that they are of the form,

$$(a) \quad \delta L = a + b T + c T^2,$$

$$(b) \quad \delta L = a + b T.$$

In a solution of this kind, it is very difficult to assign weights, on account of the existence of fluctuations. Equal weights will be given to the equations, this assumption being probably the best that can be made. The results obtained were

$$(a) \quad \delta L = +0''.01 + 0''.22 T + 0''.37 T^2,$$

$$(b) \quad \delta L = +0''.08 + 0''.11 T.$$

(T in centuries from 1850.0)

Nautical Almanac Office, U. S. Naval Observatory.

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SYSTEMATIC MOTION OF THE B-TYPE STARS, BY BENJAMIN BOSS.

THE SUN'S MEAN LONGITUDE, BY FRANK E. ROSS.

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NO. 20

INVESTIGATIONS ON THE ORBIT OF MARS,

By FRANK E. ROSS.

The "*Tables of Mars*" constructed by PROFESSOR SIMON NEWCOMB were used in the *American Ephemeris* first in 1902, and in the *British Nautical Almanac* in 1903. The geocentric positions of the planet to which they lead failed to represent observations during the next opposition, 1902-3, the discrepancy amounting to three seconds of arc in right ascension. For the opposition of 1905, the difference increased to six seconds, with an equal error in 1907.

A preliminary investigation was made to discover the origin of the apparent error in the tables. Heliocentric positions of *Mars* were computed from NEWCOMB's tables for each opposition date from 1867 to 1899, and roughly compared with observation. The result showed that the eccentricity of the orbit adopted by NEWCOMB was in error nearly a second of arc, the error of the remaining elements being small. The origin of this is clear. LEVERIER's *Tables of Mars* were adopted by NEWCOMB as the ephemeris of comparison. In these tables a small mass of the *Earth* had been used, but 0.930 of NEWCOMB's finally adopted value. From this circumstance it is not difficult to show that when elements of *Mars* are determined from positions near opposition, which is the case in great measure, an error is introduced into the eccentricity amounting to 0''.67. In general, for any assumed change δM in the mass of the *Earth*, the eccentricity of the orbit of *Mars* should be corrected by the amount

$$\delta e = 9''.0 \frac{\delta M}{M},$$

it being assumed that the perturbations are changed to conform to the change in mass.

The secular variations of the elements of *Mars* adopted by NEWCOMB in *Tables of Mars* are open to question. With the exception of the motion of perihelion, theoretical values were used throughout. For the perihelion, a quasi-theoretical value was used, based upon an assumption of gravitative force not varying exactly as the inverse square. It is well known that the secular varia-

tions of the orbits of the four inner planets differ in some cases from their theoretical values by notable amounts. With the lapse of time the differences become large, leading to serious error in the predicted position of the planets. There is no reason why values precisely determined from observation should not be used in preference to theoretical values which from many points of view are of doubtful utility.

Formally from a series of geocentric positions of a planet can be derived the elements of its orbit with their secular variations, the elements of the *Earth's* orbit and their secular variations, and the masses of disturbing planets. Such an extended solution was made by NEWCOMB in the case of *Mars*, *Venus* and *Mercurey* (*Astronomical Constants*).

It has not been considered necessary or even desirable to obtain elements of the *Earth's* orbit in the present investigation. Observations show that aside from fluctuations in the longitude, NEWCOMB's *Tables of the Sun* are not in need of correction. A reason of greater weight for excluding the solar elements is to be found in the correlation which exists between the elements of the *Earth* and *Mars*, on account of observations of the planet having been largely confined to the neighborhood of opposition. The precision of the results is in consequence seriously diminished. If use of NEWCOMB's *Tables of the Sun* is to be continued, as undoubtedly will be the case for an extended period, elements of *Mars* determined from a solution which includes corrections to the solar elements would fail to represent observations.

The method of obtaining the secular variations of the elements of *Mars* used by NEWCOMB has not been followed in the present investigation. It was deemed preferable to divide the observations into periods and solve each period separately for the elements, assumed constant for the period. The error due to this assumption should be inappreciable. It has the advantage of presenting at the mean epoch of the groups the actual values of the elements, which can be examined for any

abnormality or unusual feature. No assumption need be made as to the law of variation of the elements, as in NEWCOMB'S method of procedure. If the variation is other than linear, the fact can be established; if there is any term of long period in the mean longitude at present unknown, within certain limits as to magnitude and period, it should likewise be disclosed.

It is practicable and highly desirable to obtain from the investigation a value of the mass of the planet *Venus*, based upon the periodic perturbations it produces in *Mars*, and in the observing base, the *Earth*. No method can lead to a value of *Venus*' mass comparable with this in accuracy. Secular perturbations, while cumulative in their action, are hopeless. It is true that a good value of the mass of *Venus* can be obtained from observations of the *Sun*. The precision however is considerably less. The periodic perturbation in the geocentric longitude of *Mars* due to the action of *Venus* amounts at the maximum to 40"; in the *Sun*, to less than one-third of this. Observations of *Mercury* also lead to a determination having a precision about equal to that from *Sun* observations.

Absolute terms in the equations of condition. The system to which the observations have been reduced is NEWCOMB'S, both in right ascension and declination. The ephemeris of comparison up to 1902 was computed from LEVERRIER'S *Tables of the Sun* and of *Mars*. Reduction was made to NEWCOMB'S *Tables of the Sun* and of *Mars* by computing from the latter geocentric positions of the planet for each opposition from 1751 to 1901, at intervals of approximately two months. In all, 394 positions were computed in duplicate. These places of the planet were compared with the ephemeris in the *British Nautical Almanac* for the years 1866 to 1901. For the years 1751 to 1865, comparison was made with a manuscript ephemeris of *Mars*, based upon LEVERRIER'S tables, which had been prepared by PROFESSOR NEWCOMB. Differences NEWCOMB—LEVERRIER in both coördinates were thus formed. These differences were plotted for each opposition, and a smooth curve drawn through the points. When the number of points was not sufficient to determine the course of the curve with sufficient accuracy, additional positions were computed. From these curves the correction NEWCOMB—LEVERRIER was read off for the date of each normal place.

Division into periods. For reasons already given, the conditional equations were divided into periods. It was at first planned to begin each period with the opposition whose opposition date found the planet beyond and nearest perihelion. This would make each period about sixteen years in extent. After forming the normal equations for these periods, ten in number, it was discovered that there was a strong correlation between the coefficients of the mass of *Venus* and of the eccentricity of the orbit of

Mars, in all of these groups, which would lead to weak values for both of these unknowns. It was noticed, however, that the correlation disappeared when adjoining groups were combined into one. The original ten groups have been accordingly combined into five. The limiting dates for each period, the number of observations and equations of condition in each coördinate, are contained in the following table:

Group	Period	Limiting Oppositions	Right Ascension		Declination	
			No. Obs.	No. Eqs.	No. Obs.	No. Eqs.
A	I	1751—1766	84	39	59	28
	II	1768—1781	33	20	30	18
B	III	1783—1798	98	28	91	27
	IV	1800—1813	138	47	135	45
C	V	1815—1829	165	32	123	27
	VI	1830—1846	1229	188	1075	180
D	VII	1847—1860	810	164	798	157
	VIII	1862—1875	719	131	775	127
E	IX	1877—1892	787	142	788	141
	X	1894—1912	720	127	730	126
Totals			4783	918	4604	876

Weights. The weights assigned to the individual observations in the first solution of the normal equations are NEWCOMB'S. To obtain definitive weights, the values of the elements and their secular variations resulting from the first solution were substituted in the equations of condition, and new residuals obtained, from which the final system of weights was built up.

The results which follow depend upon the final system of weights, the results obtained from the preliminary weights being omitted.

Mass of Venus. A correction to the assumed mass of *Venus* (NEWCOMB'S) was obtained from each of the five groups, and the weighted mean value formed. The result was as follows:

$$\frac{\delta M}{M} = +0.0112^* \pm 0.0058 ;$$

or, reciprocal mass = 403490 $\pm$ 2400 (p.e.);

NEWCOMB (*Astronomical Constants*, p. 101) has obtained a value of the mass of *Venus* from the periodic perturbations of the *Sun* and of *Mercury* which apparently is more accurate than the value obtained above. The probable error which he assigns to $\frac{\delta M}{M}$ from the *Sun* observations is $\pm .0034$, and from observations of *Mercury* $\pm .0050$. But in computing these probable errors, NEWCOMB was necessarily restricted to the customary

* The maximum effect of this change of mass amounts to 0".03 in the right ascension and 0".2 in the declination of *Mars*.

weight method, whereas it has been computed here from the discordance of five independent values. Using the weight method we obtain for the probable error of the above determination ≈ 0.0020 , or but one-third the value assigned. The larger value of the probable error should, however, be accepted.

Corrections to the elements. The mean value of $\frac{\delta M}{M}$ found above is now to be substituted in the five sets of normal equations, thus reducing the number of unknowns by one. Solution of the reduced normals leads to the following results:

CORRECTIONS TO THE ELEMENTS BY GROUPS

Group	Mean Epoch	$\delta\epsilon$	Wt.	$\delta\pi_1$	Wt.	δc	Wt.	δJ	Wt.	$\sin J\delta N$	Wt.
A	1765.6	+0''.284	11	-4''.87	23	+0''.532	18	-0''.296	5	+0''.397	9
B	1798.6	+0 .338	29	-3 .31	51	+0 .525	62	+0 .157	15	-0 .161	17
C	1836.2	+0 .186	62	-2 .94	155	+0 .507	129	-0 .072	111	+0 .103	121
D	1861.7	+0 .292	204	-2 .23	378	+0 .732	477	-0 .086	241	+0 .007	226
E	1894.8	+0 .088	356	-1 .46	732	+0 .795	765	-0 .118	382	-0 .033	454

The corrections to each element in the above table are now to be developed in the form $\xi + \eta T$, where T is in centuries from 1850.0. The results are as follows:

$$\begin{aligned}\delta\epsilon &= +0''.233 & -0''.255 T \\ &= .031 & = .083 \\ \delta\pi_1 &= -2''.51 & +2''.34 T \\ &= .06 & = .15 \\ \delta c &= +0''.665 & +0''.299 T \\ &= .021 & = .057 \\ \delta J &= -0''.075 & -0''.097 T \\ &= .021 & = .060 \\ \sin J\delta N &= +0''.038 & -0''.160 T \\ &= .026 & = .071\end{aligned}$$

ϵ and π_1 are measured from an arbitrarily chosen departure point, J and N from the *Earth's* equator. Transforming to the ecliptic and equinox, the corrections become:

$$\begin{aligned}\delta\lambda &= +0''.241 & -0''.290 T \\ \delta\pi &= -2 .50 & +2 .30 T \\ \delta c &= +0 .665 & +0 .299 T \\ \delta i &= -0 .025 & -0 .183 T \\ \delta\theta &= +2 .48 & -1 .31 T \\ \delta \log a &= .0000 0000 1\end{aligned}$$

The definitive elements of *Mars* resulting from this investigation are accordingly the following:

(T is in Julian centuries from 1900 Jan. 0, Gr. M. Noon.)

Mean longitude:

$$l = 293^\circ 44' 51''.46 + (53 r. + 222 117''.33) T + 1''.1184 T^2;$$

Longitude of perihelion:

$$\pi = 334^\circ 13' 5''.53 + 6626''.73 T + 0''.4675 T^2 - 0''.0043 T^3;$$

Eccentricity:

$$e = 19247''.168 + 18''.9895 T - 0''.0158 T^2, \\ = 0.0933 1290 + 0.0000 9206 4 T - 0.0000 0007 7 T^2;$$

Longitude of node:

$$\theta = 48^\circ 47' 11''.19 + 2775''.57 T - 0''.005 T^2 - 0''.0192 T^3;$$

Inclination to ecliptic:

$$i = 1^\circ 51' 1''.20 - 2''.430 T + 0''.0454 T^2; \\ \log a = 0.1828 9703 4$$

The following table gives a comparison between observation and theory, before and after the correction of NEWCOMB's elements of *Mars* in accordance with the results above, for each opposition in which there were at least five observations within a period of a month.

RESIDUALS, $O-C$

Opp.	Right Ascension			Declination		
	NEWCOMB	ROSS	No. Obs.	NEWCOMB	ROSS	No. Obs.
1758	+0''.4	+0''.7	9	+0''.4	+0''.1	9
1760	-1 .1	+0 .1	5	+3 .2	+2 .5	5
1762	-2 .4	-0 .8	5	+0 .7	+0 .1	5
1768	+6 .8	+1 .2	14	+0 .2	-2 .2	14

Opp.	Right Ascension			Declination		
	NEWCOMB	ROSS	No. Obs.	NEWCOMB	ROSS	No. Obs.
1792	-1".0	+0".5	8	+2".2	+1".4	8
1794	-1 .6	+0 .2	15	0 .0	-0 .9	15
1796	-2 .1	-1 .2	23			
1800	+3 .9	-1 .5	15	+4 .6	+2 .8	11
1802	+2 .6	-0 .4	6	+2 .5	+2 .3	6
1805	+1 .2	+1 .0	9	-1 .0	-0 .5	8
1807	-1 .6	-0 .7	19	+0 .1	-0 .2	18
1809	-3 .4	-1 .3	9	+1 .4	+0 .5	9
1811	-2 .5	+0 .1	11	+2 .4	+1 .9	12
1813	+2 .1	0 .0	8	-0 .1	-0 .2	11
1815	+4 .6	-0 .2	10	+1 .1	-1 .0	5
1820	+5 .7	+4 .3	5	+3 .6	+3 .8	5
1822	+0 .2	+0 .6	29	-0 .4	-0 .5	17
1824	-2 .7	-0 .9	16	+0 .6	-0 .3	14
1826	-4 .8	-2 .4	20	+0 .8	+0 .1	10
1828	-0 .2	+0 .9	23	-0 .4	-0 .1	16
1830	+4 .9	+0 .4	23	+0 .7	-1 .2	12
1832	+6 .2	+1 .0	16	+1 .0	-0 .2	5
1834	+3 .7	+0 .8	19	-0 .2	0 .0	14
1837	+0 .5	+0 .1	23	+0 .1	+0 .3	22
1839	-1 .7	-0 .7	28	+1 .2	+0 .7	24
1841	-1 .9	+0 .6	23	+1 .0	+0 .1	24
1843	-2 .0	+0 .8	29	+0 .8	+0 .4	20
1845	+2 .8	+0 .4	28	+0 .9	+0 .3	22
1847	+3 .9	-1 .0	20	+1 .6	-0 .2	22
1849	+4 .5	+1 .0	16	+1 .3	+1 .2	16
1852	+1 .3	0 .0	18	-0 .6	-0 .2	14
1854	-1 .8	-1 .1	27	+1 .0	+0 .9	27
1856	-2 .2	+0 .1	19	+1 .3	+0 .4	28
1858	-2 .3	+0 .7	22	+1 .3	+0 .6	29
1860	-0 .7	+0 .3	27	+0 .2	+0 .5	28
1862	+4 .7	+0 .1	17	+2 .9	+0 .8	30
1864	+5 .0	0 .0	18	+1 .2	+0 .4	21
1867	+3 .7	+0 .9	24	-0 .2	+0 .2	26
1869	+2 .2	+2 .0	23	-0 .6	-0 .5	22
1871	-1 .5	-0 .1	18	+1 .5	+1 .0	21
1873	-3 .2	0 .0	22	+1 .2	+0 .1	23
1875	-3 .8	+0 .2	24	+0 .8	+0 .6	23
1877	+0 .7	-1 .9	23	+0 .9	-0 .2	15
1879	+5 .1	+0 .1	21	+1 .7	+0 .2	21
1882	+4 .8	+1 .5	26	-0 .6	-0 .4	17
1881	+1 .1	+0 .1	26	-0 .7	-0 .2	27
1886	-1 .5	-0 .6	20	+0 .3	+0 .1	20
1888	-1 .7	+1 .0	13	+1 .0	-0 .1	14

Opp.	Right Ascension			Declination		
	NEWCOMB	ROSS	No. Obs.	NEWCOMB	ROSS	No. Obs.
1890	-4".4	-0".4	30	+1".4	+0".5	13
1892	+0 .5	+0 .9	17	+0 .6	+0 .7	16
1894	+3 .9	-0 .8	34	+2 .5	+0 .5	35
1896	+3 .9	-1 .0	25	0 .0	-0 .3	26
1899	+2 .3	0 .0	29	-0 .5	+0 .2	31
1901	-0 .6	-0 .5	23	+0 .2	+0 .4	21
1903	-2 .2	-0 .4	34	+1 .2	+0 .6	39
1905	-5 .5	-1 .6	20	+1 .1	-0 .2	20
1907	-4 .8	-0 .2	19	-0 .8	-0 .6	20
1909	+2 .7	-0 .5	22	+1 .5	0 .0	22
1911	+6 .5	+1 .5	12	+0 .8	-0 .2	12

From the above residuals, v , we get

	R. A.	DECL.
Σr^2 (NEWCOMB)	651	117
Σr^2 (ROSS)	64	57

Secular variation of the elements. As already pointed out, the secular variations adopted by NEWCOMB in his *Tables of Mars* are theoretical and semi-theoretical, and do not agree with his own determination of their value from observation. In the following table are collected:

(a) NEWCOMB's theoretical values, adopting masses given in *Tables of Sun*;

(b) Values used by NEWCOMB in *Tables of Mars*;

(c) Values deduced by NEWCOMB from observations of *Mars*;

(d) Values resulting from the present investigation.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
D_{ρ}	+ 18".71	+ 18".71	+ 19".00	+ 19".00
$e D_{\pi}$	+148 .71	+149".23	+149 .55	+149 .45
D_{ρ}	- 2 .29	- 2 .29	- 2 .26	- 2 .47
$\sin i D_{\theta}$	- 72 .57	- 72 .57	- 72 .60	- 72 .61

Theoretical secular variations. These depend upon the adopted masses of the disturbing planets. The uncertainty as to the mass of *Venus* makes the variations doubtful to a considerable degree. The present investigation has led to a value of its mass which is midway between that found by NEWCOMB on the one hand and by LEVERRIER and COWELL on the other. Below are collected the secular variations resulting from these three values of the mass of *Venus*, and a comparison with the observed values (*d*) given above.

From mass of *Venus* by

	NEWCOMB	COWELL	ROSS
D_{ρ}	+ 18".71	+ 18".71	+ 18".71
$e D_{\pi}$	+148 .71	+148 .81	+148 .76
D_{ρ}	- 2 .29	- 2 .03	- 2 .16
$\sin i D_{\theta}$	- 72 .57	- 73 .12	- 72 .85

Outstanding residuals:

N.	C.	R.
+0".29	+0".29	+0".29
+0 .74	+0 .64	+0 .69
-0 .18	-0 .44	-0 .31
-0 .04	+0 .51	+0 .24

The equatorial correction to NEWCOMB'S declination system. It is useful to determine from the discussion the constant correction $\Delta\delta$ to the declinations of NEWCOMB'S *Fundamental Catalogue*, the system of which has been adopted in the present discussion. This correction can be obtained equally well by introducing $\Delta\delta$ as an unknown to be obtained from the solution, or from the residuals in declination remaining after the final values of the elements have been substituted in the equations of condition. The latter method has been deemed preferable. The mean value of $\Delta\delta$ for each of the five groups, with its weight, was found to be as follows:

Period	$\Delta\delta$	Wt.	T (Centuries)
A	-0''.02	28	-0.844
B	+0 .30	55	-0.514
C	-0 .13	653	-0.138
D	+0 .32	978	+0.117
E	+0 .04	1689	+0.448

solving for $\Delta\delta$,

$$\Delta\delta = +0''.087 + 0''.017 (T - 1850.0) \\ = .085 = .250 \text{ (p.e.)}$$

$\Delta\delta$ is thus uncertain by its entire amount. Strictly speaking, it is the sum of the constant equatorial declination error of the fundamental catalogue, the constant error of bisecting the disc of the planet, and a possible difference between the centers of gravity and figure of *Mars*.

It will be of interest to compare this value of $\Delta\delta$ with the value obtained by HILL from discussion of the observations of *Jupiter* and *Saturn*. He found (*Astr. Papers*, Vol. VII, Pt. 1, p. 18).

$$\Delta\delta = +0''.01 + 0''.45 (T - 1850.0),$$

in which the system is Boss's. The reduction to NEWCOMB is $+0''.10 + 0''.36 (T - 1850.0)$. (*Astr. Papers*, Vol. VIII, Pt. 2, p. 213). Reduced to NEWCOMB'S system, HILL'S value becomes

$$\Delta\delta = -0''.09 + 0''.09 (T - 1850.0).$$

The mean result of both investigations is therefore

$$\Delta\delta = 0''.00 + 0''.05 (T - 1850.0).$$

Fluctuations in the mean longitude of Mars. The question of fluctuations in the longitude of *Mars* is intimately connected with fluctuations in the *Earth*'s longitude. For the correction $\delta\epsilon$ to the longitude of *Mars* found above is strictly a correction to the difference of the longitude

of *Mars* and the *Earth*, or nearly so. Direct observations of the *Sun* appear to establish the existence of fluctuations in the *Earth*'s longitude. Observations of *Mars* should furnish valuable confirmatory evidence. If observations had not been so closely restricted to the neighborhood of opposition, it would have been possible to obtain strong values of the fluctuations of both *Mars* and the *Earth*. Because of this restriction there is a decided correlation between these fluctuations, which weakens their determination. Nevertheless valuable results can be obtained by solving the equations of condition for δl and δL , corrections to the mean longitude of the *Earth* and of *Mars*. Before making the solution, the general values of the corrections to the elements of *Mars* obtained above were substituted in the equations of condition, which were then solved separately for each opposition. Mean values of δl and δL were then formed for periods of nearly ten years. Only the equations in right ascension were used. The results, with their weights, are as follows:

Epoch	δl (<i>Mars</i>)	Wt.	δL (<i>Earth</i>)	Wt.
1754	+0''.29	89	+0''.46	28
1763	+0 .03	28	+0 .90	10
1775	+1 .24	40	+1 .50	15
1790	-0 .04	77	+0 .10	29
1802	-0 .96	29	-1 .27	11
1810	+3 .20	20	+5 .25	8
1821	-1 .41	6	-1 .67	3
1828 — 1839	+0 .18	956	+0 .64	329
1841 — 1850	-0 .13	679	-0 .45	195
1852 — 1860	+0 .52	721	+0 .83	252
1862 — 1871	+0 .11	559	+0 .01	192
1873 — 1880	+0 .11	1036	+0 .32	329
1882 — 1890	+0 .60	545	+0 .71	197
1892 — 1901	+0 .25	666	+0 .58	214
1903 — 1912	+0 .89	688	+1 .45	227

The values of δl and δL given above have been plotted along side of values of δL obtained from direct observations of the *Sun*. (See the writer's paper on "*The Sun's Mean Longitude A. J.*", No. 691, p. 156). All three curves show the existence of a major fluctuation from about 1760 to 1830, and an acceleration of the motion of both *Mars* and the *Sun* during the last two decades.

Weighted mean values of δl and δL were formed from the above table, using only results since 1829, as follows:

$$\delta l_m = +0''.31 = .08 \text{ Epoch 1870}$$

$$\delta L_m = +0 .54 = .13 \text{ Epoch 1870}$$

The probable errors assigned were obtained from the individual values, and so include the effect of fluctuations. The value of δL_m is in fair agreement with the value obtained by the writer from observations on the *Sun* (*loc. cit.*) which is, omitting values before 1829,

$$\delta L = +0''.19 + 0''.02 (T - 1900.0)$$

Systematic errors in observations of Mars. For the study of systematic errors in observing *Mars*, only ob-

servations during the period 1900 to 1912 have been considered. Results obtained for this period should be representative of the best to be expected, under present conditions. Only observations at Greenwich, Paris and Washington are available. Each observation of right ascension and declination has been compared with places computed from the final elements of this paper. New residuals, $O - C$, were thus found, whose mean values by opposition and observatory are as follows:

Opposition	$O - C$, RIGHT ASCENSION						$O - C$, DECLINATION					
	Gr.	No. Obs.	Ps.	No. Obs.	Wash.	No. Obs.	Gr.	No. Obs.	Ps.	No. Obs.	Wash.	No. Obs.
1900 — 01	— ^s .025	30	— ^s .016	14	^s .000	23	+'' ^s .39	33	+'' ^s .65	13	+'' ^s .15	24
1902 — 03	— .050	34	+ .041	43	— .042	31	+ .90	36	+ .42	52	+ .13	30
1905	— .068	21	— .040	11	— .026	30	+ .78	21	—1 .03	12	+ .69	29
1907	+ .082	49	+ .050	26	— .018	21	— .11	49	+ .65	26	+ .60	21
1909 — 10	+ .032	48	— .002	16	— .040	20	+ .36	49	+ .23	16	+ .83	20
1911 — 12	+ .155	31	+ .070	11			— .08	32	+1 .20	2		
Mean	+ ^s .021		+ ^s .017		— ^s .025		+'' ^s .37		+'' ^s .35		+'' ^s .48	

In computing the probable errors of a single observation of α and δ which follow, the systematic errors shown in the above table are not considered, being treated as accidental errors. This gives a larger but more useful value than that obtained from discordances from the mean values in the above table. For comparison, results from BRADLEY'S observations are included.

Probable error of one observation and weights.

Place	Probable Error		Weights	
	α	δ	α	δ
Greenwich	$\pm 1''.02$	$\pm 0''.67$	3	7
Paris	1 .08	0 .78	3	5
Washington	0 .77	0 .59	5	8
Bradley	1 .73	1 .23	1	2

From a study of the mean residuals by opposition given above, important conclusions can be drawn. Large systematic differences in both right ascension and declination between the mean results for different observatories are shown. These differences are greater than the probable error of a *single* observation. Under present conditions of observing, therefore, it appears useless for any one observatory to collect a large number of observations at an opposition. Fewer observations scattered over a more extended period than is now customary, both before and after opposition, would be economical in effort and more useful in results. Since a large part of the systematic differences are undoubtedly due to

personal equation in observing the planet, many observatories and observers should take part in the observations. To the writer it seems highly desirable to discard the fixed reticule in right ascension observations in the case of all planets as well as the *Sun*, substituting the transit micrometer. A new and useful field for photographic work in the determination of planetary positions is opened up. In view of recent findings that fluctuations in the positions of the planets are probable, as is known actually to be the case for the *Moon*, more accurate positions than are now available are necessary for the proper development of the subject, on account of the minuteness of the quantities involved.

The mean results $O - C$ in both coördinates in the table above for the Greenwich and Paris observations are seen to agree closely for the decade considered, notwithstanding the large differences by oppositions. The mean value for Washington stands out prominently showing an undoubted systematic difference for the twelve years. It is interesting to note that the *Sun* observations show the same difference, the Washington right ascensions being about three-quarters of a second of arc less than the Greenwich and Paris, as is here seen to be the case for *Mars*. The mean agreement in the declinations is much better, although large differences by oppositions are shown, as in the case of the right ascensions. It can be concluded from the mean value of $O - C$ that for this decade the equatorial correction to NEWCOMB'S declination system is $-0''.40$. This does not agree with the Washington determination, which is $+0''.54$. According

to the Greenwich observations (*Nine Year Catalogue*, 1900) the equatorial correction to NEWCOMB is $-0''.12$, agreeing better than the Washington result with the value determined from *Mars*. From the totality of

observations of *Mars*, the correction is $+0''.10$, as shown elsewhere in this paper. HILL's value from *Jupiter* and *Saturn* observations is $-0''.04$.

Nautical Almanac Office, U.S. Naval Observatory.

ζ SAGITTAE = A. G. C. 11.

A RAPID BINARY.

By GEORGES VAN BIESBROECK.

This 5th magnitude star (spectral type A), the position of which for 1900 is

$$19^{\text{h}} 44^{\text{m}} 32^{\text{s}} \quad +18^{\circ} 53'.5$$

was first noted as a wide double star by W. HERSCHEL in 1781. W. STRUVE classified it as Σ 2585 and estimated the companion as $S^{\text{M}}.7$. The latter has been measured frequently, but so far there is no appreciable change in

the relative position, as can be seen from this short selection of observations:

1831.10	312°.8	8''.49	6 <i>n</i>	W. STRUVE
1867.38	312 .8	8 .62	4 <i>n</i>	DEMBOWSKI
1889.85	311 .1	8 .66	8 <i>n</i>	H. STRUVE
1904.65	310 .4	8 .57	5 <i>n</i>	BOWYER
1908.71	311 .2	8 .60	3 <i>n</i>	VAN BIESBROECK

Date	OBSERVED		COMPUTED		<i>n</i>	OBSERVERS
	θ	ρ	θ	ρ		
1862.62	($\pm 337^{\circ}$)	euneiform	23°.2	0''.09	1	O. STRUVE
75.75	(316.9)	($\pm 0''.4$)	161 .9	0 .28	1	DEMBOWSKI
77.74	158.0	0 .24	157 .2	0 .25	3	BURNHAM
78.67	157.1	0 .35	155 .0	0 .23	2	BURNHAM
80.09	152.2	0 .31	149 .5	0 .19	4-3	BURNHAM
83.41	(176.6)	(0 .17)	123 .3	0 .11	6	ENGELMANN
86.79	(177.3)	oblong	44. 6	0 .07	2	H. STRUVE
86—87		simplex		<0 .07	15	SCHIAPARELLI
88.77		simplex	359 .8	0 .05	2	H. STRUVE
89.57	185.0	0 .12	186 .3	0 .12	2	BURNHAM
90.04	192.8	± 0 .20	182 .6	0 .17	3	H. STRUVE
90.53	179.7	0 .12	180 .4	0 .21	3	BURNHAM
91.61	182.8	0 .15	177 .3	0 .26	3	BURNHAM
91.71	186.0	0 .25	177 .1	0 .27	4	H. STRUVE
95.69	171.7	0 .24	170 .9	0 .33	2	LEWIS
98.41	163.9	0 .32	166 .3	0 .31	3	LEWIS
98.84	170.9	0 .47	165 .6	0 .31	2	BRYANT
99.26	158.8	0 .29	164 .9	0 .30	2	BOWYER
1899.68	159.2	0 .22	164 .2	0 .30	2	LEWIS
1900.62	162.9	0 .20	162 .5	0 .29	2	BRYANT
1.06	158.7	0 .24	161 .7	0 .28	2	AITKEN
1.71	151.9	0 .18	161 .2	0 .28	5	BRYANT
2.62	156.5	0 .16	158 .2	0 .25	2	BRYANT
3.73	158.4	(<0 .30)	155 .1	0 .23	1	VAN BIESBROECK
3.81	154.3	0 .21	154 .9	0 .23	2	BOWYER
5.87	148.2	0 .20	146 .7	0 .18	2	AITKEN
8.73	113.8	0 .18	121 .9	0 .11	4	BRYANT
8.75	(± 111)	(<0 .25)	121 .6	0 .10	2	VAN BIESBROECK
12.69	29.4	0 .14	29 .6	0 .09	2	AITKEN
1916.24	177.2	0 .22	178 .8	0 .24	1	VAN BIESBROECK

The angular precession being of $-0^{\circ}.6$ only per century, the change, if any, is less than the errors of measurement. According to the *P. G. C.*, the proper-motion of the principal star is

$$\mu = +0^{\circ}.0013 \quad \mu' = +0''.025$$

or $0''.030$ in the direction 34° . Since this motion has not affected the relative position, we must conclude that the two stars form a physical system.

In 1875, ALVAN G. CLARK, using the 12-inch now at the Lick Observatory, discovered the principal star to be a close pair, the two stars differing by about one magnitude in brightness. But it was not until 1877 that useful observations became available, mainly those of BURNHAM. The above table shows the measures on the close star.

In addition to these measures we have to record SCHIAPARELLI's statement that in 1886-87 he examined the stars on fifteen occasions, with the 18-inch Milan refractor, without separating the object or even tracing any elongation. The references for the measures till 1905 are found in BURNHAM's *General Catalogue* (No 9643), where we find the remark: "the change, if any, has apparently been in distance." But AITKEN's measures in 1912 (*Publ. Lick Obs.*, Vol. XII, p. 142) have shown that the star is changing decidedly, and this was confirmed here in the most striking way by a recent observation with the 40-inch, the position angle having diminished by more than 200 degrees in $3\frac{1}{2}$ years.

This led to a discussion of the available observations and showed that the period is about twenty-five years. A first approximation of the orbit was obtained graphically, allowing for the law of areas. This orbit was improved by steps, using the differential coefficients of the elements, so as to fit in with the observations entitled to the highest weight. The result is:

ELEMENTS OF ζ Sagittae

Period, 25.20 years ($\mu = -14^{\circ}.286$)

$$\begin{aligned} T &= 1914.11 \\ \Omega &= 4^{\circ}.6 \\ i &= 78^{\circ}.1 \\ \omega &= 295^{\circ}.0 \\ e &= 0.85 \\ a &= 0''.32 \text{ (Retrograde)} \end{aligned}$$

For the sake of comparison, the positions computed from these elements are given side by side with the observations. That comparison gives rise to the following remarks: the elongation suspected by O. STRUVE in 1862 is probably spurious, the computed distance at that time being less than $0''.1$. Likewise DEMBOWSKI's estimate in 1875 is entitled to very small weight. His 7-inch telescope could not possibly separate a star of this class. In 1876.68 he states that the star appears single to him. The same remark applies to ENGELMANN's measures (7½-inch telescope) in 1883. His position angles range from 160° to 190° and his distances from $0''.14$ to $0''.25$. The negative observations by H. STRUVE and SCHIAPARELLI in 1886-1888 are well accounted for. Periastron passage occurred at 1888.91 and at that time the distance was less than $0''.05$.

The following ephemeris shows the character of the motion for the next two decades.

EPHEMERIS FOR ζ Sagittae

Date	Position Angle	Distance	V km
1916.0	179°.4	0''.21	-16.6
18.0	174°.8	0.27	-12.1
20.0	171°.6	0.30	-9.2
22.0	168°.6	0.30	-6.8
24.0	165°.6	0.28	-4.6
26.0	162°.0	0.26	-2.5
28.0	157°.4	0.22	-0.4
30.0	151°.0	0.18	+1.8
32.0	140°.7	0.14	+4.5
34.0	119°.2	0.09	+7.9
36.0	73°.7	0.07	+13.1
1938.0	25°.5	0.08	+25.0

Assuming the mass of the system to be equal to that of the Sun, the hypothetical parallax would be $0''.013$ and the relative velocities as given in the Column V.

It is seen that the motion will be again very slow for a number of years, and that it will hardly be possible to improve the elements given here until, after some twenty years hence, the angular motion again becomes rapid.

Yerkes Observatory, April, 1916.

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ON AN EXTENSION OF THE USE OF THE NEWCOMB OPERATORS,

By R. T. A. INNES.

The late SIMON NEWCOMB gave much attention to the problem of the development of the elliptic perturbative function and made some notable advances. The most important of these was the introduction of the operators of which mention was made first by him in the *Comptes Rendus* LXX, p. 385, and a little later he developed a complete theory in Vol. III of the *American Journal of Mathematics*. He applied the operators extensively in his developments of the perturbative function in Vols. III and V of his celebrated *Astronomical Papers*, one in terms of the eccentric, the other in terms of the mean anomalies. CHESSIN has given other properties of these operators in past volumes, and in two papers published some years ago in the *Transactions of the Royal Society of South Africa*, I have extended these properties and have shown that they are such that it is simple to write down two or perhaps three of the lowest powers of the eccentricities and mutual inclination, treated as small quantities, in any term of the perturbative function, however high the order may be, thus

$$\sigma^{2s} e^j e_1^{j'} \left[\Pi_{j,j'}^s + e^2 \Pi_{j,j'}^{s,j+2,0} + e_1^2 \Pi_{j,j'}^{s,j'+2} + e^4 \Pi_{j,j}^{s,j+4,0} \right. \\ \left. + e^2 e_1^2 \Pi_{j,j'}^{s,j+2,j'+2} + e_1^4 \Pi_{j,j}^{s,j'+4} \right] \dot{A}_i \times C \left[pg' + qg + \theta \right].$$

can be found for any values of s , j and j' . LE VERRIER gave a literal development including all terms of the 7th order, and this was extended by BOQUET to the 8th order. This meant, if any of the terms indicated by $\Pi_{j,j'}^{s,j+4,0}$, $\Pi_{j,j'}^{s,j+2,j'+2}$ or $\Pi_{j,j}^{s,j'+4}$ were included, the limitation was

$$2s + j + j' + 4 = 7$$

in the case of LE VERRIER's expansion. Thus if we have to include these 4th order terms, $2s + j + j'$ must not

N.B. A leaded C stands for cosine and similarly a leaded S would be used for sine.

exceed 3. As it is very seldom indeed if ever, that we need to go beyond the 4th order, operators such as those just written, it will be obvious that the use of the NEWCOMB operators makes a great advance.

In writing of these operators NEWCOMB suggests that they "contain the germ of some principle which I have not fully grasped, and which may admit of wider and more important applications."

In a paper recently sent to the Royal Society of South Africa I have demonstrated the following theorem: —

Being given the development of the perturbing function and its derivatives, assuming one eccentricity to be zero, then by means of the Newcomb operators, this eccentricity can be introduced and the complete development attained.

The derivatives are the logarithmic differential coefficients $D = \alpha \frac{\partial}{\partial \alpha}$ with reference to α (ratio of the mean distances) and are actually required in computing perturbations.

The principle really extends somewhat further, because it can be applied to any elliptic function of the distances of two planets from the *Sun*, such as for instance

$$\frac{r}{r_1^2} H$$

in which H is the cosine of the angle between the planets as seen from the *Sun*.

Any operator such as Π_j^n is multinomial in i and D and must operate on a function of α such as the Laplacian functions

$$b_{2s+1}^{(i)} = N \alpha^{i+s} + N_1 \alpha^{i+s+2} + \text{etc.}$$

thus

$$\Pi_{1,0} = \frac{1}{2}(-2i - D) b_1^{(i)}$$

It will be seen that the operator, being a function of i must operate on its appropriate $b^{(i)}$. It would be wrong

to operate with Π for $(i+2)$ on b^2 and so on. NEWCOMB (*Astr. Papers*, V, p. 27) instructs us to use the operator $\Pi(i)$ on $a'A_i$ where

$$a'A_i = b_1^{(i)} - \frac{1}{2}\sigma^2 a(b_3^{(i-1)} + b_3^{(i+1)}) + \text{etc.}$$

which seems to violate this rule but actually $a(b_3^{(i-1)} + b_3^{(i+1)})$ is easily expressible in terms of $b_1^{(i)}$ and the procedure is correct.

NEWCOMB writes any term in the perturbative function as follows

$$\sigma^{2s} e^j e_{i'}^{j'} \left[\overset{s}{P}_{j,j'}^{j,j'} + e^2 \overset{s}{P}_{j,j'}^{j+2,j'} + e_{i'}^2 \overset{s}{P}_{j,j'}^{j,j'+2} + \text{etc.} \right] \\ \mathbf{C} [pg' + qg + \theta]$$

but that part of expression within the brackets can be written more simply, and omitting s , as

$$[\Pi_{j,j'} + e^2 \Pi_{j,j'}^{2,0} + e_{i'}^2 \Pi_{j,j'}^{0,2} + \text{etc.}] a'A_i$$

If now, we suppose e to become zero the term reduced to

$$[\Pi_{0,j'} + e_{i'}^2 \Pi_{0,j'}^{0,2} + \text{etc.}] a'A_i \quad (\text{A})$$

As

$$\Pi_{j,0}^{2n,0} \times \Pi_{0,j'}^{0,2n'} = \Pi_{j,j'}^{2n,2n'} \quad \text{which is}$$

NEWCOMB's capital discovery, we can operate upon (A) in its entirety firstly with Π_j^0 , then with Π_j^2 , then with Π_j^4 and so on as far as may be necessary. It is not necessary to deal with each element separately.

As a simple numerical example, we will develop $-a' \frac{r}{r_1^2} H$ in the case of *Jupiter* and *Saturn*, firstly assume *Jupiter's* eccentricity e to be zero; and the constants adopted will be those given in HILL's *Theory of Jupiter and Saturn* (Collected Works, III, pp. 21, 22 and 64) so that a ready check is available.

As HILL's h_1 and l are factored by $\mathbf{C} \varphi$, ($\mathbf{S} \varphi = e$) this quantity is removed so that (the [...] brackets signifying logarithms)

$$h_1 = [9.0477990] \quad l = [9.7272022]$$

As e is assumed to be zero all the quantities P_i and Q_i computed by HILL, with the exception of $P_1 = Q_1 = 1$ vanish, and the only terms which remain will be of the form

$$a' \frac{r}{r_1^2} H = \frac{1}{2} i_1^2 [h P_{i'}' \pm h_1 Q_{i'}'] \mathbf{C} [\pm i' g' - g] \\ - \frac{1}{2} i'^2 [l P_{i'}' \pm l_1 Q_{i'}'] \mathbf{S} [\pm i' g' - g]$$

(see HILL, p. 63).

When we have computed this expression, we will introduce the effect of e into the term $\pm i' g' - g$ by operating upon it with

$$P_0 = [1 + e^2 \Pi^{2,0} + e^4 \Pi^{4,0} + \text{etc.}] a$$

Next, if we operate with

$$P_1 = e [\Pi_1 + e^2 \Pi_1^2 + e^4 \Pi_1^4 + \text{etc.}] a$$

we get the coefficient of the term $\pm i' g' + (1-1)g = \pm i' g' + 0g$. Similarly the compound operator

$$P_{\pm n} = e^n [\Pi_{\pm n} + e^2 \Pi_{\pm n}^2 + e^4 \Pi_{\pm n}^4 + \text{etc.}] a$$

will yield the coefficients of

$$\pm i' g' - (1 \mp n)g$$

It will be seen immediately that the numerical work is really simpler than the explanation.

NEWCOMB (*Astr. Papers*, V, pp. 27-29) furnishes the values of the Π operators, and as in this case $D^n a = a$ we can replace each of the D 's by unity, and as the coefficient of g is -1 , the i 's by -1 . Thus we have

$$P_0 = 1 + \frac{1}{4} e^2 (-4 + 1 + 1) \\ + \frac{1}{64} e^4 (+16 - 9 + 2 - 8 - 1 - 2 + 1) + \text{etc.}$$

$$P_{-1} = \frac{1}{2} e (2 - 1) \\ + \frac{1}{16} e^3 (-8 - 10 - 2 + 4 + 5 + 3 + 2 + 1 - 1) + \text{etc.}$$

$$P_1 = \frac{1}{2} e (-2 - 1) \\ + \frac{1}{16} e^3 (+8 - 10 + 2 + 4 - 5 + 3 - 2 + 1 - 1) + \text{etc.}$$

We need not go further. It will be seen that in the case considered these reduce to

$$P_0 = 1 - \frac{1}{2} e^2 + \frac{1}{64} e^4 - \text{etc.} \\ P_{-1} = \frac{1}{2} e - \frac{3}{16} e^3 + \text{etc.} \\ P_1 = -\frac{3}{2} e \quad \text{etc.}$$

and are the coefficients of terms in the development of $\frac{r}{a} \frac{\mathbf{C}}{\mathbf{S}} f$. The others which may be required can be taken from CAYLEY's tables in the *Memoirs of the R. Ast. Soc.*, XXVI, pp. 260-262.

Thus, both literally and for $e = 0.04824277$.

$$P_0 = 1 - \frac{1}{2} e^2 + \frac{1}{64} e^4 - \text{etc.} = 0.99883623 = [9.9994943] \\ P_1 = -\frac{3}{2} e = -0.07236415 = [8.8595235]_n \\ P_{-1} = \frac{1}{2} e - \frac{3}{16} e^3 + \text{etc.} = 0.02407928 = [8.3816435] \\ P_2 = \frac{1}{8} e^2 + \frac{1}{24} e^4 + \text{etc.} = 0.00029070 = [6.463445] \\ P_{-2} = \frac{3}{8} e^2 - \frac{3}{16} e^4 + \text{etc.} = 0.00087073 = [6.939884] \\ P_3 = \frac{1}{24} e^3 + \frac{1}{64} e^5 + \text{etc.} = 0.00000468 = [4.6702] \\ P_{-3} = \frac{1}{3} e^3 - \frac{5}{12} e^5 + \text{etc.} = 0.00003732 = [5.5720] \\ P_4 = \frac{1}{32} e^4 + \text{etc.} = 0.00000013 = [3.114] \\ P_{-4} = \frac{1}{24} e^4 + \text{etc.} = 0.00000176 = [4.2455]$$

$i'g'$ $i' i$	$e = 0$	$i' i$	P_0	$i' i$	P_1	$i' i$	P_{-1}	$i' i$	P_2
-4 -1	+	4	-4 -1	+	4	-3 0	-	3	
-3 -1	+	38	-3 -1	+	38	-2 0	-	22	
-2 -1	+	310	-2 -1	+	310	-1 0	-	95	
-1 -1	+	1307	-1 -1	+	1305	1 0	+	807423	
1 -1	-	0.11157773	1 -1	-	11144786	2 0	+	90452	
2 -1	-	1249958	2 -1	-	1248504	3 0	+	8550	
3 -1	-	118148	3 -1	-	118011	4 0	+	757	
4 -1	-	10458	4 -1	-	10446	5 0	+	65	
5 -1	-	894	5 -1	-	893	6 0	+	5	
6 -1	-	75	6 -1	-	75				
7 -1	-	6	7 -1	-	6				

$i' i$	P_{-2}	$i' i$	P_3	$i' i$	P_{-3}	$i' i$	P_4	$i' i$	P_{-4}
1 -3	-9715	1 +2	-52	1 -4	-416	1 +3	-1	1 -5	-20
2 -3	-1088	2 +2	-6	2 -4	-46			2 -5	-2
3 -3	-103			3 -4	-4				
4 -3	-3								

The calculation for the first line of (B) will suffice as an example.

The numbers in the P columns are in units of the 8th decimal. Certain coefficients coalesce, those in P_1 amongst themselves, those P_0 with those in P_{-1} , etc.: collecting these, the completed table, with HILL's figures for comparison is as follows:

$i' i$	By NEWCOMB'S Operators	From HILL, p. 64
1 0	+	807328
2 0	+	90430
3 0	+	8547
4 0	+	757
5 0	+	65
6 0	+	5
-3 -1	+	4
-2 -1	-	53
-1 -1	-	1939
1 -1	-	11144787
2 -1	-	1248504
3 -1	-	118011
4 -1	-	10446
5 -1	-	893
6 -1	-	75
7 -1	-	6
-2 -2	+	1
-1 -2	-	21
1 -2	-	268671

$i' i$	By NEWCOMB'S Operators	From HILL, p. 64
2 -2	-	30098
3 -2	-	2844
4 -2	-	251.8
5 -2	-	21.52
6 -2	-	1.81
1 -3	-	9715
2 -3	-	1088
3 -3	-	103
4 -3	-	9
5 -3	-	0.8
etc.	etc.	etc.

The occasional differences in the 8th decimal place signify nothing as the working in both cases was only with 7-figure logarithms; besides this my work is unchecked. The comparison gives a numerical justification of the work of the NEWCOMB operators, which supplements the algebraical work.

In the example chosen, it was assumed that e became zero, but the choice is optional. Both cases, namely, firstly with e zero and with e' zero can be worked. After completion, the two expansions will agree if the numerical work is exact.

Union Observatory, Johannesburg, 8th April, 1916.

TWELVE FAINT STARS WITH SENSIBLE PROPER-MOTIONS.

By EDWIN P. HUBBLE.

In the table below is given an assortment of proper-motion stars, which, with one exception, were found by comparing under the Blink microscope plates taken with the 24-inch reflector by the writer during the last eight months, with earlier plates taken with the same instrument by MESSRS. RITCHEY, PARKHURST and JORDAN. The plates were usually exposed so long that the images of the brighter stars were large enough to mask any moderate displacements, and this probably accounts for the fact that the list is comprised of faint stars only.

The magnitudes are estimations, based on exposure time, density of the image, condition of the sky and comparison with whatever charts were available. On my working scale, the limit of the Franklin-Adams charts is about 13.5 to 14th magnitude, and that of the Wolf-Palisa charts about 16th magnitude.

When a star showed a decided motion under the Blink, it was measured with reference to neighboring stars, under a Gærtner machine. The accuracy of the measures is not great, due to the fact that usually the star was at some distance from the center of the field, thus was subject to large distortion, and further was blurred by the long exposure. For the same reasons, no attempt was made to give positions more accurate than is necessary

for identification. The smallest actual displacement dealt with here was 2" of arc, equal, on the scale of the reflector, to 0.023^{mm}. Under average conditions, a displacement of half this amount can readily be detected.

Attempts were made to use the stereoscopic attachment of the comparator, but the results were discouraging. Slight differences in the arrangement of grains in the images were sufficient to produce illusory relief on a large scale. Plates taken on the same evening, and even second negatives from one positive, gave the effect of pronounced relief. On the other hand, in some cases of displacements well established by the Blink and by actual measurement, no relief whatsoever could be detected in the stereoscope.

Four stars in the list are of the 15th magnitude, or fainter. So far as I am aware, these are the faintest stars in which appreciable motion has been found. Fortunately, it was possible to check these cases, by two or more plates taken at each epoch, so there can be no question as to the reality of the motions. In view of the small number of fields examined, fifteen in all, it is reasonable to suppose that considerable numbers of such faint stars exist in the immediate neighborhood of our Sun.

No.	Mag.	1900		μ	ϵ	Interval Years	Early Plate Taken by
		Right Ascension	Declination				
1	11.5	0 ^h 45 ^m 20 ^s	+57° 44'.8	1".50	72°.2	12.7	PARKHURST
2	15.0	6 33 38	+ 8 51.2	0 .27	163 .7	16.2	JORDAN (see note)
3	13.2	10 46 23	+14 31.5	1 .07	280 .5	9.9	PARKHURST
4	15.5	10 47 59	+13 29.2	0 .48	214 .6	9.9	PARKHURST
5	16.0	10 48 1	+13 40.4	0 .29	270 .0	9.9	PARKHURST
6	9.0	13 37 58	+56 32.4	0 .24	256 .0	45.0	<i>A. G. Catalogue</i>
7	9.7	14 5 5	-13 27.2	0 .38	190 .0	11.8	PARKHURST
8	14.0	14 8 46	-13 18.9	0 .73	242 .2	11.8	PARKHURST
9	15.3	14 19 25	+54 2.1	0 .22	303 .7	13.6	PARKHURST
10	11.0	14 22 8	+53 47.2	0 .52	213 .1	13.6	PARKHURST
11	12.0	18 16 0	-16 5.1	0 .23	8 .5	13.9	RITCHEY
12	13.7	21 1 24	+23 29.7	0 .20	163 .0	11.0	PARKHURST

1. This star has a decided yellowish tinge. It is in the same field as η Cassiopeia, which has a motion almost as large and in the same general direction. The agreement, however, is not sufficiently close to warrant any connection.

2. Precedes 4.4 and is 97" north of *R Monocrotis*, the nucleus of the nebula *N. G. C.* 2261. An excellent plate taken by MR. JORDAN in March, 1908, gave an

interval of 7.96 years. At the request of the Director of this observatory, MME. DOROTHEA ROBERTS had the great kindness to furnish us with copies of a plate of the nebula taken by the late MR. ISAAC ROBERTS in January, 1900. Besides confirming changes in the nebula itself, these copies afforded a more accurate determination of the motion of this star. The total displacement in the 16.2 years' interval was 4".37. Plates taken with a color-filter show the star has no appreciable color-index.

5. Precedes 4^s.89 and is 56".6 north of the 8th magnitude star *B. D.* +13°2320. Comparison of positions of the bright star in the *Weisse Bessel* and in the Leipzig *A. G. Catalogues*, apparently showed a proper-motion of the same amount and in the same direction as that of the small star. This was tested by transits taken with the 12-inch refractor, determining the $\Delta\alpha$ between *B. D.* +13°2320 and *B. D.* +13°2318.

<i>Weisse Bessel</i>	(1825)	69 ^s .78
Leipzig <i>A. G.</i>	(1869)	68.98
Mean of six transits	(1916.3)	68.90

Thus there is apparently no connection between the two stars.

6. *B. D.* +56°1675. In measuring positions of a cluster of small nebulae in this region, the star was found

to be out of its position as given in the Helsingfors-Gotha volume of the *A. G. Catalogue* (No. 7668), by 1^s.3 in α and 2".5 in δ . The discrepancy was further checked by differential measures of this star and *B. D.* +56°1674. Either the *A. G.* position is wrong, or, what is more probable, the star has a sensible proper-motion.

7. *S. D.* -13°3834, visual magnitude given as 9.7. The size of the images renders the amount of motion very uncertain.

9. 47^s.8 following and 208" north of *B. D.* +54°1668.

11. Yellowish star in the field of the nebula *M* 17. PROFESSOR BARNARD gives the position for 1915.0: 18^h 16^m 51^s.58, -16° 4' 40".6. Epoch of Obs. 1915.55.

Yerkes Observatory, March 21, 1916.

OBSERVATIONS OF COMETS,

MADE WITH THE FILAR MICROMETER OF THE 433-MM EQUATORIAL OF LA PLATA OBSERVATORY,

By P. T. DELAVAN.

COMET 1913 f (DELAVAN)

1915	La Plata M. T.	*	Comp.	α — * δ		α 's Apparent δ		Log $p'\Delta$ for α for δ	
	^h ^m ^s			^s	' "	^h ^m ^s	° ' "	^s	"
Apr. 6	14 1 50	1	10, 10	+18.97	+3 35.9	17 47 30.07	-42 9 54.4	9.6383 n	9.0342 n
12	11 25 38	2	10, 10	-20.57	+4 18.9	17 42 5.15	-44 14 23.8	9.8103 n	0.3399 n
12	12 1 18	2	10, 10	-22.18	+3 47.6	17 42 3.54	-44 14 55.1	9.7844 n	0.1608 n
14	11 43 51	4	10, 10	-27.82	+4 14.4	17 39 49.79	-44 56 22.3	9.7960 n	0.1849 n
19	13 8 36	6	10, 10	-14.40	-3 46.8	17 33 11.10	-46 39 42.8	9.6451 n	9.8109
22	13 18 10	8	10, 10	+ 4.80	+4 29.5	17 28 34.61	-47 38 59.7	9.5842 n	0.0538
24	12 37 1	10	10, 10	-40.79	+3 10.5	17 25 17.91	-48 16 46.5	9.6676 n	9.9020
25	11 49 38	11	10, 10	- 6.24	-2 13.8	17 23 36.55	-48 34 58.6	9.7490 n	9.1413
27	11 50 41	12	10, 10	-30.94	+4 19.0	17 19 57.55	-49 11 40.2	9.7336 n	9.6400
27	12 12 5	13	10, 10	+31.32	-1 26.0	17 19 55.53	-49 12 2.3	9.6918 n	9.9031
28	11 59 11	15	10, 10	+ 6.32	-4 13.0	17 18 2.42	-49 29 46.2	9.7092 n	9.8592
28	13 2 12	17	10, 10	-25.78	-3 20.3	17 17 57.61	-49 30 36.1	9.5394 n	0.2058
29	13 11 59	18	10, 10	+ 5.17	-7 37.9	17 15 59.59	-49 48 20.7	9.4813 n	0.2582
29	13 27 20	19	10, 10	-17.55	-1 4.2	17 15 58.04	-49 48 28.9	9.4096 n	0.2903
May 1	13 8 26	20	10, 10	-27.33	-2 4.5	17 11 56.02	-50 22 29.1	9.4494 n	0.2944
10	10 22 38	21	10, 10	-21.34	+0 48.8	16 51 43.41	-52 35 9.4	9.7818 n	9.8235
10	10 34 24	22	10, 10	+37.27	-6 43.8	16 51 42.79	-52 35 18.4	9.7579 n	9.9663
11	11 59 12	24	10, 10	+ 5.91	-7 44.8	16 49 6.26	-52 48 45.4	9.5065 n	0.3507
12	10 29 1	26	10, 10	+16.28	-5 27.5	16 46 45.65	-53 0 11.0	9.7528 n	0.0276
14	14 31 18	27	10, 10	+16.97	+5 5.1	16 41 15.32	-53 25 3.9	9.4256	0.3966
14	15 5 42	27	10, 10	+12.93	+4 50.4	16 41 11.28	-53 25 18.6	9.5791	0.3281
16	8 22 9	28	10, 10	+21.69	-3 11.3	16 36 45.88	-53 43 18.7	9.8835 n	0.0722
16	13 37 43	28	10, 10	-12.93	-5 26.0	16 36 11.28	-53 45 33.7	9.1170	0.4458

1915	La Plata M. T.	*	Comp.	$\delta - *$		δ 's Apparent		Log $p'\Delta$	
				α	δ	α	δ	for α	for δ
COMET 1913 f (DELAN) (Continued)									
May 16	h m s			s	' "	h m s	° ' "	s	"
21	14 40 47	29	10, 10	+24.39	+4 3.4	16 36 4.22	-53 45 58.0	9.5377	0.3659
22	15 50 34	30	10, 10	+11.39	-7 13.9	16 22 48.79	-54 29 23.0	9.8079	9.9086
June 1	14 4 22	31	10, 10	+30.59	-0 37.7	16 20 22.13	-54 35 55.1	9.5579	0.3807
4	14 0 5	32	10, 10	+ 8.09	+4 1.6	15 54 33.15	-55 19 43.4	9.7448	0.2138
7	14 1 20	34	10, 10	+42.00	-5 7.9	15 47 15.92	-55 24 6.3	9.7859	0.1077
11	11 42 40	36	10, 10	-28.98	+7 36.0	15 40 31.23	-55 24 56.9	9.3691	0.4586
17	15 3 29	38	10, 10	-21.04	+6 56.1	15 31 29.29	-55 21 11.5	9.9082	0.0253 _n
25	13 28 51	39	10, 10	+ 3.66	-5 38.3	15 19 58.66	-55 7 21.5	9.8517	9.5004
26	14 52 4	40	10, 10	+25.07	-2 16.2	15 7 11.81	-54 37 40.1	9.9200	0.4546 _n
28	14 12 29	41	10, 10	-12.48	+1 52.6	15 5 52.52	-54 33 34.1	9.9139	0.2944 _n
30	9 36 15	42	10, 10	-12.43	-5 32.0	15 3 33.42	-54 25 39.1	9.3093	0.4446
July 3	10 31 4	44	10, 10	- 9.93	-7 48.0	15 1 8.18	-54 10 4.4	9.6216	0.3245
8	13 39 40	46	10, 10	+33.07	+5 4.6	14 57 46.85	-54 1 50.3	9.9091	0.3172 _n
Aug. 31	13 4 47	48	10, 10	- 2.13	+4 23.6	14 53 24.56	-53 38 30.8	9.9017	0.2672 _n
Sept. 1	9 15 49	49	10, 10	-12.76	+1 3.0	14 58 11.25	-51 9 44.1	9.8672	0.1955 _n
	9 12 29	49	10, 10	+27.75	+1 21.5	14 58 51.74	-51 9 25.4	9.8672	0.1955 _n
COMET Tempel II 1915e									
May 16	17 0 30	51	10, 10	+14.02	-0 10.8	0 33 0.99	- 2 5 31.4	9.6163 _n	0.6874 _n
17	17 7 18	53	10, 10	+ 8.60	+2 30.7	0 36 6.00	- 1 53 34.4	9.6076 _n	0.6880 _n
19	17 0 11	55	10, 10	+16.74	-5 10.7	0 42 10.02	- 1 29 37.6	9.6138 _n	0.6910 _n
21	16 59 26	57	10, 10	- 3.95	-3 46.1	0 48 12.13	- 1 6 26.1	9.6125 _n	0.6931 _n
June 4	17 8 51	59	10, 10	- 4.19	+5 59.7	1 28 44.05	+ 1 23 52.6	9.5807 _n	0.7087 _n
22	17 52 19	61	10, 10	+18.84	+1 13.8	2 16 18.57	+ 3 56 37.3	9.4469 _n	0.7325 _n

Mean Places of Comparison Stars.

*	α 1915.0	Red. to App. Pl.	δ 1915.0	Red. to App. Pl.	Authority
1	h m s	s	° ' "	"	Cordoba Zone Catalogue 2994.
2	17 47 8.58	+2.52	-42 13 28.1	- 2.2	Argentine General Catalogue 24117.
3	17 42 22.89	+2.83	-44 18 40.4	- 2.3	C. Z. 2676.
4	17 42 22.90	+2.93	-44 59 52.1	- 2.1	Connected with Star 3.
5	17 40 14.67	+2.94	-45 0 34.4	- 2.3	C. Z. 1947.
6	17 31 37.47	+3.24	-46 31 55.6	- 2.7	Connected with Star 5.
7	17 33 22.22	+3.23	-46 35 53.5	- 2.5	Argentine General Catalogue 23790.
8	17 28 25.20	+3.45	-47 49 13.8	- 2.9	Connected with Star 7.
9	17 28 26.39	+3.42	-47 43 26.4	- 2.8	C. Z. 1565.
10	17 26 11.29	+3.53	-48 19 22.8	- 3.2	Connected with Star 9.
11	17 25 55.18	+3.52	-48 19 53.7	- 3.3	Cordoba Zone Catalogue 1394.
12	17 23 39.21	+3.58	-48 32 41.3	- 3.5	C. Z. 1173.
13	17 20 24.79	+3.70	-49 15 55.4	- 3.8	C. Z. 1103.
14	17 19 20.52	+3.69	-49 10 32.5	- 3.8	C. Z. 992.
15	17 17 33.49	+3.74	-49 18 30.8	- 4.1	Connected with Star 14.
16	17 17 52.35	+3.75	-49 25 29.2	- 4.0	Argentine General Catalogue 23510.
17	17 18 52.05	+3.75	-49 22 20.4	- 4.0	Connected with Star 16.
18	17 18 19.64	+3.75	-49 27 11.8	- 4.0	

Mean Places of Comparison Stars (Continued)

*	α 1915.0	Red. to App. Pl	δ 1915.0	Red. to App. Pl	Authority
	^h ^m ^s	^s	[°] ['] ["]	["]	
18	17 15 50.61	+3.81	-49 40 38.6	- 4.2	Argentine General Catalogue 23462.
19	17 16 11.78	+3.81	-49 47 20.5	- 4.2	C. Z. 898.
20	17 12 19.42	+3.93	-50 20 20.0	- 4.6	C. Z. 610.
21	16 52 0.36	+4.39	-52 35 50.5	- 7.7	La Plata M. O.* C. P. D. -52° 10360.
22	16 51 1.14	+4.38	-52 28 26.8	- 7.8	La Plata M. O. C. P. D. -52° 10346.
23	16 50 13.33	+4.43	-52 36 46.3	- 7.9	La Plata M. O. C. P. D. -52° 10335.
24	16 48 55.92	+4.43	-52 40 52.6	- 8.0	Connected with Star 23.
25	16 45 43.67	+4.46	-52 51 16.6	- 8.5	La Plata M. O. C. P. D. -52° 10278.
26	16 46 24.91	+4.46	-52 54 35.1	- 8.4	Connected with Star 25.
27	16 40 53.78	+4.57	-53 29 59.8	- 9.2	La Plata M. O. C. P. D. -53° 8176.
28	16 36 19.58	+4.61	-53 39 57.8	- 9.6	La Plata M. O. C. P. D. -53° 8147.
29	16 35 35.18	+4.65	-53 49 51.3	-10.1	La Plata M. O. C. P. D. -53° 8140.
30	16 22 32.68	+4.72	-54 21 56.5	-12.6	La Plata M. O. C. P. D. -54° 7719.
31	16 19 46.76	+4.78	-54 35 4.4	-13.0	La Plata M. O. C. P. D. -54° 7691.
32	15 54 20.18	+4.88	-55 23 27.3	-17.7	La Plata M. O. C. P. D. -55° 6916.
33	15 45 55.18	+4.82	-55 18 1.3	-19.2	La Plata M. O. C. P. D. -55° 6741.
34	15 46 29.09	+4.83	-55 18 39.3	-19.1	Connected with Star 33.
35	15 41 2.08	+4.82	-55 37 5.2	-20.2	La Plata M. O. C. P. D. 55° 6704.
36	15 40 55.40	+4.81	-55 32 12.8	-20.1	Connected with Star 35.
37	15 31 52.36	+4.75	-55 32 48.2	-21.8	La Plata M. O. C. P. D. 55° 6649.
38	15 31 45.59	+4.74	-55 27 45.9	-21.7	Connected with Star 37.
39	15 19 50.44	+4.56	-55 1 19.2	-24.0	La Plata M. O. C. P. D. 54° 6501.
40	15 6 42.42	+4.32	-54 34 57.8	-26.1	La Plata M. O. C. P. D. 54° 6385.
41	15 6 0.69	+4.31	-54 35 0.4	-26.3	Connected with Star 40.
42	15 3 41.61	+4.24	-54 19 40.3	-26.8	Cordoba Zones 49.
43	15 1 20.44	+4.16	-54 0 14.1	-27.2	La Plata M. O. C. P. D. 53° 6288.
44	15 1 13.95	+4.16	-54 1 49.1	-27.3	Connected with Star 43.
45	14 56 22.43	+4.04	-54 9 59.0	-27.9	La Plata M. O. C. P. D. 54° 6277.
46	14 57 9.74	+4.04	-54 6 27.1	-27.8	Connected with Star 45.
47	14 53 32.93	+3.92	-53 48 54.1	-28.4	La Plata M. O. C. P. D. 53° 6181.
48	14 53 22.77	+3.92	-53 42 26.0	-28.4	Connected with Star 47.
49	14 58 21.29	+2.72	-51 10 20.1	-27.0	Cordoba Zones 3528.
50	0 33 52.16	+0.86	- 2 10 38.7	+ 8.1	A. G. Zones Strassburg 128.
51	0 32 46.11	+0.86	- 2 5 28.8	+ 8.2	Connected with Star 50.
52	0 36 49.96	+0.87	- 1 59 2.6	+ 8.2	A. G. Strassburg 145.
53	0 35 56.53	+0.87	- 1 55 56.9	+ 8.2	Connected with Star 52.
54	0 41 34.33	+0.89	- 1 16 2.9	+ 8.4	A. G. Nicolajew 134.
55	0 41 52.39	+0.89	- 1 24 35.2	+ 8.3	Connected with Star 54.
56	0 48 33.63	+0.91	- 0 54 32.7	+ 9.0	A. G. Nicolajew 162.
57	0 48 15.16	+0.92	- 1 2 49.0	+ 9.0	Connected with Star 56.
58	1 30 44.62	+1.08	+ 1 19 17.1	+11.5	A. G. Nicolajew 305.
59	1 28 47.16	+1.08	+ 1 17 41.4	+11.5	Connected with Star 58.
60	2 14 56.94	+1.39	+ 3 49 17.6	+14.0	A. G. Albany 654.
61	2 15 58.34	+1.39	+ 3 55 9.5	+14.0	Connected with Star 60.

* M. O. stands for meridian observations.

All comet observations of $\Delta\alpha$ have been made by direct micrometer measures.

All measures have been corrected for differential refraction.

La Plata Argentina, April, 1916.

OBSERVATIONS OF COMET 1913 *f* (DELANVAN),

MADE WITH THE 26-INCH AND 12-INCH EQUATORIALS OF THE U. S. NAVAL OBSERVATORY,

[Communicated by Captain J. A. HOOGEWERFF, U. S. Navy, Superintendent.]

Date Wash. M. T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	$\frac{\log p\Delta}{\alpha \delta}$	Red. to App. Pl.
¹⁹¹⁴ July 28 15 39 42	1	18. 6	-0 36.57	+2 55.3	^{h m s} 5 54 24.23	+37 46 57.2	9.762 _n	0.619
Aug. 15 15 2 38	2	20. 4	+1 28.23	+3 1.1	6 56 51.20	+43 52 27.8	9.801 _n	0.638
Sept. 9 15 27 47	3	25. 5	-1 21.64	-7 4.7	9 19 46.22	+49 56 42.1	9.846 _n	0.643
Sept. 17 16 24 30	4	25. 5	+1 42.16	-1 19.5	10 20 19.18	+49 30 26.7	9.847 _n	0.579
Sept. 26 16 52 11	5	25. 5	+3 28.53	+1 27.9	11 28 46.80	+46 45 10.4	9.823 _n	0.612
Oct. 2 17 7 24	6	25. 5	-1 34.31	+5 12.4	12 10 53.14	+43 38 52.6	9.799 _n	0.639
Oct. 23 7 0 10	7	25. 5	+2 48.27	-3 15.9	14 0 16.13	+28 45 59.3	9.699	0.760
Oct. 30 6 42 24	8	15. 3	-2 36.32	+6 7.8	14 26 41.00	+23 24 44.1	9.687	0.750
Oct. 31 6 25 27	9	35. 7	+1 25.96	+2 19.1	14 30 5.71	+22 40 20.6	9.690	0.737
¹⁹¹⁵ Jan. 20 18 0 54	10	25. 5	+4 28.13	+1 23.4	17 12 0.05	-17 28 48.8	9.552 _n	0.821

Observers: *C. B. WATTS, †H. E. BURTON.

Observations on July 28, August 15 and January 20 were made with 26-inch, the others with 12-inch equatorial.

Mean Places of Comparison Stars for the Beginning of the Year

*	α	δ	Authority	*	α	δ	Authority
1	^{h m s} 5 54 59.24	+37 43 56.4	A.G. Lund 3046	6	^{h m s} 12 12 25.93	+43 33 54.3	A.G. Bonn 8423
2	6 55 20.93	+43 49 25.5	A.G. Bonn 5680	7	13 57 26.50	+28 49 29.9	A.G. Camb.(Eng.) 6690
3	9 21 5.67	+50 3 55.2	A.G. Camb. (U.S.) 3392	8	14 29 15.93	+23 18 50.9	A.G. Berlin B. 5098
4	10 18 34.99	+49 31 57.5	A.G. Bonn 7590	9	14 28 38.34	+22 38 16.3	A.G. Berlin B. 5095
5	11 25 16.55	+46 43 56.0	A.G. Bonn 8069	10	17 7 32.12	-17 30 3.7	A.G. Washington 6153

NOTES

- July 28. Very bright nucleus and conspicuous tail. Seeing poor.
 Aug. 15. Very bright nucleus and conspicuous tail. Seeing poor. Hazy.
 Sept. 9. Very bright nucleus. Smooth, well defined tail. Seeing poor.
 Sept. 17. Appearance similar to that on Sept. 9. Estimated magnitude of nucleus 4 to 4.5. Seeing poor.
 Sept. 26. Bright nucleus. Seeing fair. Chronograph record poor.
 Oct. 2. Seeing good. Haze. Chronograph record poor.
 Oct. 23. Seeing poor. Chronograph record poor.
 Oct. 30. Star very faint during part of observation. Seeing poor.
 Oct. 31. Seeing fair.
 Jan. 20. Fairly bright nucleus and faint tail. Barely visible in 5-inch finder. Seeing poor.

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OBSERVATIONS OF COMET 1913 *f* (Delavan), BY C. B. WATTS AND H. E. BURTON.

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MASS-RATIOS IN VISUAL BINARY STARS,

By G. VAN BIESBROECK.

In recent years several successful attempts have been made to find the masses of binary stars. Combining visual, spectroscopic and even photometric measures or the fairly well known parallaxes of some of them the linear dimensions have been determined in a number of binaries and consequently we have good approximations to the total mass of the system. The question that we are considering here is another one: namely, the partition of that total mass among the two components opening thus a way to find masses of individual stars. The solution of this problem is found by measuring the fraction of the total relative motion that is to be attributed to each or at least to one of the components. It is a very simple one theoretically. Let us call A and B the masses of the two components of a binary, A referring to the brighter one or the one that is taken as the origin for the relative motion. Consider their absolute positions (eventually corrected for parallax,) on the celestial sphere. We measure these positions from a fixed origin and in a certain fixed direction, be it in right ascension, in declination, or in any other chosen direction. We will suppose

$$A + B = 1$$

the masses meaning then the fractions of the mass of the system that belongs to each component. We will call a and b the absolute coördinates of the two components — if necessary corrected for parallax — and r the relative coördinate, — B relative to A , — at the same time t . Then we shall have:

$$a = \mu t - (1 - A)r + K \quad \text{or} \quad b = \mu t + Ar + K$$

with the condition $r = C - a$.

In these relations μ represents the proper-motion of the center of gravity of the system, measured also along the same direction as the one chosen for a and C ; K is

a constant term depending on the initial conditions. From measures of one or other of the components at different epochs we deduce linear equations of condition containing the three unknowns: μ , A and K , and we see that there will be no indeterminateness in the problem as long as there is no proportionality between r and t . Three sufficiently distant epochs will fulfill all requirements in such a case.

The question is further simplified in the majority of the stars to be considered because μ is sufficiently well known from a long series of meridian observations. If so two unknowns only are to be considered: A and K . Two epochs, let us call them 1 and 2, will furnish all the required data. From the equations in C for the two epochs we find:

$$A = \frac{\mu(t_2 - t_1) - (C_2 - C_1)}{r_2 - r_1}$$

The best determination of A will be obtained when the denominator $r_2 - r_1$ is as large as possible. That condition fixes the most advantageous direction for the measurement. The relative displacement $r_2 - r_1$, will be a maximum when the measures are made in a direction nearly parallel to the relative displacement between the two epochs. If for instance at the time considered the companion is moving principally in declination relative to the principal star the measures in declination will give the best possible result. Measures in right ascension would be of no value, as in that case the denominator in the relation giving A will be nearly equal to zero.

We see that the mass A is given as a simple ratio of directly measured quantities, and does not contain the elements of the orbital motion of the binary considered when the two components can be measured so as to give the values of r . Even when the period is entirely unknown the formula can give good results provided the relative motion, that enters in the denominator, is

large enough during the time covered by the two epochs. The influence of an error in the assumed proper-motion will be small if the interval of time $t_2 - t_1$ is relatively short, but as the differences $C_2 - C_1$, and $r_2 - r_1$, become small at the same time the highest accuracy is required in the measurement of these quantities. For that purpose measures of photographs obtained with long-focus instruments, such as are used in stellar parallax work, give an enormous advantage over meridian or visual micrometer work. AITKEN briefly called attention to that point when he published his "Observing List for the Determination of the Relative Masses in Visual Binary Systems," [*Lick Obs. Bull.* No. 208, Vol. VII, p. 7]. It is the aim of this note to present some results obtained from plates available at this observatory, most of them not taken for this particular purpose. The striking advantage of photography in this case is shown especially by the small number of years required for obtaining data that are far superior to long series of meridian or micrometric observations where personal and systematic errors conceal most of the small displacements. In photographic work the determinations can be made in a purely differential way, as is done in parallax work, and the chances of vitiating the results by systematic or personal errors are reduced to a minimum.

In looking over the plate material available at this observatory, a number of stars were found where determinations of relative masses seemed possible. By a more careful consideration of the observational data some of these stars had to be discarded for various reasons, the most frequent one being the shortness of time covered by the plates; in other cases the two components were sufficiently bright to produce an image, but their distance was too small (such as $0\sigma 235$, ξ *Scorpii*, β 80) making measures on the individual components difficult. In some instances (61 *Cygni*, $\sigma 2398$,) the time covered by the plates was large enough so far as the relative motion is concerned, but as in such cases the proper-motion of the center of gravity cannot be assumed, longer intervals of time proved to be necessary. Finally four binary stars were retained as is illustrated hereafter. The first

star is a case where the period is still very doubtful and where the relative motion was found from the measures of the companion. For the three other systems generally only one component could be measured — at least at the time when the plates were taken — so that the elliptic elements of the motion had to be taken from the best sources. For the last star it was found necessary to recompute better elements of the orbital motion for establishing the coefficients in the equations of condition.

$$\eta \text{ Cassiopeiae } \Sigma 60 \text{ (3}^m.6 - 7^m.9)$$

$$0^h 43^m \quad +57^\circ 17' \quad (1900)$$

As a first application of the preceding remarks we consider this well known system, that has been observed many years. The period is long and still uncertain by at least a century. The relative motion in recent years was larger in right ascension than in declination. But as we found on hand among the unfinished series of parallax-plates taken for this star, two good plates separated by an eleven year interval the relative motion was thought to be large enough in both coördinates. The two plates, which show measurable images of both components, were therefore measured in both right ascension and declination, so as to yield two independent determinations.

The following plates have been used:

Number	Date	Gr. M. T.	Parallax Factors	
			In α	In δ
Schles. 473	1904 Sept. 26	17 ^h 44 ^m	+0.161	-0.839
π 2194	1915 Aug. 18	20 44	+0.678	-0.479

As the proper-motion of this star is large in both coördinates, it is important to orient the plate carefully: this was done by means of AG stars reduced to the mean of the two dates 1910.2. Five comparison stars have been used and the measures reduced by the dependence method. The solutions or measured displacements of the two components were found:

	MEASURED DISPLACEMENT		CORRECTED FOR PARALLAX	
	Principal Star	Companion	Principal Star	Companion
In right ascension	+12".81	+10".51	+12".72	+10".42
In declination	- 5 .99	- 1 .40	- 6 .05	- 5 .69

For the parallax we adopted PETER's value $0''.18$ deduced from heliometer measures. From photographic determinations RUSSELL found $0''.19$. Finally for the

proper-motion of the system we have adopted the values given in BOSS' *P. G. C.* The corresponding displacements in the 10.897 years' interval between

the two plates are given at the same time:

$$\begin{array}{lcl} \text{Proper motions (1910.2)} & \left\{ \begin{array}{l} \mu = +0''.1392 \\ \mu' = -0''.523 \end{array} \right. & \text{Displacements} \quad \left\{ \begin{array}{l} +12''.28 \\ -5''.69 \end{array} \right. \end{array}$$

From these data we find:

$$\begin{array}{lcl} A = \frac{+12.28 - 10.42}{12.72 - 10.42} = 0.81 \text{ from right ascensions} & & \\ A = \frac{-5.69 + 4.46}{-6.05 + 4.46} = 0.77 \text{ from declinations} & & \\ & \left. \vphantom{\begin{array}{l} A = \\ A = \end{array}} \right\} \text{Mean } \mathbf{0.79} \end{array}$$

As only one plate has been used for each epoch the agreement must be considered as satisfactory. A larger number of plates taken so as to avoid or to determine the parallax would give a higher weight to the result.

As far back as 1881 L. STRUVE (*Melanges Math. et Astr.*, Vol. V) found from a discussion of the meridian observations compared with the contemporaneous micrometric measures $\frac{B}{A} = 0.268$ or $A = 0.789$. A later discussion by T. LEWIS (*Mem. R. A. S.*, Vol. LVI, p. 18) gives $\frac{B}{A} = \frac{1}{2}$ or $A = 0.67$.

A more thorough discussion of meridian observations has been made by L. BOSS, as is found in the appendix of his *P. G. C.* (p. 264):

$$\begin{array}{lcl} \text{From right ascensions} & A = 0.38 & (\text{Weight } 60) \\ \text{From declinations} & A = 0.71 & (\text{Weight } 85) \end{array}$$

whence he adopts 0.57, but the result is considered as very uncertain "owing to the independence of the expressions for orbital and proper-motion respectively."

In the present determination the proper-motion was assumed; a possible error in this value can however have only a small influence on the result because of the shortness of the time covered.

$$70 \text{ Ophiuchi } \Sigma 2272 \text{ (4}^m.3 - 5^m.8)$$

$$18^h 0^m \quad +2^\circ 31' \quad (1900)$$

The parallax of this binary ($\pi = 0''.21$) has been measured by SLOOTM (*Aph. J.* 41, 241). The plates measured by him covering too short an interval of time for the present purposes, I have taken two additional plates in 1916, the earlier epoch being made up of four plates as seen in the following table:

Plate No.	Date	Time in Decimal Fraction of Year	SOLUTION (Right Ascension)		Correction for Parallax
			Principal Star	Companion	
247	1910 Apr. 10	1910.274	0.371	...	-0.074
272	10 July 30	10.577	0.326	...	+0.048
410	11 Apr. 22	11.306	0.458	1.124	-0.067
468	11 Aug. 19	11.631	0.327	...	+0.066
2533	1916 May 3	1916.339	0.685	1.871	-0.058
2534	16 May 3	16.339	0.687	1.873	-0.058

The measures, the result of which is given in values of the revolutions of the screw (1 R = 2''.665) in the column "Solution," were made in right ascension only, the motion being small in declination between the first and last epoch. The proper-motion of the system is so well known that it ought not to be introduced as an additional unknown. Its value was taken from BOSS' *P. G. C.* and is based on a series of meridian observations covering a longer time than the period of revolution, so that the uncertainty of the position of the center of gravity has but little influence.

In the present series of plates the apparent magnitude of 70 *Ophiuchi* was reduced by about five magnitudes by means of the rotating sector. This made the image of the companion weak and difficult to measure unless the plates were exceptionally good. This image was well measurable on only one of the earlier plates. Instead of using it to find the relative position, upon which depends the coefficients of the mass in the equations of condition, it was thought advisable to deduce them from LOHSE's elliptical elements as given in *Publik. des Astroph. Obs. Potsdam*, Vol. XX, p. 146.

For the five dates considered we have:

	θ (1900.0)	ρ
1910.274	153°.59	3".425
1910.577	152°.61	3".505
11.306	150°.44	3".693
11.631	149°.55	3".775
16.339	139°.73	4".845

The direction of the measurement on the plates is that of a trail on the first plate used. The computed position-angles have been corrected for precession ($-0^{\circ}.06$) from 1900 to that first date. Hence the relative positions $r = \rho \sin \theta$ expressed in the same unit as the measured quantities become:

a Corrected for π	C Corrected for π	μt	r
+0.297		+0.206	+0.573
0.370		.055	.606
0.391	+1.057	.125	.685
0.393		.156	.719
0.627	1.813	.607	1.177
0.629	1.815	.607	.177

The six equations of condition for the principal star give:

$$A = 0.53 \pm 0.02$$

An independent determination from the measures of the companion:

$$A = 0.56$$

is entitled to much less weight because only one earlier plate is available.

Previous determinations by A. PREY [*Astr. Nachr.*, Vol. CLXV, p. 158 (1904)] from meridian observations 1821 — 1894 had given:

From right ascensions	$A = 1 - Z = 0.19$
From declinations	0.27
From both	0.20

COMSTOCK [*Astr. Nachr.*, Vol. CLXXVIII, p. 18 (1908)] from an independent discussion of the meridian observations shows that one of PREY's normal places is in error, and finds:

$$\begin{aligned} B &= 0.456 \quad \text{or} \quad A = 0.544 \quad \text{from right ascensions} \\ B &= 0.507 \quad \text{or} \quad A = 0.493 \quad \text{from declinations.} \end{aligned}$$

About the same time LAT [*Bull. Astr.*, Vol. XXV, p. 141 (1908)] comes to a similar conclusion about PREY's computation and finds from a discussion of micrometric measures of two stars in the vicinity:

From Star a	$A = 0.63 \pm 0.12$
From Star C	$A = 0.69 \pm 0.18$

L. BOSS finds the following values in the appendix of the *P. G. C.*, p. 275:

From right ascensions	$A = 0.52 \pm 0.02$
From declinations	$A = 0.58 \pm 0.03$

It is seen that the present photographic result confirms entirely this last value. Although based on only six plates covering a six year interval of time, the probable error is as small as the one that resulted from the elaborate discussion of the long series of meridian observations.

$$\begin{aligned} \tau \text{ Cygni, } A. G. C. 13 \quad (3^{\text{M}}.8 - 7^{\text{M}}.4) \\ 21^{\text{h}} 11^{\text{m}} + 37^{\circ} 37' \quad (1900) \end{aligned}$$

The brightness of the two components of this binary differ by about four magnitudes. The image of the system on the parallax-plates is built up by the brighter component only. The relative motion has been largely in declination during the time covered and consequently the plates have been measured in that direction only. The reductions are shown in the table following.

The parallax of this star is small. The value $\pi = 0''.006 \pm 0''.016$ by SLOCUM and MITCHELL [*Aph. J.*, Vol. XXXVIII, p. 23 (1913)] has been adopted in computing the corresponding small correction. The proper-motion is taken from the *P. G. C.*: $\mu' = +0''.427$.

The corrected measures, converted into seconds of arc, are given under a . The last columns give the orbital motion in declination as resulting from two recent sets of elements: the first column is deduced from AITKEN's Elements [*Publ. Lick Obs.*, Vol. XII, p. 161, (1911)] the last one from ABERTI's Elements [*Mem. Spett. Ital.*, Vol. I, p. 178 (1912)]. Some of these elements are quite different; e.g., $e = 0.22$ in the former, $e = 0.36$ in the latter. Yet at the present time the difference between the two systems is small for a star always difficult to measure. However, recent observations favor AITKEN's orbit.

Plate No.	Date	Solution in Declination	Correction for π	Correction for μ	α resulting in seconds	Orbital Motion r	
						I	II
39	1907 Oct. 12	-0.418	-0.001	+0.500	+0''.216	0''.200	0''.194
112	09 Sept. 18	-0.092	- .001	+0.190	.259	.378	.377
259	10 June 18	+0.022	- .002	+0.070	.240	.443	.444
268	10 June 25	+0.056	- .002	+0.067	.322	.444	.446
299	10 Sept. 9	+0.078	- .001	+0.033	.293	.464	.465
793	12 June 22	+0.398	- .002	-0.253	.381	.605	.606
794	12 June 22	+0.394	- .002	-0.253	.370	.605	.606
797	12 June 23	+0.398	- .002	-0.253	.381	.605	.606
2573	1916 June 4	+1.083	- .002	-0.886	0 .520	0 .850	0 .913

The mass-ratio has been computed under both assumptions. The same weight was given to all the plates:

From AITKEN's Elements

$$A = 0.52 \pm 0.03$$

From ABETTI's Elements

$$A = 0.55 \pm 0.03$$

A preliminary solution from the first eight plates had given in both cases:

$$A = 0.56 \pm 0.04$$

Considering that AITKEN's elements seem to be nearer to the truth we adopt as a final value:

$$A = 0.53$$

From a discussion of the measures of a C star, obtained by O. STRUVE (1851) and COMSTOCK, (1903) HADLEY finds (*Pop. Astr.* 13.325)

$$\text{From right ascensions } A = 1 - k = 0.570 \pm 0.450$$

$$\text{From declinations } A = 1 - k = 0.832 \pm 0.359$$

In the paper of ABETTI, cited above, we find an interesting discussion of the measures of two distant companions C and D that have been measured repeatedly, for thirty-five and sixty-one years, respectively, with the purpose of finding the mass-ratio and the motion of the center of gravity. His results for A are:

	From C	From D
From right ascensions	0.47 $\pm$ 0.39	0.51 $\pm$ 0.28
From declinations	0.19 $\pm$ 0.40	0.47 $\pm$ 0.29

These probable errors, compared with those of the photographic determinations, illustrate well the advantage of the latter method. In a much shorter time — nine or even five years — values of decidedly higher accuracy are obtained in the present case. The interval of time is small enough for considering μ as a known quantity, while in the discussion of micrometric measures covering a long interval of time, it is not possible to take advantage of that circumstance.

$$85 \text{ Pegasi} = \beta 733 \text{ (5}^{\text{M}}.8 - 11^{\text{M}}.0)$$

$$23^{\text{h}} 57^{\text{m}} + 26^{\circ} 33' \text{ (1900)}$$

This is another binary star where the difference of magnitude between the two components is very large, so that only the brightest component appears on the plates. SLOCUM and MITCHELL [*Aph. J.*, Vol. LXI, p. 243 (1915)] have discussed these for parallax, but the plates that they have used cover an interval of less than two years. However fast the motion of the system (the period is less than twenty-seven years) that interval is too short for producing a measurable displacement. Therefore, we have taken as a first epoch two early plates that have not been used in the parallax discussion. The orbital motion being almost as large in right ascension as in declination between the two considered epochs, the following plates have been measured in both coördinates:

Plate No.	Date	RIGHT ASCENSION			DECLINATION		
		Solution	Correction for π	Correction for μ	Solution	Correction for π	Correction for μ
134	1909 Oct. 30	-3''.475	+0''.048	0''.000	+3''.887	-0''.012	0''.000
157	1909 Nov. 13	-3 .448	+0 .060	-0 .032	+3 .857	-0 .000	+0 .038
1323	1913 July 24	-0 .550	-0 .066	-3 .113	-0 .638	-0 .046	+3 .680
1324	1913 July 24	-0 .564	-0 .066	-3 .113	-0 .588	-0 .046	+3 .680

The plates were measured in the direction of the equator for 1900.0 which was found by comparison with the rectangular coördinates from the *Oxford Astrographic Catalogue*. The parallax $\pi = 0''.084$ was adopted from the paper referred to. The proper-motion was taken from Boss' *P. G. C.* Taking the means of the two earlier and the two later plates, we find:

	Right Ascension	Declination
1909.849	$-3''.423$	$+3''.885$
1913.563	-3.736	$+4.245$
Displacement	-0.313	$+0.360$

The companion not being measurable on the plates the total orbital motion is either to be taken from micrometric measures, or to be computed from the elliptic elements if these are sufficiently well known. The measures of this important star are scarce and liable to be largely in error because the star is always a difficult object.

Recent observations have shown that the best elements available need revision. Using all the available measures up to 1915, I have deduced the following elements:

$P = 26.70$ years	$(n = +13^\circ.483)$
$T = 1909.88$	$\Omega = 281.2$
$e = 0.390$	$\omega = 98.6$
$a = 0''.81$	$i = 52.8$

On account of the character of the observational material only position angles have been used in the discussion, the major-axis being adjusted afterwards so as to represent as well as possible the very discordant distances.

Measures of the radial velocity at this observatory by PROFESSOR FROST show larger discordances than would be expected in a star of this type. This induced him to suspect the brighter star of being a spectroscopic binary [*Aph. J.*, Vol. XXV, p. 59 (1907)]. The question is still open, but as the velocities indicate a change towards the positive side in the last ten years, the ascending node of the visual orbit must be placed in the fourth quadrant.

The elements represent all the observations as well as could be expected at present. Although the position angle has changed by about 580 degrees since the discovery by BURNHAM in 1878, some of the elements are still very uncertain, due mainly to the lack of observations near periastron. Rough as they are, these elements will give a fairly good value of the relative

motion during the four years covered by the plates. We find for the two epochs:

	θ	ρ	In Right Ascension	In Declination
1909.849	$33^\circ 35'$	$0''.303$	$+0''.168$	$+0''.253$
1913.563	$118^\circ 53$	0.695	$+0.609$	-0.336
Relative change			$+0.441$	-0.589

Hence the value of the mass of the principal star:

$$\text{From right ascensions } A = \frac{0.441 - 0.313}{0.441} = 0.29$$

$$\text{From declinations } A = \frac{0.589 - 0.360}{0.589} = 0.39$$

A large part of the uncertainty of these figures is probably due to the defects in the adopted orbit. The first epoch corresponds very nearly to the time of periastron, when the angular motion is very rapid (over 50° a year). A slight error in the time of periastron passage will change specially the right ascension by an appreciable quantity, so that the first value deserves the lesser weight. Giving double weight to the second value we find the resulting value:

$$A = 0.36$$

The mass-ratio of this system has already been investigated several times:

From measures of the position of a distant star C. COMSTOCK [*Aph. J.*, Vol. XVII, p. 220 (1903)] deduces:

$$\begin{array}{l} \text{From right ascensions } A = 0.396 \\ \text{From declinations } A = 0.369 \end{array} \quad A = 0.38$$

In *Mem. R. A. Soc.*, Vol. LXI, p. 21, LEWIS states that from a discussion of meridian observations, it was found that

$$A = 0.20 \text{ (LEWIS)} \quad A = 0.25 \text{ (FURNER)}$$

About the same time BOWYER and FURNER give in detail the discussion of meridian observations [*Monthly Not.*, Vol. LXVI, p. 425, (1906)] and deduce:

$$\begin{array}{l} \text{From right ascensions } A = 0.15 \\ \text{From declinations } A = 0.24 \end{array}$$

From a discussion of the measures of the *C* star the same authors give:

From right ascensions	$A = 0.26$
From declinations	$A = 0.29$

The more recent discussion by L. Boss gave (*P. G. C.*, p. 278):

From right ascensions	$A = 0.49 \pm 0.16$
From declinations	$A = 0.16 \pm 0.20$

But he accepts reluctantly the conclusion that the brighter component has the smaller mass. "There is no other instance in which it has been possible to demonstrate with a strong preponderance of probability that the fainter component of a binary star has the greater mass, while the strong natural inference must be in favor of greater mass for the brighter component, especially when the discrepancy in brightness is very great, as in the present instance. Accordingly, it is here assumed that the two masses are sensibly equal."

The photographic result unquestionably confirms the small mass of the brighter component, that had been found repeatedly by earlier investigators.

As a result of the parallax-campaign started here by SCHLESINGER it can be considered that the plates taken with the 40-inch telescope give semi-absolute positions; *i. e.*, referred to a number of faint comparison stars considered as a fixed background, with a probable error of about $0''.03$ from one plate. Analogous results have been found with other large instruments. Not considering the influence of proper-motion we see that if the relative motion in the time covered by the plates is only $0''.50$, the probable error of the determination of A will be about 6%: a very small amount, considering the nature of the problem and the results obtained so far in this direction.

We have collected in the following list a number of stars where conditions seem to be favorable for a successful determination of mass-ratio. The list was limited to stars where the motion is fast enough to yield measurable quantities by the photographic method after a relatively short interval of time (ten or fifteen years). Besides this limit of time resulting from the slowness of the motion, we have adopted $1^m.5$ as the limit of difference of magnitude, inside of which it is unsafe to assume that the companion does not interfere with the image of the principal star when the distance is small, for instance, less than $1''$. For large distances there is, of course, no limitation of that kind.

The successive columns give the names of the stars and their position for 1900, the magnitudes of the components and the observatories where the star is on the parallax program [*Public. of the American Astr. Society*. Report of the committee on stellar parallaxes, Aug. (1915)]. The observatories are designated as follows:

A	Allegheny	S	Sproul
D	Dearborn	W	Mount Wilson
M	McCormick	Y	Yerkes

The two last columns give the number of years required for obtaining a minimum orbital displacement of about $0''.50$, both in right ascension and declination. This shows after what lapse of time and in what direction of measurement good results are to be expected. In some of the more rapidly moving stars these indications will of course have to be changed after some years. With very few exceptions the stars are already on the parallax program of one or more observatories. After a decade or so material will be available for about three dozen stars.

1. *α Fornacis*. AITKEN (Loc. cit.) writes: "measures, begun now, will yield a good value for m'/m within a century." By means of photography the determination could be made after some ten years both in right ascension and in declination. If measured obliquely, the displacement of $0''.50$ that we consider as the lower necessary limit is reached in about seven years.

2. *β 552*. This star has shown considerable orbital motion, but for years the companion has been invisible, so that it is almost impossible to predict now which one of the two coördinates is the most favorable. As soon as the companion is again measurable the question will be settled, but very likely a ten or fifteen years' interval is all that is needed.

3. *Sirius*. The apparent orbit is so large that good results can be obtained from meridian observations. But as an independent photographic determination requires only a couple of years, this can go hand in hand with the determination of the parallax.

4. *ζ Cancri*. In case of the $\frac{A+B}{2} - C$ system, the interval of time ought to be about fifteen years.

5. *ϵ Hydrae*. If the determination was started in 1919 the declination could no longer be used; after four or five years the right ascension direction would then give the necessary data.

6. *Σ 1306*. This star ought to be placed at once on the parallax programs.

7. *α Urs. Maj.* The direction of the motion is very uncertain at the present time.

LIST OF STARS FOR MASS-RATIO DETERMINATIONS

BURNHAM G. C.	Star	1925.0		Magnitude	Observatories	Time		Notes
		h	m			In α	In δ	
426	η Cassiopeiæ	0	44.5	+57 25	4.0- 7.6	A, M, S, Y	4	4
1036	β 513	1	55.8	+70 33	5.0- 7.5	S, Y	8	
1612	α Fornacis	3	8.9	-29 17	3.9- 6.9		10	10
2007	Σ 483	3	59.1	+39 18	8.0- 9.5	S, Y	..	10
2109	40 Eridani	4	11.8	- 7 46	9.1-10.8	M, S, Y	10	5
2134	0 Σ 79	4	15.6	+16 21	7.0- 8.8	A, S	15	..
2154	0 Σ 82	4	18.5	+14 53	7.0- 9.0	A, S	..	15
2383	β 552	4	47.6	+13 32	7.0-11.0	S	10?	15?
3596	Sirius	6	41.8	-16 37	-2.0- 9.0	M, Y	2	3
4120	Castor	7	29.8	+32 3	2.7- 3.7	A, D, M, Y	8	..
4187	Procyon	7	35.3	+ 5 25	0.2-12	A, M, S, Y	..	1
4477	ζ Cancri	8	7.9	+17 57	5.0-5.5-5.7	A, S, W, Y	10	10
4714	β 208	8	35.8	-22 24	6.0- 9.0	Y	15	10
4771	ϵ Hydra	8	42.8	+ 6 42	3.8- 5.5	A, M, S, Y	..	4
4923	Σ 1306	9	3.8	+67 26	5.0- 8.2		5	6
5652	α Urs. Maj.	10	59.1	+62 9	1.7-11.1	A, Y	..	10
5734	ξ Urs. Maj.	11	14.2	+31 57	4.0- 4.9	A, S	10	4
6566	25 Can. Venat.	13	34.1	+36 41	5.0- 7.0	S, Y	15	..
6641	Σ 1785	13	45.6	+27 21	7.2- 7.5	M, S, Y	5	..
..	α Centauri	14	34.5	-60 32	0.0- 1.5		5	2
7034	ξ Bootis	14	47.9	+19 25	4.7- 6.6	A, M, S, Y	..	2
7259	μ Bootis	15	21.7	+37 38	6.7- 7.3	A, M, S, Y	..	15
7563	σ Cor. Bor.	16	11.9	+34 3	5.7- 6.8	A, S	12	..
7649	λ Ophiuchi	16	27.1	+ 2 9	4.0- 6.1	A, M, S, Y	..	15
7717	ζ Herculis	16	38.5	+31 44	2.8- 6.5	A, S	..	5
7783	Σ 2107	16	48.9	+28 47	6.5- 8.0	A, S	12	..
7878	μ Draconis	17	3.8	+54 34	5.0- 5.1	S	15	15
7929	β 416	17	13.8	-34 54	6.0- 8.0		5	..
8099	26 Draconis	17	34.2	+61 56	5.4 10.1	A, D, S, Y	10	10
8340	70 Ophiuchi	18	1.7	+ 2 31	4.3- 6.3	A, M, S, Y	4	..
8372	99 Herculis	18	4.2	+30 33	5.2-10.5	A, M, S	10	8
8933	β 648	18	54.2	+32 48	5.5- 9.5	A, S	5	..
9605	δ Cygni	19	42.6	+44 57	2.8- 7.9	A, S	..	12
10846	γ Cygni	21	11.8	+37 43	3.8- 7.5	A, S, Y	6	..
11214	μ Cygni	21	40.8	+28 24	4.9- 5.9	M, S, Y	9	..
11743	ζ Aquarii	22	25.0	- 0 24	4.3- 4.5	A, M, Y	..	12
11761	Kruger 60	22	25.4	+57 20	9.0-12.0	D	..	3
12196	π Cephei	23	5.5	+74 59	4.6- 7.5	A, S	..	15
12701	85 Pegasi	23	58.2	+26 41	5.8-11.0	A, D, S, Y	6	6
12755	Σ 3062	0	2.3	+58 1	6.2- 7.3	S, Y	12	..

Yerkes Observatory, Williams Bay, Wis., June, 1916.

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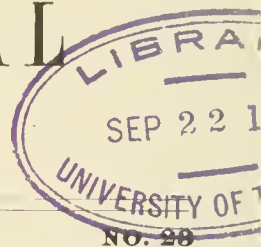
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A SMALL STAR WITH LARGE PROPER-MOTION,

By E. E. BARNARD.

I have found on my photographs a small star of the 11th (photographic) magnitude which has a large proper-motion of about $10''.3$ a year in a direction almost due north. The star is yellowish, while its near neighbor, BD $+4^{\circ} 3560$ ($8^m.7$), is bluish white. On account of its color it is brighter visually than photographically, being about the $10\frac{1}{2}$ visual magnitude. The position of the star is:

Epoch 1916.423 α $17^h 53^m 43^s.60$ $\delta + 4^{\circ} 27' 48''.0$

It is $9^s.1$ following and $51''$ north of $+4^{\circ} 3560$, which is Albany A. G. C. 6005, and which in this paper I have called a .

I have tied the star up visually with the 40-inch telescope with the following stars:

*	Mag.	α	δ
a	8.7	1916.0 $17^h 53^m 34^s.51$	$+4^{\circ} 26' 56''.4$
c	14 $\pm$	17 53 43 .84	$+4 25 41 .6$
k	14 $\pm$	17 53 48 .25	$+4 28 7 .1$
n	13 $\pm$	17 53 43 .37	$+4 24 49 .4$

The positions of c , k and n are derived from that of a . We will call the proper-motion star P .

P and a

		P. A.	Dist.
1916.423	June 3	$249^{\circ}.20$	$145''.30$
.442	10	$249 .10$	$145 .21$
.453	14	$249 .07$	$145 .13$
.461	17	$248 .99$	$145 .16$

P and c

1916.423	June 3	$178^{\circ}.61$	$126''.34$
.453	14	$178 .35$	$126 .25$
.461	17	$178 .42$	$126 .55$
1916.446		$178^{\circ}.46$	$126''.38$

P and k

		P. A.	Dist.
1916.453	June 14	$74^{\circ}.87$	$72''.40$
.461	17	$74 .79$	$72 .30$
1916.457		$74^{\circ}.83$	$72''.35$

A lesser star $22''$ south of k .

P and n

1916.461	June 17	$180^{\circ}.97$	$179''.10$
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a and c

1916.461	June 17	$118^{\circ}.31$	$158''.05$
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a and k

1916.461	June 17	$70^{\circ}.94$	$217''.14$
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P and a (P north following)

		$\Delta\alpha \cos \delta$	$\Delta\alpha$	$\Delta\delta$
1916.423	June 3	$136''.19 =$	$9^s.11$	$51''.46$
.442	10	$135 .52$	$9 .06$	$51 .57$

1916.432		$135''.85 =$	$9^s.08$	$51''.51$
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a and c (c south following)

1916.442	June 10	$139''.58 =$	$9^s.33$	$74''.75$
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a and k (k north following)

1916.442	June 10	$205''.50 =$	$13^s.74$	$70''.69$
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By comparison with the stars a , c and k on a negative made by the writer 1894 August 24, with the 6-inch Willard lens of the Lick Observatory, I have obtained the following positions of P for that date:

(a)	1894.648	α $17^h 52^m 38^s.93$	$\delta + 4^{\circ} 24' 17''.1$
(c)		$17 52 39 .30$	$+ 4 24 15 .4$
(k)		$17 52 39 .03$	$+ 4 24 17 .6$
	1894.648	α $17^h 52^m 39^s.09$	$\delta + 4^{\circ} 24' 16''.7$

STARS MEASURED WITH RESPECT TO THE PROPER-MOTION STAR *P*

The arrow indicates the amount and direction of the annual motion. Epoch of the diagram 1916.4.

The small scale of the plate and the quality of the images do not seem to permit a more accurate position than the above. Bringing this up to 1916, we have:

1916.648 *a* $17^{\text{h}} 53^{\text{m}} 44^{\text{s}}.43$ $\delta + 4^{\circ} 24' 3''.7$

By comparing this with the position from the visual measures of 1916.423 we get the annual motion $10''.30$ in the direction $359^{\circ}.7$.

A comparison with a photograph taken with the 10-inch Bruce telescope on June 11, 1904, gives the annual motion $10''.36$. At present this is the largest known proper-motion of a star.

This star is shown on Plate 53, Vol. XI, *Publications of the Lick Observatory*, where it is easily

identified. It is 1.77 inches (45mm.) from the right side and 2.64 inches (67mm.) from the top of the plate, south following the star BD +4° 3560. It is interesting to compare its position on that plate with the present place in the sky.

Yerkes Observatory, Williams Bay, Wis., June 21, 1916.

ADDITIONAL OBSERVATIONS OF THE PROPER-MOTION STAR.

The following additional observations of the proper-motion star have been made. Two other stars, not on the chart, have been measured: one of these, *b*, is in the direction of motion of *P*, which is approaching it, and will serve admirably for some years for micrometer observations of the motion of *P*.

The rapid motion of the star is shown in the micrometer measures, though the interval is only about one month. The angle is changing in the case of *a* and *k*, while the distance from *n* and *c* is increasing.

On June 24, with good seeing, *P* was tested for any peculiarity of focus. A second, almost stellar, crimson image was formed 7^{mm} outside the regular focus. This image was surrounded by a broken greenish blue or grayish glow.

The following spectroscopic results were obtained with the 60-inch reflector at Mount Wilson by MR. ADAMS, who has kindly given permission to use them.

Radial velocity	−91 km
Spectral type	<i>M b</i> (Harvard classification)
Parallax from spectrum	+0''.2

The visual magnitude as determined at Mt. Wilson by MR. JOY is 9.64.

My own visual observations (rather discordant) with the 40-inch telescope and its finder would make *P* 1.3 magnitude less than *a*, or of the 10.0 magnitude. Direct estimates made it $10\frac{1}{2}$ magnitude. The relative brightness between it and *a* is much affected by bad seeing or moonlight, because of the difference in color of the two stars. Photographically it is of the 11th magnitude. The measures follow:

<i>P</i> and <i>a</i>					
		P. A.	Dist.	Mag.	Mag.
1916.492	June 28	248°.93	145''.24	...	...
.519	July 8	248°.91	145°.12	...	...
1916.505		248°.92	145''.18	...	...

P and *b*.

		P. A.	Dist.	Mag.	Mag.
1916.492	June 28	8°.75	250''.38	...	12
.511	July 5	8°.78	250°.11	...	12.3
.519	8	8°.83	249°.98	...	...

1916.507		8°.79	250''.16	...	12.1
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A $12\frac{1}{4}$ magnitude star 1' north following *b*.

P and *c*

1916.472	June 21	178°.34	126''.81	...	...
.481	24	178°.36	126°.80	...	13.5
.492	28	178°.31	126°.91	...	...
.511	July 5	178°.25	127°.15	...	...
.519	8	178°.20	127°.10	...	...
1916.495		178°.29	126''.95	...	13.5

P and *k*

1916.472	June 21	75°.20	72''.51	...	...
.481	24	75°.21	72°.35	...	12.9
.511	July 5	75°.51	72°.52	...	...
.519	8	75°.52	72°.54	...	...
1916.496		75°.36	72''.48	...	12.9

P and *n*

1916.472	June 21	181°.22	179''.21	...	...
.481	24	181°.10	179°.35	...	13
.492	28	181°.06	179°.45	...	...
.519	July 8	180°.96	179°.57	...	...
1916.491		181°.09	179''.39	...	13

a and *c*

1916.481	June 24	118°.35	158''.34	...	...
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a and *k*

1916.481	June 24	71°.04	216''.99	...	...
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<i>a</i> and <i>n</i>						<i>k</i> and <i>s</i>					
1916.472	June 21	134°.03	183''.80	...	...	1916.472	June 21	204°.25	21''.22	...	13 1/2
.481	24	134.09	183.75	...	...	.492	28	203.10	21.21	12.8	13.4
.492	28	133.88	183.93	...	...						
1916.482		134°.00	183''.83	...	...	1916.482		203°.67	21''.21	12.8	13.5
<i>Yerkes Observatory, Williams Bay, Wis., July 12, 1916.</i>											

Yerkes Observatory, Williams Bay, Wis., July 12, 1916.

OBSERVATIONS OF COMETS AT YERKES OBSERVATORY,

By G. VAN BIESBROECK.

The following observations of comets have been made here with the 12-inch telescope (1 Rev. = 9''.533). Only a few, as indicated in the remarks have been obtained occasionally with the 40-inch refractor. Whenever necessary the measures have been corrected for differential refraction. In the column headed "Comp." the number of settings is given for each coördinate. An equal number of comparisons in both coördinates indicates a measure made while the telescope was driven by the clock. When comparisons in right ascension are more numerous than the ones in declination, it is an indication that transits over the wires have been taken, the telescope being fixed in hour-angle.

The abbreviations proposed by AUWERS (*A. N.* 4176) have been used for the catalogues of reference stars. When the stars' positions are taken from the astrographic catalogues the rectilinear coördinates have been converted into equatorial coördinates by means of HNATEK's tables (*A. N.* 4329). These give a shorter computation than most of the special tables that are found in the respective volumes and have the important advantage of being uniform for all declinations.

For comets that became bright enough, I estimated the total light intensity on moonless nights by comparing them extra-focally with photometrically known stars, or simply with BD stars. The three-inch finder was used for the purpose and the eye-piece shifted far enough out of focus so as to make the star images nearly comparable in size with the image of the comet. The differences are then estimated according to ARGLANDER's step method. In this way the integrated light of the comet — nucleus, head and eventually brightest part of the tail — is referred to known standards of light. Even rough as the BD estimates are, this way of indicating the magnitudes of comets seems to me to have many advantages over a direct, focal estimation, where the brightness of the nucleus alone is likely to be considered as the total luminosity, or at least to bias the result in that direction. This would avoid the apparent anomaly that sometimes comets estimated as faint as 7th or 8th magnitude have been visible to the naked eye. Independent estimates by different observers using the way described here show remarkably consistent results.

Date	Gr. M. T.	$\Delta\alpha$	$\Delta\delta$	Comp.	App. α	App. δ	$\frac{\log p \Delta}{\alpha \delta}$	*
Comet 1915 a (MELLISH)								
1915-1916								
Sept. 13	22 ^h 11 ^m 21 ^s	-0 ^m 45 ^s .53	-4' 1''.8	10-10	6 ^h 26 ^m 59 ^s .12	-20° 37' 45''.1	9.517 _n	0.652 1
18	20 43 33	+0 6.08	+2 11.9	6-5	24 43.41	-19 41 48.4	9.558 _n	0.697 2
20	21 2 18	-0 29.88	+4 5.3	8-8	23 27.11	-19 19 37.7	9.518 _n	0.667 3
29	21 9 49	-0 1.97	-0 16.5	6-6	16 14.56	-17 38 44.8	9.418 _n	0.612 4
Oct. 1	21 27 7	-0 11.43	-1 10.1	6-6	6 14 8.21	-17 15 35.6	9.364 _n	0.588 5
18	19 51 48	+0 14.87	+2 39.8	8-8	5 48 56.30	-13 35 11.8	9.362 _n	0.575 6
Nov. 4	19 23 8	+0 57.25	+2 21.2	10-8	11 44.32	-8 40 11.7	8.968 _n	0.501 7
8	18 6 8	-2 50.54	-3 56.0	30-6	5 2 4.42	-7 20 8.2	9.267 _n	0.523 8
Dec. 1	16 30 28	+1 28.87	+0 9.5	30-6	4 8 42.22	+0 57 43.8	8.994 _n	0.435 9
7	15 46 25	-1 29.67	+4 3.6	30-6	3 57 30.03	3 1 33.6	9.064 _n	0.421 10

Date	Gr. M. T.	$\Delta\alpha$	$\Delta\delta$	Comp.	App. α	App. δ	$\log p \Delta$		*
$\begin{array}{cc} a & \delta \end{array}$									
Comet 1915 a (MELLISH) (Continued)									
1915-1916									
Dec. 13	18 ^h 31 ^m 48 ^s	-1 ^m 26 ^s .13	-4' 29''.0	25-5	3 ^h 47 ^m 40 ^s .29	5° 0' 54''.7	9.389	0.475	11
Jan. 5	12 55 28	-0 7.65	-4 11 .9	8-8	25 39.68	11 10 2 .3	9.215 _n	0.470	12
	31 14 36 52	-0 12.74	-0 38 .0	8-8	22 42.53	16 21 45 .5	9.353	0.334	13
Feb. 2	13 34 57	-0 47.98	+0 0 .9	30-5	23 10.67	16 41 34 .6	9.103	0.266	14
	6 14 3 56	+1 12.60	+0 26 .4	30-5	24 21.78	17 20 56 .6	9.317	0.307	15
	27 14 41 30	-1 23.31	+1 32 .8	6-6	3 37 36.71	+20 25 18 .9	9.568	0.467	16
Comet 1915 d (MELLISH)									
1915									
Sept. 18	22 ^h 45 ^m 0 ^s	+0 ^m 38 ^s .54	-4' 46''.3	2-2	10 ^h 28 ^m 42 ^s .24	+26° 38' 37''.1	9.684 _n	0.713	17
	20 22 35 40	+0 55.77	-4 53 .3	15-5	44 18.51	25 49 32 .0	9.682 _n	0.733	18
	21 22 22 5	-0 0.02	-0 57 .6	8-8	10 52 11.80	+25 21 49 .6	9.679 _n	0.746	19
Comet 1915 e (TAYLOR)									
1915-1916									
Dec. 6	18 ^h 14 ^m 0 ^s	-0 ^m 15 ^s .32	-1' 19''.0	6-6	5 ^h 22 ^m 40 ^s .31	+ 0° 2' 10''.5	7.763 _n	0.430	20
	7 16 24 24	+0 4.09	+0 53 .0	8-8	22 10.77	0 12 23 .2	9.295 _n	0.481	21
	7 18 31 30	+0 0.88	+1 53 .5	8-8	22 7.56	0 13 23 .7	8.554	0.430	21
	9 16 38 35	-0 24.27	-2 57 .0	8-8	21 2.60	0 36 4 .8	9.197 _n	0.458	22
	13 16 27 30	-0 2.71	-6 44 .2	6-6	18 39.83	1 30 25 .7	9.162 _n	0.445	23
	14 16 14 23	+0 5.68	-0 39 .4	8-8	18 3.58	1 45 26 .5	9.205 _n	0.450	24
Jan. 2	14 38 20	+2 36.14	+5 3 .3	30-6	8 3.13	8 9 24 .8	9.260 _n	0.402	26
	3 15 31 53	+0 16.83	+2 12 .4	8-8	7 42.84	8 34 41 .1	8.925 _n	0.357	27
	5 14 0 26	+1 47.87	+0 11 .9	30-6	7 13.30	9 22 35 .7	9.353 _n	0.418	28
	22 12 34 20	+0 6.45	+3 45 .5	6-6	9 37.89	16 38 46 .1	9.428 _n	0.371	29
	23 13 24 28	-0 6.91	+0 5 .6	6-6	10 13.00	17 4 59 .6	9.247 _n	0.287	30
	31 15 48 11	-2 18.97	+4 42 .8	15-3	16 38.93	20 23 42 .8	9.185	0.218	31
Feb. 2	14 11 27	+0 9.67	+3 20 .6	6-6	18 40.46	21 8 40 .0	8.442 _n	0.162	32
	3 14 18 29	+0 6.36	+5 24 .9	6-6	19 47.74	21 31 34 .1	7.889 _n	0.153	33
	9 15 3 47	+0 13.80	+0 29 .3	6-6	5 27 31.89	23 43 14 .6	9.070	0.134	34
Mar. 4	13 9 45	-0 8.62	-1 43 .1	8-8	6 12 27.28	30 47 34 .4	8.943 _n	9.905	35
	22 15 25 25	+0 2.93	+3 29 .5	6-6	6 57 57.41	32 37 5 .8	9.508	0.116	36
April 2	17 9 47	-0 11.36	+1 22 .8	6-6	7 27 40.76	+33 3 57 .0	9.681	0.105	37
Comet 1916 a (NEUMIN)									
1916									
Feb. 29	14 ^h 41 ^m 20 ^s	-0 ^m 21 ^s .35	-0' 3''.3	6-6	8 ^h 58 ^m 46 ^s .28	+13° 35' 18''.2	9.264 _n	0.340	38
Mar. 3	16 11 27	-0 58.84	+3 52 .1	30-6	8 59 26.84	11 58 12 .9	7.904	0.306	40
	4 13 40 28	+0 18.31	+0 54 .8	6-6	8 59 44.67	11 30 3 .6	9.413 _n	0.425	41
	8 13 35 58	-0 29.99	+1 54 .2	8-8	9 1 32.26	9 25 8 .2	9.392 _n	0.435	42
	10 18 2 43	-0 22.70	-0 17 .9	8-8	9 2 49.92	8 18 24 .1	9.399	0.450	43
	22 14 30 38	+4 6.60	-0 41 .2	30-6	9 14 8.78	+ 2 39 17 .2	8.840 _n	0.447	44
April 2	18 4 21	+1 26.24	+3 10 .3	12-4	9 30 11.34	- 1 53 13 .8	9.528	0.614	45
	4 15 4 17	+0 10.36	+2 35 .3	8-8	9 32 58.26	- 2 34 31 .7	8.744	0.454	46
	24 14 47 28	-0 36.63	-6 0 .6	8-8	10 12 15.55	- 8 43 40 .0	8.994	0.503	47
May 3	14 57 36	+1 54.57	-4 3 .9	30-6	10 31 57.82	-10 54 41 .7	9.168	0.552	48
	4 16 43 51	+1 4.78	-4 21 .4	6-6	10 34 4.64	-11 8 58 .8	9.506	0.629	49

Date		Gr. M. T.	$\Delta\alpha$	$\Delta\delta$	Comp.	App. α	App. δ	$\frac{\log p \Delta}{\alpha \qquad \delta}$		*
Comet 1916 b (WOLF)										
1916 May	4	20 ^h 33 ^m 20 ^s	-1 ^m 29 ^s .04	-2' 9''.8	10-5	12 ^h 36 ^m 55 ^s .65	+ 2° 59' 50''.8	9.618	0.724	50
	11	16 4 13	-0 17.46	+3 30.8	8-8	34 12.85	3 28 15.3	9.007	0.413	51
	18	16 1 55	+0 30.81	+0 46.0	8-8	31 55.68	3 53 9.3	9.180	0.427	52
	23	17 7 25	+6 37.21	-1 13.1	6-3	30 37.21	4 8 12.8	9.464	0.525	53
	28	15 41 9	+1 49.06	-0 39.5	6-4	29 38.83	4 20 27.9	9.274	0.442	54
	30	16 49 21	+1 30.13	+3 42.9	10-4	29 19.88	4 24 50.5	9.427	0.450	55
June	4	17 16 26	-2 5.33	+3 42.5	8-4	28 47.94	4 33 47.4	9.559	0.613	56

Comparison Stars

Mean Coördinates for Beginning of Year and Reductions to Apparent Places

	α	δ	Red. α	Red. δ	Authority
1	6 ^h 27 ^m 42 ^s .65	-20° 34' 1''.9	+2.00	+18''.6	Ci. Z. 1039.
2	6 24 35.08	-19 44 19.2	2.25	18.9	Ci. Z. 1029.
3	6 23 54.73	-19 24 1.9	2.29	18.9	Ci. Z. 1023.
4	6 16 14.16	-17 38 47.4	2.37	19.1	Wash. 2016.
5	6 14 16.97	-17 14 44.5	2.70	19.0	Wash. 1996.
6	5 49 44.43	-13 28 53.3	3.44	18.7	Cbr. M. 1665.
7	5 10 42.91	- 8 42 51.3	4.16	18.4	Ott. 1397.
8	5 4 50.66	- 7 16 30.2	4.30	18.0	Ott. 1355.
9	4 7 8.46	+ 0 57 15.4	4.89	18.9	Kü. 1795.
10	3 58 54.73	2 57 11.9	4.97	19.1	Alb. 1183.
11	3 49 1.40	5 5 4.1	5.02	19.6	Kü. 1658.
12	3 25 55.04	11 14 5.3	1.75	8.9	Lpz. I 1028.
13	3 22 53.78	16 22 14.1	1.49	9.4	Berl. A. 923.
14	3 23 57.17	16 41 24.3	1.48	9.4	Bord. ph. 3 ^h 24 ^m , + 17° N ^o . 156.
15	3 23 7.77	17 20 20.7	1.41	9.5	Bord. ph. 3 ^h 24 ^m , +17° N ^o . 24.
16	3 36 12.22	20 23 36.6	1.18	+ 9.5	Berl. B. 1095.
17	10 28 1.64	26 43 33.5	2.06	-10.1	Cbr. E. 5396.
18	10 43 20.72	25 54 36.1	2.02	-10.8	Ox. ph. 25° 38' 53.2 = 26° 29' 27.3.
19	10 52 9.82	25 22 58.3	2.00	-11.1	Cbr. E. 5506.
20	5 22 50.69	0 3 17.9	4.94	+11.6	Alg. ph. 5 ^h 20 ^m , 0° N ^o . 120.
21	5 22 1.72	0 11 18.7	4.96	11.5	Alg. ph. 5 ^h 20 ^m , 0° N ^o . 102.
22	5 21 21.87	0 38 50.5	5.00	11.3	Alg. ph. 5 ^h 20 ^m , 0° N ^o . 85.
23	5 18 37.46	1 36 58.9	5.08	11.0	BD + 1° 991 from 3 obs. 1910 in Abbadia.
24	5 16 52.81	1 45 54.9	5.09	11.0	BD + 1° 989 referred to Star 25.
25	5 20 20.03	1 46 8.9	5.09	11.0	Boss' P. G. C. 1302.
26	5 5 24.86	8 4 15.4	2.13	6.1	Lpz. II 2027.
27	5 7 23.87	8 32 22.7	2.14	6.0	Tou. ph. 5 ^h 8 ^m , +9° N ^o . 107.
28	5 5 23.28	9 22 17.7	2.15	6.1	Lpz. II 2026.
29	5 9 29.24	16 34 54.0	2.20	6.6	Berl. A. 1433.
30	5 10 17.70	17 4 47.3	2.21	6.7	Bord. ph. 5 ^h 8 ^m , +17° N ^o . 66 = 5 ^h 12 ^m , +16° N ^o . 22.
31	5 18 55.66	20 19 11.1	2.24	7.0	Berl. B. 1723.
32	5 18 28.56	21 5 12.2	2.23	7.2	Par. ph. 5 ^h 20 ^m , +22° N ^o . 183.
33	5 19 39.15	21 26 1.9	2.23	7.3	Berl. B. 1728.

	α	δ	Red. α	Red. δ	Authority
34	5 ^h 27 ^m 15 ^s .86	23° 42' 37".6	2 ^s .23	7".7	Par. ph. 5 ^h 28 ^m , +24° N ^o . 154.
35	6 12 33.59	30 49 9 .7	2.31	7 .8	Ox. ph. 30° 14384 = Ox. ph. 31° 15857.
36	6 57 52.16	32 33 30 .0	2.32	6 .3	Pots. ph. Bd. 4, S. 202, N ^o . 268 = Bd. 6, S. 32, N ^o . 75.
37	7 27 49.80	33 2 29 .2	2.32	+ 5 .0	Lei. 3176.
38	8 59 4.90	13 35 29 .6	2.73	- 8 .1	BD 13° 2032 referred to Star 39.
39	9 0 21.20	13 40 45 .0	2.73	- 8 .1	BD 13° 2036 (<i>Annales Bordeaux</i> , Vol. XV).
40	9 0 22.97	11 54 29 .0	2.71	- 8 .2	BD 12° 1963 (<i>Annales Bordeaux</i> , Vol. XV).
41	8 59 23.66	11 29 17 .0	2.70	- 8 .2	Kü. 4009.
42	9 1 59.60	9 23 22 .8	2.65	- 8 .8	Kü. 4030.
43	9 3 9.98	8 18 50 .6	2.64	- 8 .6	Tou. ph. 9 ^h 0 ^m , +9° N ^o . 250.
44	9 9 59.72	+ 2 40 9 .3	2.46	-10 .9	Boss' <i>P. G. C.</i> 2479 (θ <i>Hydræ</i>).
45	9 28 42.71	- 1 56 10 .9	2.39	-13 .2	Nic. 2887.
46	9 32 45.56	- 2 36 53 .5	2.34	-13 .5	Alg. ph. 9 ^h 28 ^m , -2° N ^o . 165 = 9 ^h 36 ^m , -2° N ^o . 174.
47	10 12 49.94	- 8 37 22 .3	2.24	-17 .1	Ott. 3950.
48	10 30 1.04	-10 50 19 .4	2.21	-18 .4	Cbr. M. 4002.
49	10 33 7.62	-11 4 18 .9	2.22	-18 .5	Cbr. M. 4027.
50	12 38 21.79	+ 3 2 18 .3	2.90	-17 .9	BD +3° 2681 from 3 obs. Abbadia.
51	12 33 52.53	3 25 1 .7	2.86	-17 .3	Kü. 5582.
52	12 31 22.06	3 52 40 .0	2.81	-16 .7	Alb. 4519.
53	12 23 59.29	4 9 42 .0	2.75	-16 .1	Alb. 4493.
54	12 27 47.04	4 21 23 .2	2.75	-15 .8	BD +4° 2626 from 5 obs. Abbadia.
55	12 27 47.04	4 21 23 .2	2.71	-15 .6	BD +4° 2626 from 5 obs. Abbadia.
56	12 30 50.58	4 30 20 .2	2.69	-15 .3	Lpz. II 6174.

REMARKS

Comet 1915 *a*. Sept. 18. Observation interrupted by clouds.

Nov. 4. Nucleus unsharp.

Dec. 1. Nucleus unsharp. About 10^m.5. Tail at least 1° long.

Jan. 5. Sharp nucleus about 12^m.

Feb. 2. Settings difficult with 12-inch. Brightness about 11^m.5.

Feb. 27. With 40-inch. Brightness 13^m. Nucleus only 14^m.

Extrafocal estimation of brightness in the 3-inch finder:

Nov. 4. $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$ - 2 - BD - 8° 1073; BD - 8° 1078 - 4 - $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$. Concluded $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$ = 8^m.7.

Nov. 8. $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$ - 1 - BD - 7° 985; BD - 7° 974 - 2 - $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$. Concluded $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$ = 8 .7.

Dec. 1. $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$ - 5 - BD + 0° 711; BD + 0° 707 - 4 - $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$. Concluded $\begin{smallmatrix} \diagup \\ \diagdown \end{smallmatrix}$ = 8 .5.

Comet 1915 *d*. Sept. 18. Poor observation. Object hardly visible on bright morning sky.

Sept. 20. Difficult. Object diffuse.

Sept. 21. Difficult in morning sky.

Comet 1915 *e*. Dec. 6. Interrupted by clouds.

Dec. 7. Tail about 10' long in position angle 310°. Brightness 10^m±.

Jan. 22. Diameter decreased. Brightness 10^m.5±.

Feb. 2. Nucleus unsharp. Settings difficult in δ on account of elongation in that direction.
Total brightness 10^m.5±.

Feb. 3. Same elongation.

Feb. 9. Setting on principal nucleus, the companion being hardly visible in 12-inch.

Mar. 4. Second nucleus invisible in 12-inch.

Apr. 2. With 40-inch. Setting on the eccentric brightest part of the fan-shaped nebulosity.
Total brightness 13^m.5±. Nucleus single 14^m.5±.

Extrafocal estimations of brightness in the 3-inch finder:

- Dec. 9. $\text{☿} - 2 - \text{BD } + 0^\circ 1055$; $\text{BD } 0^\circ 1045 - 4 - \text{☿}$; $\text{☿} = 9^m.3$.
 Jan. 2. $\text{☿} = \text{BD } 8^\circ 878$; $\text{☿} - 5 - \text{BD } 8^\circ 880$; $\text{BD } 8^\circ 880$; $\text{BD } 8^\circ 888 - 4 - \text{☿}$; $\text{☿} = 8^m.8$.
 Jan. 3. $\text{☿} = \text{BD } 8^\circ 881$; $\text{☿} = 9^m.2$.
 Jan. 5. $\text{☿} = \text{BD } 9^\circ 760$; $\text{BD } 9^\circ 747 - 2 - \text{☿}$; $\text{☿} = 9^m.0$.

- Comet 1916 a. Mar. 3. Round nebulosity about 2' in diameter.
 Mar. 8. Total brightness about 10^m.
 Mar. 10. Faint stellar nucleus (13^m).
 Mar. 22. Total brightness 11^m±. Faint nucleus 13^m.5.
 Apr. 24. Diffuse and about 3' in diameter.
 May 3. Total brightness 12^m±.

- Comet 1916 C. May 4. Observed at low altitude after the object had been identified on a plate taken by
 M. HUBBLE with the 2-ft. reflector.
 May 11. Small object with eccentric nucleus. Total brightness 14^m±.
 May 18-23. Brightness and appearance unchanged.
 May 28-30. Luminosity is increased. About 13^m.

Williams Bay, Wis., June, 1916.

ELLIPTIC ELEMENTS OF TAYLOR'S COMET,

BY FRANK E. SEAGRAVE.

Geocentric positions (normals) that elements are based on:

G. M. T.

$t = (1915) \text{ Dec. } 7.7719$	$\lambda = 79^\circ 41' 34''.46$	$\beta = -22^\circ 53' 33''.44$
$t' = (1916) \text{ Jan. } 8.6177$	$\lambda' = 76^\circ 36' 22''.52$	$\beta' = -12^\circ 10' 37''.14$
$t'' = (1916) \text{ Feb. } 9.5742$	$\lambda'' = 82^\circ 33' 10''.06$	$\beta'' = + 0^\circ 27' 40''.59$
$\odot = 255^\circ 4' 43''.00$	$\text{Log } R = 9.993434$	$\psi' = 146^\circ 59' 22''.56$
$\odot' = 287^\circ 31' 39''.47$	$\text{Log } R' = 9.992715$	$z' = 19^\circ 48' 36''.21$
$\odot'' = 320^\circ 0' 33''.00$	$\text{Log } R'' = 9.994218$	

CONSTANTS

$x = r (9.986314)$	$\text{Sin } (204^\circ 35' 3''.65 + u)$
$y = r (9.981829)$	$\text{Sin } (118^\circ 54' 21''.24 + u)$
$z = r (9.575159)$	$\text{Sin } (75^\circ 38' 8''.01 + u)$

Epoch = Jan. 8.6177 G.M.T. 1916

$M = 356^\circ 40' 17''.21$
$\omega = 354^\circ 50' 2''.12$
$\pi = 108^\circ 36' 27''.60$
$\Omega = 113^\circ 46' 25''.48$
$i = 15^\circ 40' 5''.42$
$\text{Log } e = 9.745210$
$\text{Log } a = 0.547051$
$\text{Log } q = 0.194270$
$\mu = 536''.3251$

Heliocentric coördinates from 5th and last hypothesis.

$l = 76^\circ 54' 55''.65$	$b = -9^\circ 33' 1''.48$
$l' = 95^\circ 19' 29''.15$	$b' = -5^\circ 4' 20''.76$
$l'' = 114^\circ 36' 37''.96$	$b'' = +0^\circ 14' 4''.93$
$\text{Log } r = 0.219655$	$u = 322^\circ 5' 42''.47$
$\text{Log } r' = 0.198869$	$u' = 340^\circ 53' 23''.37$
$\text{Log } r'' = 0.195130$	$u'' = 0^\circ 52' 8''.72$

On March 18, 1916, 13^h 40^m 5^s. G.M.T., PROF. BARNARD of the Yerkes Observatory observed the comet in the following position $\alpha = 6^h 47^m 19^s.14$. $\delta = +32^\circ 16' 0''.40$. These elements give the following position of the comet for the same time $\alpha = 6^h 47^m 9^s.72$. $\delta = +32^\circ 16' 9''.40$. Fairly good.

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NO. 24

ON EVIDENCES OF SYSTEMATIC VARIATION IN RECENT LONGER SERIES
OF DETERMINATIONS OF STELLAR PARALLAX,

By ALBERT S. FLINT.

The investigation from which the following is taken forms a part of the subsequent discussion of the second series of parallax observations made at the Washburn Observatory of the University of Wisconsin, the results of which were published as noted below. In view of the large work in parallax determination in progress at several observatories, it seems desirable to publish at this time the principal results of the present discussion. In the previous publication only a brief comparison was made with other authorities, as presented in the *Kapteyn-Wersma Catalogue* of 1910. Since then there have been published the complete Yale series, containing a number of stars not included in that catalogue and revised results for others of the Yale stars. ABETTI and JEWDOKIMOV also have published each an important series of results; and the RUSSELL-HINKS list and the especially refined determinations by SCHLESINGER-SLOCUM-MITCHELL have all been published definitively in the journals. Apart from differences somewhat constant and due perhaps to peculiarities of method, apparatus, or observers, it may be assumed that systematic error is most liable to occur relatively to the right ascension, or season of the year, and the present examination is confined to such possible relation. The data for the main discussion were derived from a comparison of each of the authorities on a given star with the mean of all the selected authorities that occur on that star. Absolute parallax values, observed and determined as such, have been omitted. For example, on α *Ursæ Minoris* only two authorities are represented, PRITCHARD and FLINT II, since these alone have made differential determinations for this star.

No data were omitted from any selected authority except as described in the treatment of certain authorities in preparation for the comparisons or, as noted thereafter, in connection with special supplementary comparisons. No data were omitted anywhere except upon *a priori* grounds. Some attempts were made at the beginning

to employ numerical solutions, especially in deriving the preliminary correction to certain authorities; but this proved to be unsatisfactory and the graphical method was adopted throughout. This lacks the definite measure of the accuracy of numerical results, such as is furnished by the method of least squares, but it is especially valuable for detecting tendencies to aberration and for following the smaller and less regular variations.

The list of authorities covered by the discussion, together with an account of the preparation of each for the comparison, is as follows. The values of observed parallax for the larger series were obtained from the original publications or computations. The parallaxes were taken to thousandths of a second except for JEWDOKIMOV, published only to hundredths, and a number of the Yale results of larger probable error similarly published. PETER's results alone of the larger series were copied from the column of absolute parallax in the *Kapteyn-Wersma Catalogue*. The few values from each of a number of miscellaneous authorities were adopted from the *Kapteyn-Wersma Catalogue* or that of *Bigourdan*. The observed parallaxes of all the authorities were reduced to absolute values by means of the Groningen table, *Publications* No. 24. The abbreviation adopted for each authority is given in parentheses after the title. The authorities are presented here in the order in which they were taken up in the discussion.

YALE, (Y.). Vol. II, Part IV, 1912; 243 parallaxes. With the heliometer. The mean magnitudes of the comparison stars, taken as observed in pairs, all fall between 6.0 and 8.8, and the reductions to absolute values for individual parallax stars differ from $+0''.005$ to $+0''.015$.

Upon inspection, there appeared to be a systematic variation of a periodic nature in the parallaxes running with the right ascension, and a preliminary computation was undertaken to obtain the corresponding reductions to an approximate general mean value of $+0''.0400$.

In the first computation several values were omitted in accordance with the rules adopted in the final computation, as described below. Among them were α *Bootis*, 61 *Cygni*, and *Groombridge* 1830, which have very abnormal parallaxes with reference to proper-motion. The great majority of Yale stars are included under spectral classes *G* and *K*, and all were reduced to the mean of these two classes* by the application of the difference between this mean and the means of the respective spectral classes*. Group means were then computed in order of proper-motion with nine to thirteen stars in each group and with the stars divided into two series, those of magnitude 5.5 or brighter, and those fainter than magnitude 5.5. The two series were plotted separately and a straight line drawn for each as best representing the points. All of the stars included have recorded proper-motions except four and the proper-motions of these four were assumed to be zero. The difference between the ordinate of the line at each star's right ascension and the approximate mean value of the parallaxes, $+0''.0400$, in the sense latter minus former, was applied as a correction to the star's parallax. The entire list of 243 parallaxes reduced with reference to spectral class and proper-motion, were written in order of right ascension and the values adopted were divided into groups of nine or ten each. The group means were plotted and a curve was drawn free-hand to represent the points. The original parallaxes were corrected by means of the ordinates of the curve and the entire computation was repeated, so that the reductions obtained with reference to right ascension were employed in redetermining the reductions with reference to spectral class and proper-motion and these in turn for redetermining the reductions with reference to right ascension.

An ideal series of parallaxes for such a comparison between its own members, would be one of faint stars known to have very small proper-motion and small parallax. From this consideration and from others apparent in the items stated below, it seemed advisable to omit certain stars in the second computation, as follows. Nearly all of these were omitted in the first computation also. The numbers at the right show the consequent reduction in the number of stars of the complete list.

- | | |
|--|----------|
| 1. Those having the observed probable error of parallax greater than $\pm 0''.055$. | 6 stars. |
| 2. Those which, after the application of the reduction depending upon proper-motion, still had abnormally high and therefore probably large real parallaxes. | 3 stars. |
| 3. Those having parallaxes abnormally low with reference to proper-motion. | 3 stars. |

*For Yale.

- | | |
|--|----------|
| 4. Those whose comparison stars have a marked proper-motion (Yale, Vol. II, p. 381). | 4 stars. |
| 5. Comparison stars observed as parallax stars, or stars in very close association with others already adopted in the list: Stars <i>e</i> and <i>f</i> near $17^h 20^m$; also β_1 <i>Cygni</i> , $19^h 26^m$, non-descript as to spectrum, and having β_2 to represent that point in the sky. | 3 stars. |
| 6. The parallaxes of the <i>B</i> stars in <i>Orion</i> were taken in one mean value, as a single star. | 4 stars. |
| 7. The three (faint) <i>Pleiades</i> were taken in one mean value, as a single star. | 2 stars. |

These omissions reduced the number of adopted parallaxes from 243 to 218.

The adopted expressions for the parallax as varying with the proper-motion are as follows:

Mag. 5.5 or brighter —

$$\pi = +0''.0400 + 0.11912 (\mu - 0''.1555) \quad 65 \text{ stars.}$$

Mag. fainter than 5.5 —

$$\pi = +0''.0400 + 0.08336 (\mu - 0''.5550) \quad 153 \text{ stars.}$$

The 218 parallaxes were corrected in accordance with the curve and these equations, and were divided into twenty-two groups in order of right ascension, with nine to ten stars in each group. Then the residual reductions of the individual stars with reference to their respective group means were computed. The largest positive residual in each group was written down and likewise the largest negative residual, so that these largest residuals were collected in order of numerical magnitude in two columns, the one positive, the other negative. It was found that the highest five residuals in the positive series rose above the preceding residuals in the column by ordinary differences, but that the highest five residuals in the negative series rose above the preceding residuals in their column by decidedly increased differences. These highest five values of each series were then thrown out from their respective groups and the corresponding group means recomputed for the six groups from which the ten residuals came. The resulting groups did not differ materially from those of the remaining sixteen groups, of average appearance as regards the residuals. There are undoubtedly a number of sensible real parallaxes in the list, and therefore, the larger negative residuals must be expected to predominate over the larger positive residuals. Accordingly it was decided to omit from the final computation, the stars having the four highest negative residuals and those having the two highest positive residuals. The former, the four residuals,

range from $-0''.097$ to $-0''.140$ and the latter, the two residuals, are $+0''.112$ and $+0''.114$. The total number of parallaxes entering into the determination of the systematic variation with right ascension was thus further reduced from 218 to 212.

The final plot with reference to right ascension is shown in Fig. 1. The number of points appears as eighteen, which was the number of groups occurring in the final computation, with eleven to twelve stars in each group. The reductions of the ordinates of the curve to the standard mean, $+0''.0400$, were read for every half-hour of right ascension and are given in the third column of Table I.

These were the only corrections applied to the Yale absolute values as they were entered in the comparisons with other authorities.

A general average value of the observed probable error of a Yale parallax may be taken as $\pm 0''.0296$. This is the mean for the 115 stars that occur in common with any others of the authorities selected for the subsequent portion of the discussion. There is a great

variety of magnitudes, however, in the published Yale probable errors, of which further note will be made. The average number of parallaxes entering into one group mean of the plot is 11.8; so that the consequent approximate average probable error of a single point in the plot is $\pm 0''.0093$. It is seen that the corrections given in the table range from $+0''.029$ to $-0''.015$.

In the comparisons of Yale with other authorities, no omissions were made except under the first and fourth of the items described above. That is, the omitted stars were those having the probable error of a parallax greater than $\pm 0''.055$ and in addition the four cases of sensible proper-motion in the comparison stars.

FLINT I, (FL. I). First Series. Publications of the Washburn Observatory, Vol. XI, Table IX, 1902; 96 parallaxes. Meridian transits with fixed vertical threads. The values adopted here are those of column five of Table IX, the original observed parallaxes uncorrected except as reduced here to absolute values.

This series of results is very uneven, mainly because of the small number of observations on some of the stars.

TABLE I

Systematic Corrections with Reference to Right Ascension, to be Applied to the YALE Observed Parallaxes

R. A.	Parallax Ordinate	Correction	Diff.	R. A.	Parallax Ordinate	Correction	Diff.
0 ^h .0	$+0''.044$	$-0''.004$		12 ^h .0	$+0''.028$	$+0''.012$	
0.5	050	- 010	-6	12.5	020	+ 010	- 2
1.0	051	- 011	-1	13.0	032	+ 008	- 2
1.5	051	- 011	0	13.5	034	+ 006	- 2
2.0	050	- 010	+1	14.0	035	+ 005	- 1
2.5	049	- 009	+1	14.5	038	+ 002	- 3
3.0	048	- 008	+1	15.0	040	000	- 2
3.5	046	- 006	+2	15.5	043	- 003	- 3
4.0	044	- 004	+2	16.0	045	- 005	- 2
4.5	040	000	+4	16.5	049	- 009	- 4
5.0	038	+ 002	+2	17.0	051	- 011	- 2
5.5	034	+ 006	+4	17.5	053	- 013	- 2
6.0	030	+ 010	+4	18.0	055	- 015	- 2
6.5	027	+ 013	+3	18.5	052	- 012	+ 3
7.0	024	+ 016	+3	19.0	045	- 005	+ 7
7.5	021	+ 019	+3	19.5	039	+ 001	+ 6
8.0	020	+ 020	+1	20.0	031	+ 009	+ 8
8.5	020	+ 020	0	20.5	024	+ 016	+ 7
9.0	020	+ 020	0	21.0	019	+ 021	+ 5
9.5	020	+ 020	0	21.5	012	+ 028	+ 7
10.0	021	+ 019	-1	22.0	011	+ 029	+ 1
10.5	022	+ 018	-1	22.5	013	+ 027	- 2
11.0	024	+ 016	-2	23.0	020	+ 020	- 7
11.5	026	+ 014	-2	23.5	032	+ 008	-12
12.0	$+0''.028$	$+0''.012$	-2	24 ^h .0	$+0''.044$	$-0''.004$	-12

There seems also to have been a large disturbing factor, especially in the last hours of right ascension. As a matter of expedition and experiment, observations were made on many nights when the star images were extremely poor or the temperature very low. Besides this, there was a frequent and very marked disturbance, worst near the zenith, due evidently to currents of warm air from a large attic space above the observing room which was open to the air above the instrument. This loft did not get sufficiently ventilated as shown by comparisons of thermometers above and below in the late afternoon.

Parallaxes of this list whose probable errors exceeded $\pm 0''.055$ are excluded. This leaves seventy-four values and their average probable error is $\pm 0''.0389$. Among these are the twenty-seven stars of Vol. XI, Table VIII, from whose comparison stars were derived the quantities e of Table IX, designed to correct the observed parallaxes for personal equation, depending upon difference of magnitude between the parallax and comparison stars. Several attempts were made to determine this correction more satisfactorily. For this purpose, all of those stars were included for which separate solutions had been made with the individual comparison stars. There were four of these stars which were not included in the discussion in Vol. XI, because the differences of magnitude were so small. With the addition of these four stars, the total number available for the discussion became thirty-one. These values of apparent parallax were divided into six groups, in order of algebraic difference of magnitude. The group means, when plotted, were very discordant and did not seem to have any marked significance as a series. A simple equation was then derived from a comparison of the mean of the first three groups with the mean of the last three groups. A comparison of the full parallax results, uncorrected for magnitude equation, was made also with all the other authorities available. The adopted equations were as follows:

From comparison stars of Fl. I, 31 pairs:

$$\Delta \pi = -0''.0072 + 0''.06483 \quad (1/2 Dm).$$

From comparison with other authorities, 57 stars:

$$\Delta \pi = +0''.0155 + 0''.06125 \quad (1/2 Dm).$$

Here $1/2 Dm$ is the difference in apparent magnitude between parallax star and mean of comparison stars, in the sense latter minus former. Notwithstanding the close agreement of the magnitude coefficients in these two equations, the aberrations of individual values and group means of several values do not appear capable of any entirely satisfactory explanation. The observed parallaxes were corrected by means of the first of the

two equations above before they were entered in the comparison of authorities.

FLINT II, (Fl. II). Second Series, Results, *Astronomical Journal*, No. 631, with one correction in No. 636, 1912; 124 independent parallaxes. Meridian transits with hand-driven traveling threads, star images set between double threads. No screens were employed on the fainter parallax stars—17 in number—and the remaining 107 stars were almost equally divided between the two methods of reducing apparent magnitude, one by means of the separate wire screens as in the FLINT I series, and the other by means of rotating slats mounted on the objective end of the telescope. The probable errors rise over $\pm 0''.050$ in only two cases, and in these there were especially unfavorable conditions. With the omission of these two stars, the average probable error is $\pm 0''.0320$.

An investigation of magnitude equation was carried out in the same manner as for Series I, except that the plotted points were well represented by a straight line; but the differences of magnitude were on the average only one-third as much. From the thirty-one cases for which solutions had been made for apparent parallax of comparison stars, one was omitted, θ Centauri at $-35^\circ.9$ declination. The resulting equations were as follows:

From comparison stars, 30 pairs,

$$\Delta \pi = -0''.0183 + 0''.01036 \quad (1/2 Dm).$$

From comparison with other authorities, 59 stars,

$$\Delta \pi = -0''.0040 + 0''.01418 \quad (1/2 Dm).$$

The observed parallaxes were corrected by means of the first of these equations before they were entered in the comparison with other authorities.

RUSSELL-HINKS, (Russ.). *Astronomical Journal*, Nos. 618, 619 (1910); 40 independent parallaxes. Photographic. Comparison stars all of about the ninth magnitude.

SCHLESINGER-SLOCUM-MITCHELL, (Schl.). *Astrophysical Journal*, Vol. XXXIV, p. 28 (1910-1911); *Astronomische Nachrichten*, No. 4709 and 4960 (1914); 54 independent parallaxes. Photographic.

JEWDOKIMOV, (Jelko.). *Annales de l'Obs., Astr. de l'Univ. Imp. de Kharkow*, Tome III, (1912); 59 parallaxes. Meridian transits with traveling threads fitted with driving clock but driven at times by hand.

PRITCHARD, (Pr.). *Astr. Obsns.*, Univ. Obs. Oxford, No. IV, Part II (1892); 28 parallaxes. Photographic. This series was undertaken mainly to determine the parallaxes of second-magnitude stars and incidentally to demonstrate the efficacy of the photographic method.

A reflecting telescope was employed. Of the list of twenty-eight stars there are twenty-five that occur in the FLINT II list, all as second-magnitude stars, and of these there are eighteen stars which have no other authorities. Previous determinations had been made on only two of PRITCHARD's stars, the values of absolute parallax for *a Ursæ Minoris*, and the well-known values of relative parallax for 61 *Cygni*.

PRITCHARD published the parallax and probable error from a solution made with each comparison star separately. The simple mean of the several values of parallax for a given star was adopted here; and the lowest probable error in the separate solutions was adopted, for the sake of simplicity, as the observed probable error of the adopted parallax for that star.

ABETTI, (Abbt.). (Heidelberg, 1906—1908); *Memorie del R. Oss. Astr. al Coll. Rom.*, Ser. III, Vol. V., Parte II (1912). Forty-one independent parallaxes. Meridian transits with clock-driven threads; star images set between double threads. Otherwise the instrument the same as employed by JOST in his series.

PETER, Leipzig. Helimeter; 13 parallaxes, (1892—1908). Results taken here from the *Kapteyn-Wersma Catalogue*.

JOST, Heidelberg, Vol. IV (1906); 29 parallaxes. Meridian transits with fixed vertical threads.

In addition the following authorities were included in the mean values of parallax for individual stars.

	No. of Stars		No. of Stars		No. of Stars
AUWERS	4	HUSSEY	1	RAMBAUT	2
BARNARD	1	KAPTEYN	10	DE SITTER	2
BELOPOLSKI	1	KAPTEYN and DE SITTER	5	H. STRUVE	1
BERGSTRAND	1	KOSTINSKY	6	TIKHOFF	1
DÖLLEN	1	KRUEGER	3		
(Schlüter- Wichmann)		LAU	1	WILSING	1
		MITCHELL	2	WINNECKE	1
GILL	1	(McCormick Observatory)		ZEIPEL	1
GILL-ELKIN	1			(Kostinsky)	
A. HALL	5				

The total numbers of parallax values from each authority that enter into the comparisons for the several authorities are given in Table IV, as the sums of the column $\bar{m}$. They appear also in Table V., third column.

No data were omitted in the comparisons of authorities as represented in Tables II — VI, and the accompanying plots, except in accordance with the first and fourth items of the list of exceptions in the preliminary treatment of YALE. The first rule was applied strictly to YALE and FLINT I. In the case of FLINT II and RUSSELL,

there was one star for each in excess as to probable error, but both were included. In the case of JEWDOKIMOV seven out of the fifty-four stars that are found in other authorities have probable errors in excess but were included. The remaining authorities had no cases of probable error greater than $\pm 0''.055$.

As the object in view was not to obtain the most probable mean values of parallax for the individual stars, but to obtain for each authority a series of differences from standard mean values which should be as free as possible from the dominating influence of any one authority, the several values of parallax for each star were combined with equal weight. Then the residuals for each authority, in the sense Mean — Authority, were collected in order of right ascension and group means computed to be plotted for evidence of systematic error. In this process regard was had to natural divisions due to gaps in the abscissae (right ascensions) rather than to the even distribution of the stars in number amongst the several groups.

An attempt was made to determine from the average probable errors of the several authorities the number of differences of parallax that should go into each group mean in order that all the group means, as between several authorities, should have approximately the same weight. This would mean that all the points in the accompanying plots would have approximately the same probable error. The average of the observed probable errors for the ten selected authorities, is $\pm 0''.0278$, and the general average of the number of authorities that occur on a single star may be taken as 3.65. Then we may express the general probable error of a single difference of parallax values in the discussion as

$$r_d = \frac{\sqrt{2.65}}{3.65} \sqrt{\{2.65r_1^2 + (0.0278)^2\}};$$

where r_1 is the average probable error of a parallax for the given authority. But the consideration of gaps in the series in order of right ascension, for a given authority, and the advisability of dividing up so as to secure a larger count of groups and so of plotted points in the case of an authority with a smaller number of parallax differences, led to considerable modification of the theoretical numbers. The data here concerned are given in Table II.

FULL DESIGNATIONS OF COLUMNS OF TABLE II

1. Authority.
2. Average probable error of a single observed parallax.

TABLE II.

Miscellaneous Data of the Comparisons.

1	2	3	4	5	6	7	8	9	10	11	12
Authority	Prob. Err. Obs'd Par.	No. of Auth.	Prob. Err. Diff. (Formula)	Theor. No. Diffs.	Adopted No. Diffs.	Pr. Err. of a Point (Formula)	Pr. Err. of a Diff. (Comp.)	Pr. Err. of a Point (Comp.)	No. Res. > 2.79 × Pr. Err.	No. such Residuals by Theory	Av. Red. to Mean. of Auths.
YALE	±0''.0296	2.90	±0''.0248	10.0	10.5	±0''.0076	±0''.0176	±0''.0054	13	6.9	-0''.0026
FLINT I	0380	3.59	302	14.8	7.2	0113	0338	0126	8	3.5	+ 0183
FLINT II	0320	2.59	264	11.3	8.1	0093	0222	0078	11	4.9	+ 0066
RUSSELL	0248	4.27	219	7.8	3.6	0115	0274	0144	3	1.8	+ 0165
SCHLESINGER	0131	3.91	156	4.0	4.6	0073	0168	0078	5	1.9	+ 0120
JEWDOKIMOV	0474	3.65	366	21.7	6.8	0140	0293	0112	4	3.2	- 0135
PRITCHARD	0250	3.14	220	7.8	3.5	0118	0204	0109	3	1.7	- 0235
ABETTI	0238	3.86	213	7.4	3.5	0114	0230	0123	2	0.8	- 0050
PETER	0112	4.69	148	3.6	3.2	0083	0179	0109	1	0.8	- 0093
JOST	0332	3.92	271	11.9	3.3	±0 .0149	±0 .0194	±0 .0107	2	0.8	-0 .0126
Gen. Avg.	±0''.0278	3.652									-0''.00131

The total number of stars that each authority has in common with other authorities may be seen readily in Table IV at the foot of the column m for the respective authorities; also in Table V, 3d column.

3. Average number of authorities on a single star, including the authority named.

4. Approximate average probable error of a single reduction to mean of all authorities on a single star. By formula.

5. Theoretical number of differences to one group mean or plotted point, as depending upon the numbers in column 4.

6. Adopted number of differences to one group mean or plotted point.

7. Resulting theoretical probable error of a plotted point, as depending upon the numbers in columns 4 and 6.

8. Average probable error of a single reduction to mean of all authorities on a single star computed for each list of individual differences. To compare with column 4.

9. Probable error of a plotted point, representing an average number of differences, as depending upon the numbers of column 8. To compare with column 7.

10. Number of residuals > 2.79 × the probable error of column 8; where the residuals are the differences for

each authority between the reductions referred to in column 8 and the general mean of such reductions for that authority.

11. Number of such residuals according to theory of least squares, 6 per cent. of total number.

12. General average reduction of absolute parallaxes to the mean of all authorities on the individual stars.

The case of the YALE probable errors, as noted earlier, calls for further remark. The larger values in general are excessive, in all probability, when compared with published probable errors of other authorities. These larger values appear to be due in the great majority of cases to a "systematic error" so called, which was determined for individual stars from solutions with different pairs of comparison stars. This error was compounded with the probable errors resulting from the least squares solutions and the resulting number published in the catalogue. As a test of the significance of these larger and smaller probable errors in relation to the parallax values themselves, a comparison was made with FLINT II alone as the series most comparable with YALE. The differences Y - Fl. II were written out in two series, one for stars which had the probable error for Y ≤ ±0''.025, the other for stars which had the probable error for Y > ±0''.025. Each series was divided into

four groups and group means computed. The results are given in Table III.

TABLE III

Comparison of YALE with FLINT II as a Test of the Published Probable Errors of YALE

Differences of Absolute Parallaxes

YALE Prob. Err. $\leq \pm 0''.025$			YALE Prob. Err. $> \pm 0''.025$		
Average = $\pm 0''.0160$			Average = $\pm 0''.0391$		
R.A.	Y. - Fl. II	No. of Stars	R.A.	Y. - Fl. II	No. of Stars
3 ^h .77	+0''.0118	6	3 ^h .05	+0''.0160	5
5 .95	- .0412	5	10 .29	+ .0298	6
11 .64	+ .0115	6	18 .86	+ .0247	6
19 .06	+ .0268	5	22 .24	+ .0125	6
Total		22	Total		23

The large negative value at 5^h.95 happens to include the largest two negative individual values: $-0''.120$ for δ *Orionis* and $-0''.110$ for α *Geminorum*. The only parallax values for the former are Y. $-0''.064$ and Fl. II $+0''.056$; for the latter Y. has $+0''.048$ and Fl. II $+0''.150$. The only other authorities for this star are Fl. I $-0''.135$ and Russ. $+0''.108$. The second class of YALE stars presented above does not seem to suffer in comparison with the first class as regards agreement with FLINT II.

The group means from the comparisons of authorities are given in Table IV. The number of stars included in each mean is indicated by m and the aggregate number of parallax values in each group by n , including those of the authority under comparison. The totals of these numbers for each sub-division of the table are given below their respective columns. There are six stars on which Fl. I and Fl. II occur together along with other authorities, but the mean of Fl. I and Fl. II is adopted as a single authority in computing the mean. There are four stars for which Fl. I and Fl. II occur together and are the only authorities. In all of these ten cases, of course, the full difference of parallax values, Mean - Fl. I and Mean - Fl. II, enter along with the differences for other stars into the corresponding group means of Table IV.

TABLE IV

Comparison of the Individual Authorities with the Mean of Several Authorities which Occur on Each Star and with Special Authorities. Group Means.

YALE				OMITTING FL. I, RUSS., SCHL., AND JDKO. ¹			
WITH ALL CONCURRENT AUTHORITIES							
R.A.	Mean - Y.	m	n	R.A.	Mean - Y.	m	n
1 ^h .26	$-0''.0003$	10	30	2 ^h .25	$+0''.0103$	6	13
3 .20	+ .0040	10	25	3 .88	- .0072	6	15
4 .66	+ .0100	11	37	5 .48	- .0006	7	17
6 .47	- .0088	10	27	7 .84	+ .0027	6	12
8 .84	+ .0135	11	24	9 .11	+ .0040	6	12
11 .81	- .0132	10	31	11 .08	- .0072	6	18
14 .77	- .0059	11	27	13 .27	+ .0097	6	13
17 .35	- .0088	10	28	16 .76	- .0017	6	12
19 .73	- .0120	11	37	19 .46	- .0140	6	15
20 .79	- .0016	10	36	20 .25	- .0063	6	13
22 .96	- .0065	11	31	21 .19	+ .0072	6	18
115 333				23 .16	+ .0057	6	12
				73 170			

FLINT I				WITH YALE ALONE		
WITH ALL CONCURRENT AUTHORITIES				R.A.	Y. - Fl. I	m
R.A.	Mean - Fl. I	m	n	R.A.	Y. - Fl. I	m
0 ^h .35	$-0''.0020$	7	29	3 ^h .38	$+0''.0233$	6
3 .48	+ .0054	7	28	7 .83	+ .0489	7
6 .62	+ .0505	8	26	13 .95	+ .0210	8
9 .24	+ .0241	7	26	17 .54	+ .0227	7
13 .75	+ .0299	7	21	21 .79	+ .0516	7
16 .66	- .0309	8	17	—		
18 .35	+ .0044	7	28	35		
20 .41	+ .0676	7	33			
58 208						

FLINT II						
R.A.	Mean—Fl. II	<i>m</i>	<i>n</i>	R. A.	Y. — Fl. II	<i>m</i>
1 ^h .03	+0 ^{''} .0265	8	19	1 ^h .54	−0 ^{''} .0155	8
3 .10	+ .0005	8	22	4 .99	+ .0100	7
5 .50	+ .0149	8	21	8 .09	− .0121	8
8 .38	− .0162	8	19	12 .49	+ .0105	8
10 .71	− .0002	8	19	18 .38	+ .0232	8
12 .44	+ .0001	9	20	21 .21	+ .0294	7
15 .31	+ .0044	8	17			—
18 .54	+ .0224	8	32			46
20 .76	+ .0169	8	20			
22 .62	− .0018	8	21			
		—	—			
		81	210			

¹On 61 *Cygni* PRITCHARD also omitted.

RUSSELL

R. A.	Mean—Russ.	<i>m</i>	<i>n</i>	R. A.	Y.—Russ	<i>m</i>	
0 ^h .18	−0″.0085	4	19	0 ^h .00	−0″.0020	3	
3 .85	+	0532	4	3 .85	+	0400	4
(7 .47	−	052.	1	(7 .47	−	060.	1)
10 .93	+	0212	4	11 .37	−	0057	3
12 .94	−	0115	4	15 .33	+	0730	4
15 .88	+	0747	3	18 .07	+	0113	3
17 .24	+	0800	3	22 .24	−	0347	3
18 .92	−	0083	3				—
22 .23	−	0278	4				21
		—	—				
		30	128				

SCHLESINGER-SLOCUM-MITCHELL

R. A.	Mean—Schl.	<i>m</i>	<i>n</i>	R. A.	Y. — Schl.	<i>m</i>
0 ^h .40	+0″.0194	5	21	0 ^h .41	+0″.0427	3
3 .48	+ 0382	4	18	3 .55	+ 0103	3
7 .17	— 0262	5	16	7 .35	+ 0045	2
13 .61	+ 0104	5	13	13 .76	+ 0187	3
18 .46	— 0102	5	22	19 .50	— 0108	4
20 .50	+ 0442	4	22	22 .25	+ 0450	4
21 .96	+ 0215	4	13			—
		—	—			19
		32	125			

JEWDOKIMOV

R. A.	Mean—Jdko.	<i>m</i>	<i>n</i>	R. A.	Y—Jdko.	<i>m</i>
0 ^h .30	−0″.0051	7	28	2 ^h .23	−0″.0042	8
1 .77	− 0033	7	21	4 .60	− 0584	8
3 .51	− 0263	7	25	18 .91	− 0179	7
5 .27	− 0361	7	24	20 .83	+ 0152	6
18 .30	− 0106	7	26	23 .38	− 0079	8
19 .68	− 0238	6	25			—
21 .30	+ 0016	7	33			37
22 .92	+ 0222	6	15			
		—	—			
		54	197			

FLINT I has a number of large differences for individual stars; but when arranged in algebraic order none stand out conspicuously in the series.

RUSSELL. α *Geminorum*, 7^h 17^m, stands quite alone in this series, as regards right ascension, with a gap of 2^h.73 preceding and one of 2^h.90 following, and is not included in a group mean. Its residual reduction is shown in the plot by a dot enclosed in parentheses.

PRITCHARD

R. A.	Mean — Pr.	<i>m</i>	<i>n</i>	R. A.	Fl. II — Pr.	<i>m</i>
0 ^h .98	+0 ^{''} .0103	3	10	0 ^h .64	— 0 ^{''} .0598	4
2 .43	— 0270	4	12	2 .43	— 0312	4
5 .91	— 0170	3	9	5 .91	— 0480	3
11 .36	+ 0095	4	8	11 .21	— 0080	3
13 .80	— 0220	3	6	12 .79	— 0253	3
17 .91	— 0090	3	9	17 .86	— 0668 ²	5
21 .17	— 0740 ¹	4	23	21 .98	— 0447	8
23 .92	— 0450	4	11			—
		—	—			25
		28	88			

¹Omitting ϵ *Cygni*, 20^h.70, and 61 *Cygni*, 21^h.04:

²Omitting ϵ *Cygni*, 20^h.70:

21 ^h .46	−0″.0160	2	4	17 ^h .14	−0″.0408	4
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ABETTI

R. A.	Mean—Abbt.	<i>m</i>	<i>n</i>	R. A.	Y.—Abbt.	<i>m</i>
6 ^h .56	−0′′.0048	4	12	5 ^h .38	+0′′.0295	2
9 .93	+ 0200	3	12	10 .04	− 0105	2
20 .09	− 0438	4	10	20 .09	− 0257	3
21 .24	+ 0187	3	20	21 .09	− 0270	3
		—	—			—
		14	54			10

PETER

R. A.	Mean—Pet.	<i>m</i>	<i>n</i>	R. A.	Y. —Pet.	<i>m</i>
0 ^h .30	−0 [″] .0187	3	12	(7 ^h .79	+0 [″] .080.	1)
9 .12	− 0140	4	16	13 .77	− 0030	3
13 .77	− 0157	3	10	19 .97	+ 0197	3
19 .97	+ 0110	3	23			—
		—	—			7
		13	61			

JOST

R. A.	Mean—Jost.	m	n	R. A.	Y. — Jost.	m	
8 ^h .26	−0 ^{''} .0103	3	7	8 ^h .26	−0 ^{''} .0167	3	
10 .55	−	0022	4	10 .75	−	0070	3
19 .67	−	0117	3	20 .22	+	0078	4
20 .90	−	0323	3				—
		—	—				10
		13	51				

JEWDOKIMOV has conspicuously large values on two stars: 141, 6889 prec., 3^h 67^m, 41° 2, $\mu = 1″.38$, Mean—Jdko. = −0″.178; and 85 *Pegasi*, 23^h.95, 26° 46, $\mu = 1″.30$, Mean—Jdko. = −0″.177.

PRITCHARD has conspicuously large values on three stars:

Parallax Star	R. A.	Decl.	Prop. Mot.	Mean - Pr.
μ Cassiopeiæ	1 ^h .03	54° 4	3".75	+0".064
ϵ Cygni	20 .70	33 .6	0 .49	-0 .141
61 Cygni	21 .04	38 .3	5 .25	-0 .123

The average reduction of Pr. to the mean of all authorities is $-0''.0235$. If this is adopted as the mean systematic difference for Pr., then the reductions peculiar to these three stars become $+0''.088$, $-0''.118$, and $-0''.100$ respectively. PRITCHARD has a large deviation near 21^h due to the second and third of these stars. It should be noted, however, that on ϵ Cygni he meets a curious combination of three negative values of parallax from other authorities: Fl. I, $-0''.175$; Fl. II, $-0''.038$; Jdco., $-0''.05$. Thus the aberration here appears to be theirs and not PRITCHARD's. The only value of parallax in PRITCHARD's list that appears decidedly abnormal, is that for 61 Cygni on which he has the highest value amongst the fifteen selected authorities. Whatever the influence at work in his determination of this parallax, it happily was such as to offset exactly the low value from FLINT I and so preserve the integrity of the general mean for the star. There is evidence, however, as noted in previous publications, that there are material errors in his published equations, and it is very desirable that they be revised.

In the accompanying plates, Figs. 2 *et seq.*, are shown the plots of the group means of Table IV, and the curves drawn to represent them or straight lines drawn to connect them. To assist the reader to judge the nature of the variations in the first or full lines, two short horizontal lines are drawn near the ordinate at 0^h of right ascension. The distance apart of these lines is equal to twice the approximate probable error of a single plotted point for the given authority as given in Table II, column 9. The reader must note, with reference to the text and plots, that the terms "high" and "low" apply not to parallax values themselves but to their reductions to a mean of authorities or to one other authority. ABETTI, PETER and JOST are omitted here as less important, owing to the smallness of their lists of stars and the large gaps in their series of right ascensions.

The YALE parallaxes are of such number and were found in the investigation to be of such excellent quality that it seemed desirable to exhibit also a comparison of each of the other authorities with YALE alone. The group means of these differences, in the sense YALE - Authority, are given in the second division of Table IV under each authority. In the case of PRITCHARD, there are only four stars in common with YALE, while twenty-five stars

of his twenty-eight stars are found in FLINT II, all from 1.8 to 2.6 in magnitude. Accordingly in this second comparison the differences are FLINT II - PRITCHARD. In the column *m* as before, is indicated the number of stars that enter into each group mean. The differences Mean - Authority are represented in the plots by circular points connected by full lines and the differences YALE - Authority by square points connected by broken lines.

A further simple device was employed as a test whether the apparent systematic deviations of an authority are real. It might be expected that an occasional wild value, due to some interfering factor acting through a small extent of the abscissæ of reference, as the right ascensions, would occur in any given series of observed quantities. But, if such a series be divided into two alternate series, one of the odd-numbered values, the other of the even-numbered values, and still the deviations of the one series coincide sensibly with those of the other, the probability is very strong that the deviations are real and systematic; and this qualitative test requires no knowledge of the probable errors involved. Accordingly a second set of plots are shown, the first of which is Fig. 5, for the six authorities which have the larger series of values, beginning with FLINT I. The group means for the one series are represented by circular points connected by full lines, and those for the other series by square points connected by open lines.

The YALE parallaxes form an admirable example of the value of a long series of determinations carried on persistently under similar conditions of physical surroundings, method, and care. They are also well distributed around the heavens in right ascension and between the equatorial region and within about 15° of the pole. The plot (Fig. 2) exhibits a remarkably close agreement with the means of all authorities and very small departure of the ordinates from the zero line. These suggest a remaining systematic difference, perhaps, in the intervals 2^h - 10^h and 12^h - 22^h; but these slight deviations appear to have no relation, in phase, maxima and minima, to the corrections of Table I.

FLINT I and RUSSELL exhibit the greatest oscillations in their reductions to mean of authorities, indicating some irregular causes of variation, yet common to several stars in particular regions of right ascension. The former, however, shows a marked improvement when compared with YALE alone on the thirty-five stars in common. This is most evident in the interval 14^h.0 to 4^h.0. In the plots for the two alternate series the two lines for FLINT I rise and fall together in fair agreement. The same is true of RUSSELL except for the sharp rise in the second line at 17^h. This is due to the fact that the highest two reductions happen to fall together in the

second series. One of these, $+0''.170$, is a difference from FLINT I alone and appears especially abnormal, since the next highest two are $+0''.071$ and $+0''.064$. The omission of the highest two values, however, does not change materially the character of the first plot, that for RUSSELL's single list (Fig. 8), where the triangular point, at $16^h.7$, represents the group mean of the four stars remaining in the two consecutive groups after omitting these two highest values.

FLINT II shows a marked improvement over FLINT I, both as regards the average departure from the zero line and also as regards the differences between the successive points. There seems to be a tendency to larger irregularity from 20^h to 4^h . The lines of comparison with YALE follow those of comparison with all authorities very closely, with small departures above and below from 20^h to 4^h . In the plot for the two alternate series the two lines follow each other closely or depart in small, well-balanced oscillations.

SCHLESINGER exhibits unexpected differences in the relative positions of his group means. The probable error of a single plotted point is $\pm 0''.0078$, according to Table II, while the largest and smallest ordinates of the points algebraically are $+0''.046$ and $-0''.027$ respectively, with a range of $0''.073$ or 8.5 times the probable error. The comparisons with YALE alone follow the first, or full line, plot in a general way, with a diminution of the range in the ordinates to $0''.054$. It seems apparent that there is on the whole a marked elevation of the plotted points, through the interval from 20^h to 4^h and a depression through the remaining hours. The lines of the plot for the two alternate series follow each other very well and confirm the characteristics of the first plot.

JEWDOKIMOV, in his limited extent in right ascension, appears to be subject to somewhat larger oscillations and to have a marked descent from 22^h to 5^h . The lines of the plot for the two alternate series agree fairly well with each other in their direction through the successive hours, with the curious feature that the second line runs below the first by a sensible difference throughout.

PRITCHARD makes a very fair showing, even at first sight. There is a marked depression in the interval $15^h - 22^h$; but, if the two peculiar values noted above, those for ϵ Cygni and 61 Cygni, be omitted, the consistency of the plotted points becomes at least as good as for the average of the selected authorities. Two group means are changed by these omissions and have their consequent values represented in the plot by the two triangular points, while the changed connections are shown by means of the dash-and-dot lines. In the second plot, that for FL. II - Pr., the low point at $17^h.86$ is due first to the abnormally low value of parallax for ϵ Cygni from FLINT II, as already noted, and second,

to a low value from FLINT II for γ Cygni, $20^h.31$, $39^\circ.9$. Here YALE occurs also, and the only values of parallax for this star are Y. $+0''.121$, FL. II $-0''.018$, Pr. $+0''.108$. The reduction of PRITCHARD to YALE, therefore, on this star is only $+0''.013$. 61 Cygni, on which PRITCHARD has such a high value, does not occur in FLINT II. Here again the group mean as changed by the omission of ϵ Cygni is represented by a triangular point and the consequent connections are shown by means of dash-and-dot lines. The plots for the two alternate series show a good agreement throughout, although there are only from two to three stars included in each point. The depressions at 20^h and 22^h are due mainly to the peculiar reductions to means of authorities for ϵ Cygni and 61 Cygni which have already been noted in connection with Table IV and which happen to enter here in different series. As above, the group means which are changed by the omission of these two stars, are represented by triangular points and the new connections by dash-and-dot lines.

ABETTI's low ordinate at $20^h - 21^h$ seems to be real. The individual numbers for the four stars that make up the group are $-0''.036$, $-0''.005$, $-0''.028$ and $-0''.106$. YALE occurs on all of these stars, and the corresponding differences from YALE alone are $-0''.011$, $-0''.011$, $-0''.055$ and $-0''.136$. The fourth star, showing the largest difference is η Cephei, $20^h.72$, $61^\circ.5$.

PETERS' refined determinations make an excellent showing in these comparisons. The number of stars is small but they are distributed over the twenty-four hours with two considerable gaps, which occur from $1^h.0$ to $7^h.8$ and from $14^h.9$ to $19^h.3$.

JOST likewise has a drop at 21^h , which seems to be real. The three individual differences that make up this group are $-0''.018$, $-0''.002$, and $-0''.069$. YALE occurs on only the first two of these stars; the reductions to YALE are $-0''.035$ and $-0''.008$. The third star is π Cygni, $21^h.18$, $37^\circ.6$, and the values of parallax for the three authorities on the star are Abbt. $+0''.038$, Schl. $+0''.011$, Jost $+0''.128$.

In Table II, last column, is given for each authority the average reduction to the mean of all authorities. This is the simple mean of the list of differences for the individual stars which enter into the group means presented in the second column of each division of Table IV. The same numbers are given in the first division of Table V, but in order of algebraic magnitude. In the second division is given the corresponding average reduction to YALE but, of course, with a less number of stars for each authority. In the case of PRITCHARD, who has only five stars in common with YALE, the reduction was obtained through FLINT II as follows:

$$\begin{aligned}\text{Fl. II.} - \text{Pr.} &= -0''.0431, & 25 \text{ stars.} \\ \text{Y} - \text{Fl. II} &= +0''.0071, & 46 \text{ stars.}\end{aligned}$$

$$\text{Whence} \quad \text{Y} - \text{Pr.} = -0''.0360$$

For three of the authorities the number of stars in common with YALE is too small to give any meaning to the apparent reductions. In the case of PETER, for instance, who has the largest change, an examination showed that it is due to a chance distribution of the individual parallaxes, larger and smaller, as between the stars that are in common with YALE alone and those that are in common with other authorities. The signs and comparative magnitudes of the numbers in the second column agree well with the judgments formed from an inspection of the diagrams. These numbers should be an approximation to the average systematic corrections of the respective authorities. An inspection of the diagrams show that these numbers, with four exceptions, are of no affirmative significance inasmuch as they are affected so largely by the accidental or systematic oscillations of the successive group means. The exceptions are PRITCHARD, JOST, RUSSELL and FLINT I, for all of which the averages appear to be real.

The differences given in the last column are theoretically the reduction of YALE to the mean of all authorities, as deduced from the comparisons for the individual authorities. These cannot be expected to exhibit much apparent consistency since they are so small; but the simple mean of the ten numbers is $-0''.0039$, while the original value of Mean - YALE is $-0''.0026$, as given in Table V and in the last column of Table II.

TABLE V

Auth.	Reduction to Mean of All	No. Stars	Reduction to Yale	No. Stars	2d Column - 4th Column
Pr.	$-0''.0235$	28	$-0''.0360$	25, 46	$+0''.0125$
Jdko.	$-0''.0135$	54	$-0''.0162$	37	$+0''.0027$
Jost	$-0''.0126$	13	$-0''.0040$	10	$-0''.0086$
Peter	$-0''.0093$	13	$+0''.0186$	7	$-0''.0279$
Abtt.	$-0''.0050$	14	$-0''.0120$	10	$+0''.0070$
Y.	$-0''.0026$	115	$0''.0000$	115	$-0''.0026$
Fl. II	$+0''.0066$	81	$+0''.0071$	46	$-0''.0005$
Schl.	$+0''.0120$	32	$+0''.0190$	19	$-0''.0070$
Russ.	$+0''.0165$	30	$+0''.0142$	21	$+0''.0023$
Fl. I	$+0''.0183$	58	$+0''.0354$	35	$-0''.0171$
Mean	$-0''.00131$		$+0''.00261$		$-0''.00392$

An inspection of the diagrams shows that, if ϵ Cygni and 61 Cygni be omitted from PRITCHARD's series, there

remain four authorities that appear to be subject to marked variations of the ordinates, and that may be regarded as disturbing members of the group of authorities. These are FLINT I, RUSSELL, JEWDOKIMOV, and SCHLESINGER. The photographic method, as applied by the last named, seems so promising and is probably to be applied so extensively in the immediate future, as to make it of special interest. It seems desirable, therefore, to compare SCHLESINGER with the mean of all the remaining authorities after omitting the three others named above. The results are given in Table VI and a plot of the differences, Mean - Schl., in Fig. 10 by means of the triangular points. In the second division of the table are shown the individual differences arranged in order of algebraic magnitude. There appear to be no marked irregularities in the progression of these differences. Yet the plot shows substantially the same large oscillations in the ordinate as exhibited in the three previous plots for this authority, those of comparison with the mean of all authorities, with YALE alone, and between the alternate series. A table of corrections read from a smooth curve drawn to represent the plot of comparisons with the mean of all the selected authorities was constructed for the SCHLESINGER parallaxes. The writer would not present it yet as fully established, but it may be of interest to the reader to have here the maximum and minimum values of the table. These are as follows:

Maxima	Minima
$0^h + 0''.039$; $14^h + 0''.011$	$7^h - 0''.029$; $18^h - 0''.011$

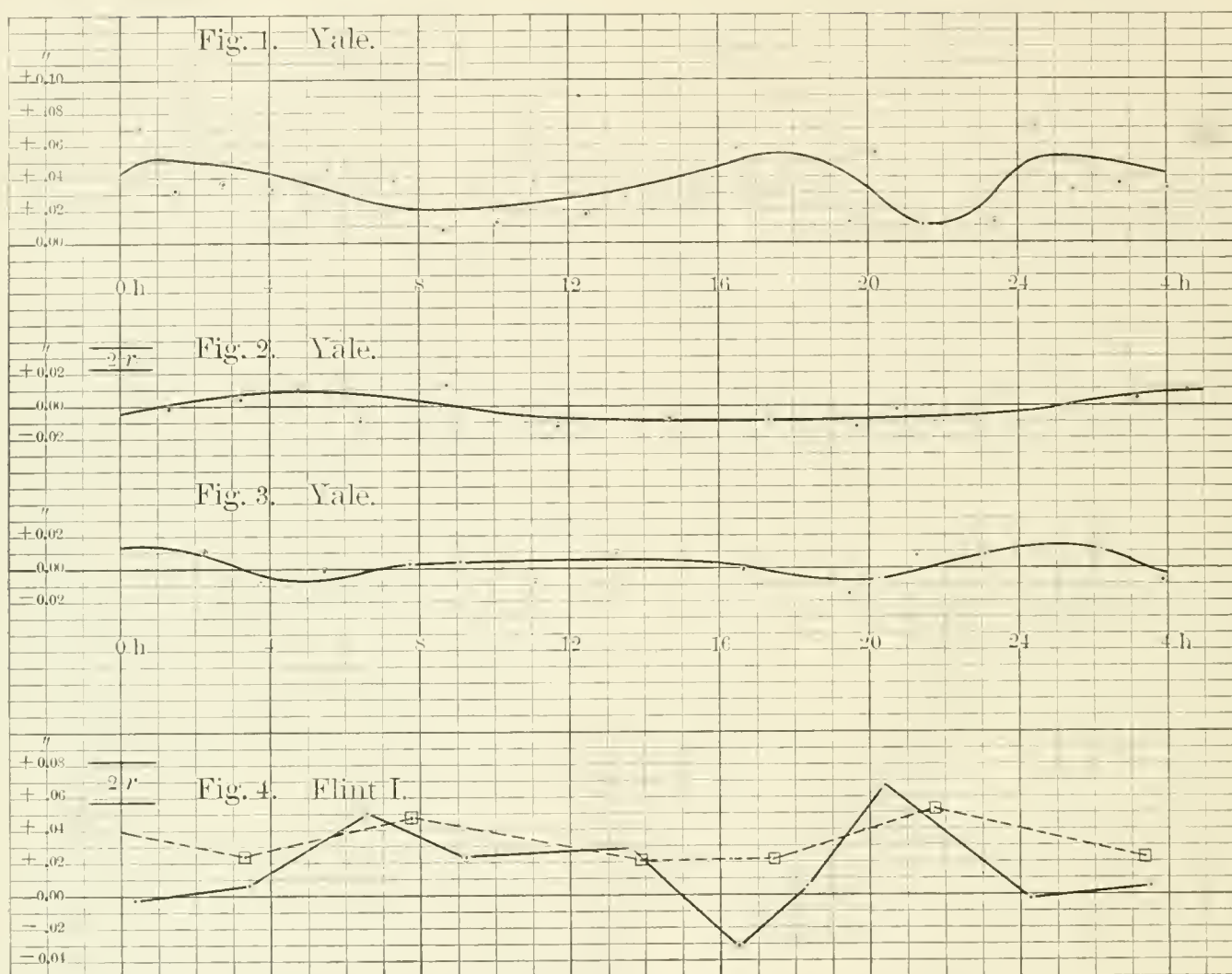
The conclusion seems unavoidable that these finely determined parallaxes are subject to considerable systematic error.

TABLE VI

Comparison of SCHLESINGER with the Mean of the More Self-Consistent Authorities

R. A.	Mean - SCHL.	m	n
$0^h.40$	$+0''.0148$	5	12
3.48	$+0''.0342$	4	12
7.60	$-0''.0130$	4	11
13.59	$+0''.0140$	4	9
18.35	$-0''.0142$	4	10
20.50	$+0''.0428$	4	18 ¹
21.96	$+0''.0160$	4	11
		29	83

¹ On 61 Cygni, Pr. also omitted.



INDIVIDUAL DIFFERENCES, MEAN—SCHL. IN ALGEBRAIC ORDER

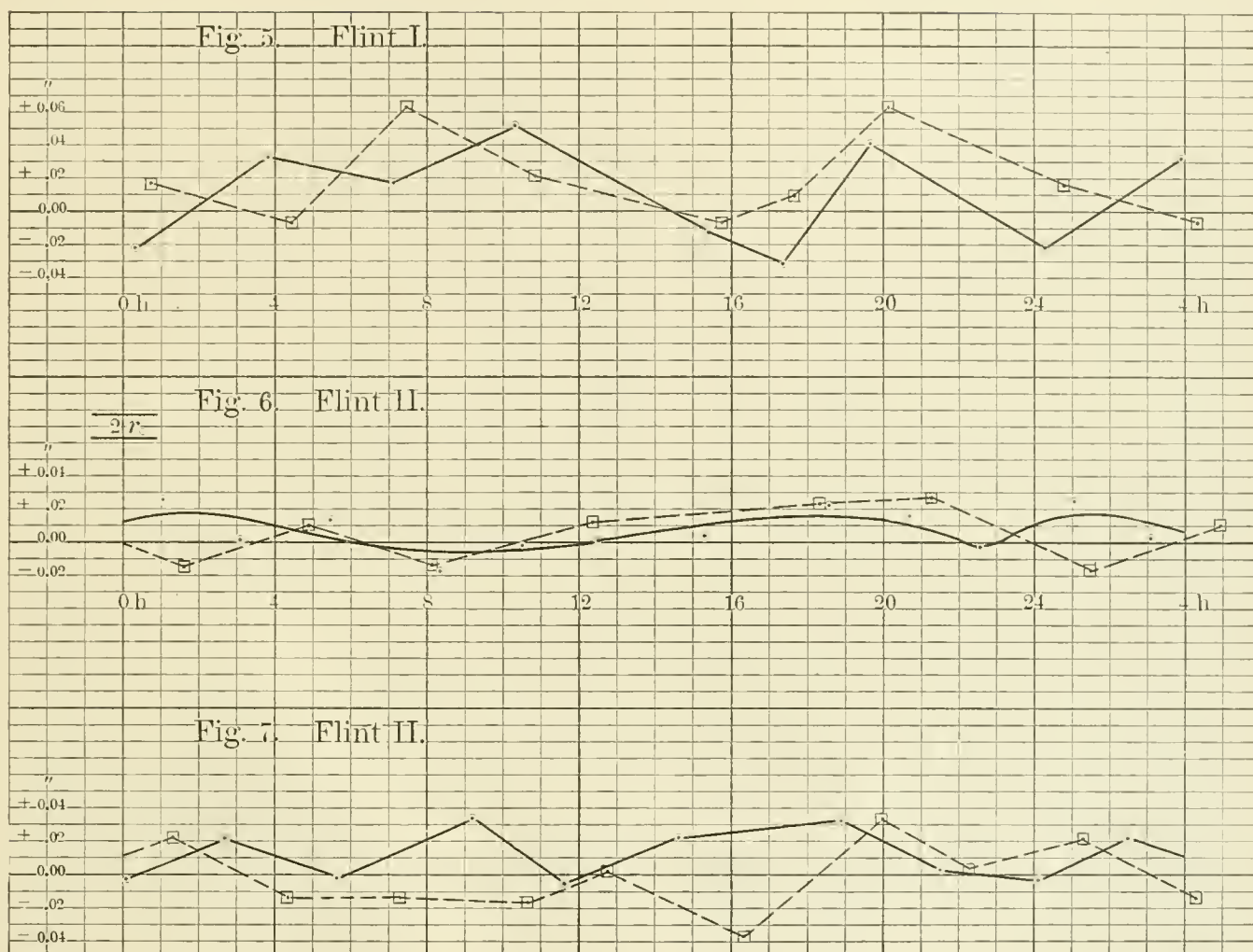
R. A.		R. A.	
19 ^h .3	-0 ^{''} .039	17 ^h .4	+0 ^{''} .008
9 .1	- .033	13 .7	+ .014
18 .0	- .030	23 .7	+ .019
1 .0	- .013	3 .0	+ .020
22 .0	- .013	13 .1	+ .020
22 .4	- .011	14 .9	+ .022
6 .9	- .010	0 .2	+ .025
7 .8	- .007	21 .0	+ .035
6 .6	-0 ^{''} .002	1 .1	+ .036
12 .7	.000	22 .2	+ .040
3 .9	+ .001	21 .2	+ .048
20 .6	+ .002	3 .7	+ .053
18 .7	+ .004	19 .6	+ .059
0 .0	+0 ^{''} .007	3 .3	+ .063
		20 .7	+0 ^{''} .075

While parallax determinations by any method are more or less liable to the occurrence of abnormal results that appear discouraging, the author, upon completing this comparison of measured parallaxes, feels a largely increased confidence in the general value of all the methods hitherto applied to the measurement of relative parallax.

SUMMARY OF PRINCIPLES AND METHOD

The present investigation is based upon a comparison of each of several parallax authorities having a larger number of determinations, with the mean of all of a number of selected authorities, as they happen to occur on individual stars.

The purpose is to test the principal authorities in regard to both average and variable systematic error, but especially the latter, with reference to a series in right ascension.



The original parallax results of the several authorities are adopted, except for three of the authorities. For YALE, they are corrected for a small systematic error, varying with the right ascension (Table I). For FLINT I and FLINT II, they are corrected for a personal error due to difference of apparent magnitude between parallax star and mean of comparison stars; but the correction in the case of the latter authority is very small.

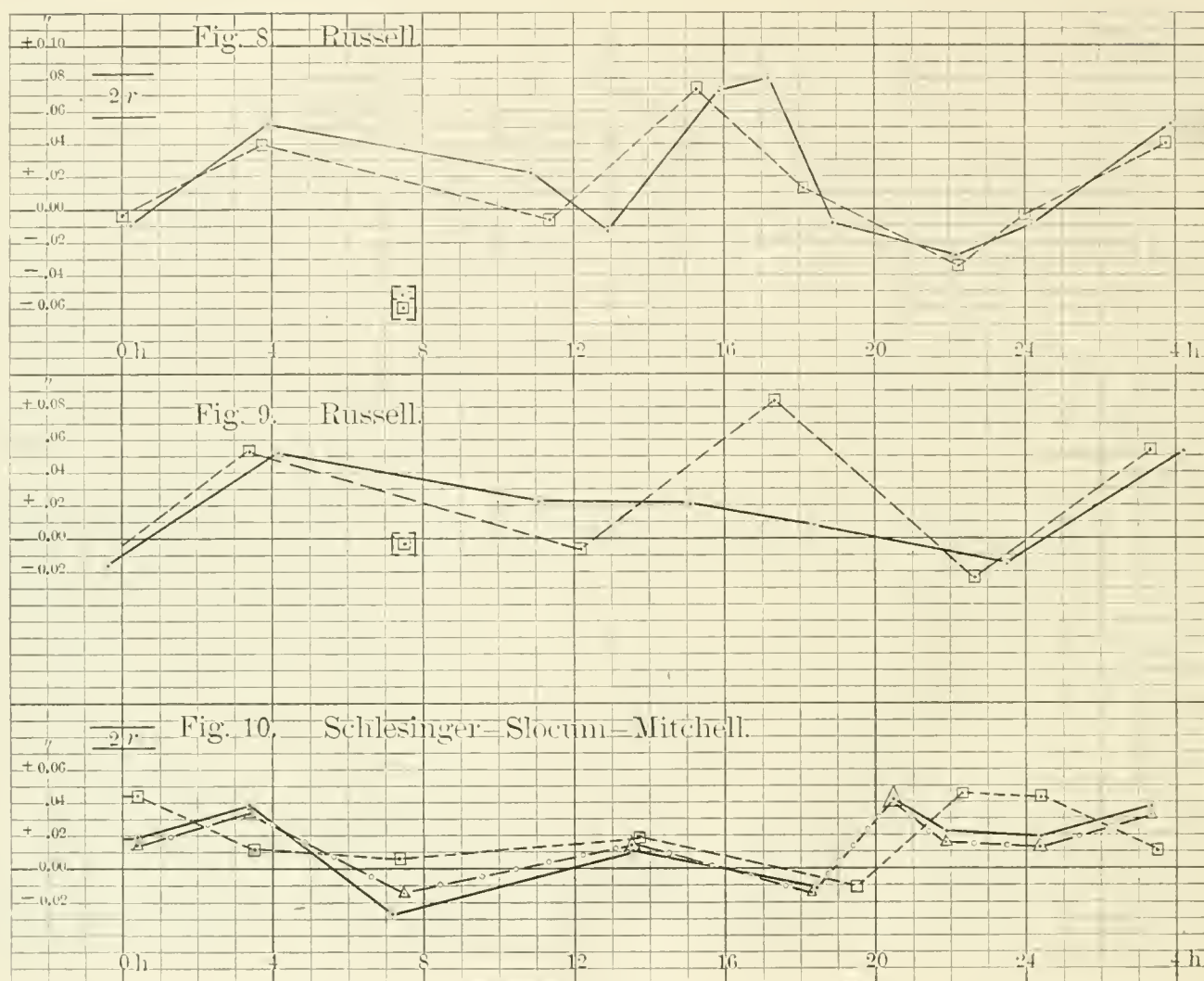
The individual reductions of each authority to the mean of all are grouped in mean values, with regard to gaps in the series of right ascensions as natural divisions between the groups. (Table IV.) These group means are represented in a series of plots with right ascensions as abscissae.

A similar comparison is made next with YALE alone as a single standard because of its small deviations, the large number of its stars in common with the other authorities,

and the good distribution of its stars over the twenty-four hours of right ascension. These differences are represented by parallel plots in the same diagrams with the preceding sets of differences.

As a further test the series of differences — reductions to mean of authorities — for each authority, is divided into two alternate sub-series, one of the odd-numbered values, the other of the even-numbered values. The group means are plotted in a second parallel series of diagrams and confirm the marked variations on the part of certain authorities.

Dependence is placed on a fair distribution in right ascension of the stars on which any authority occurs with other authorities, and also on the fair distribution of the authorities among the stars of the several groups. The character of each distribution may be inferred from the numbers in columns R. A., m , and n , of Table IV.



CONCLUSIONS

1. Parallaxes observed with the heliometer are liable to variable systematic error rising to somewhat more than the probable error of their better determined values or over twice that of their most refined values.

2. Parallaxes observed with the heliometer, with such correction as may be required by the preceding consideration, are superior as regards both constant and variable systematic error.

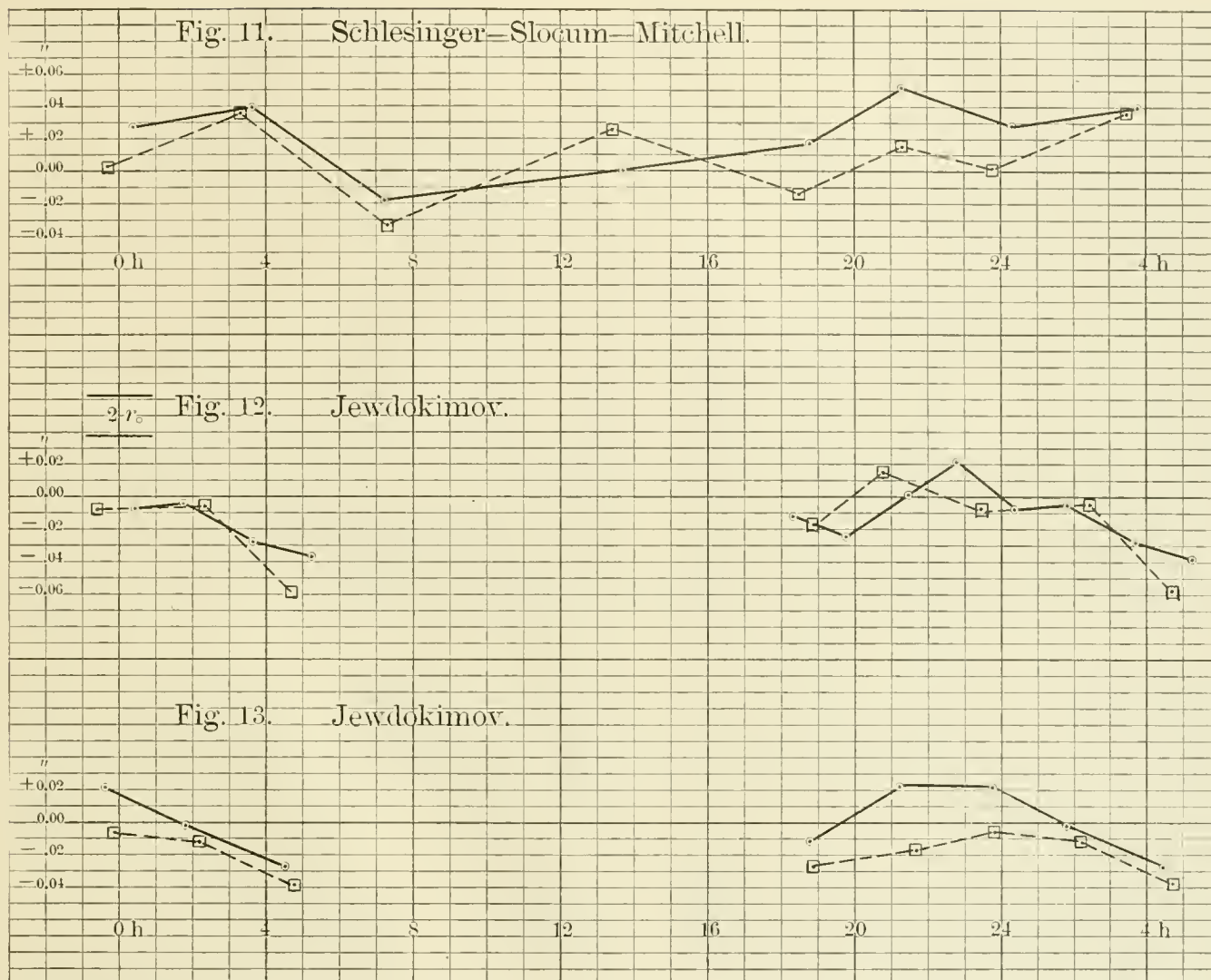
3. Parallaxes determined by the method of meridian transits, especially with the aid of the traveling thread, and graduated screens, are but little inferior to the results of heliometer observations as regards both constant and variable systematic error.

4. Recent photographic determinations, although the

most refined of all as regards individual parallax stars, are subject to variable systematic errors as large as those that occur in results obtained by the method of meridian transits under less guarded conditions for the latter. These errors of the photographic method rise to three times their probable error of a single determination of parallax.

5. The YALE parallaxes, because of their general consistency, number, and wide and uniform distribution, should be adopted as a standard by which to test future series of determinations in the northern hemisphere.

6. The YALE observations should be continued, under the same general conditions as hitherto, in order also to furnish contemporary results for comparison with other future series.



7. Similar heliometer determinations should be maintained in the southern hemisphere.

8. In order to test further the parallax values obtained by all the methods of promise, especially by the heliometer, the filar micrometer, meridian transits, and photography, some observations by all of these methods should be maintained at or near the same stations so as to obtain the closest similarity of conditions climatic, atmospheric, and in other respects.

PLOTS OF COMPARISONS BETWEEN AUTHORITIES

Fig. 1. YALE. Standard parallaxes for determining the variation with right ascension, 212 stars in 18 groups.

Fig. 2. YALE. Reduction to mean of all authorities, 115 stars in 11 groups.

Fig. 3. YALE. Reduction to mean of all authorities except Fl. I, Russ., Schl., and Jdco., 73 stars in 12 groups.

Fig. 4. FLINT I. Reduction to mean of all authorities, 57 stars in 8 groups, $\odot$ —————; to YALE alone, 35 stars in 5 groups, $\square$ — — — — —.

Fig. 5. FLINT I. Reduction to mean of all. In alternate series.

Fig. 6. FLINT II. Reduction to mean of all authorities, 80 stars in 10 groups, —————; $\odot$ to YALE alone, 46 stars in 6 groups, $\square$ — — — — —.

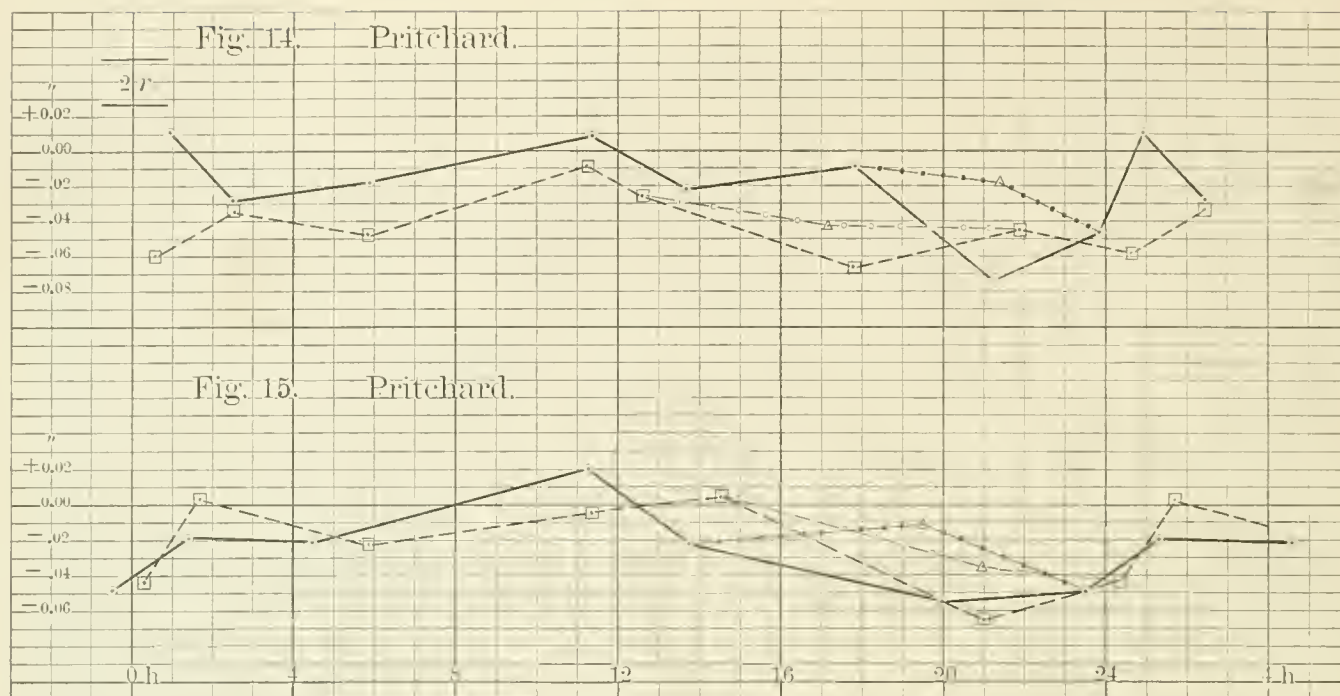


Fig. 7. FLINT II. Reduction to mean of all. In alternate series.

Fig. 8. RUSSELL. Reduction to mean of all authorities, 30 stars in 8 groups, $\odot$ —————; to YALE alone, 21 stars in 6 groups, $\square$ — — — — —.

Fig. 9. RUSSELL. Reduction to mean of all. In alternate series.

Fig. 10. SCHLESINGER-SLOCUM-MITCHELL. Reduction to mean of all authorities, 29 stars in 8 groups, $\odot$ —————; to YALE alone, 18 stars in 6 groups, $\square$ — — — — —; to mean of all omitting Fl. I, Russ., and Jdko., 26 stars in 8 groups, $\triangle$ — o — o —.

Fig. 11. SCHLESINGER-SLOCUM-MITCHELL. Reduction to mean of all. In alternate series.

Fig. 12. JEWOKIMOV. Reduction to mean of all authorities, 54 stars in 8 groups, $\odot$ —————; to YALE alone, 37 stars in 5 groups, $\square$ — — — — —.

Fig. 13. JEWOKIMOV. Reduction to mean of all. In alternate series.

Fig. 14. PRITCHARD. Reduction to mean of all authorities, 28 stars in 8 groups, $\odot$ —————; omitting ϵ Cygni, 20^h.70, and 61 Cygni, 21^h.04, 26 stars in 8 groups, $\triangle$ — — — — —; to Fl. II alone, 25 stars in 6 groups, $\square$ — — — — —; omitting ϵ Cygni, 24 stars in 6 groups, $\triangle$ — o — o —.

Fig. 15. PRITCHARD. Reduction to mean of all. In alternate series. Omitting ϵ Cygni and 61 Cygni, changes indicated in same manner as in Fig. 14.

NOTE. The pair of short horizontal lines across the zero ordinate in the first plot for each authority are distant apart by twice the average probable error of a point in the plot. These probable errors were computed for the respective authorities from their lists of differences from the means of all authorities on the individual stars.

In the making of the plates for the original drawings, sensible changes have occurred in the intervals in certain cases. The required corrections to the several authorities represented are as follows:

Y.	-0".0034	SCHL.	+0".0023
FL. I	+ .0014	JDKO.	- .0008
FL. II	+ .0003	PR.	- .0032
Russ.	+ .0034		

Washburn Observatory, Madison, Wisconsin, 1916, May.

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